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NUMERICAL ANALYTIC AIRFOIL STABILITY INVESTIGATION BASED ON VORTEX ELEMENT METHOD

ILIA K. MARCHEVSKY*

Abstract. Numerical analytic method which could be effectively used for airfoil stability in the flow investigation is suggested. The method is based on joint usage of numerical vortex element method for incompressible flow simulation and airfoil aerodynamic coefficients calculation with analytic instability sufficient conditions, which depend only on drag force and lift force stationary aerodynamic coefficients.

Key words. Stability criteria, instability sufficient condition, airfoil, vortex element method, vortex flow, Cauchy-Lagrange integral, numerical analytic method.

AMS subject classifications. 34D20, 76B47, 76D10

1. The general statement of the problem and the review of known results.

1.1. Some examples of airfoil instability. The basic method of an airfoil behavior in air flow investigation and its aerodynamic characteristics determination is experiment with the airfoil or its model in wind tunnel. For some airfoils engineers repeatedly note the following phenomenon: at some angles of incidence the airfoil equilibrium in air flow becomes unstable and there are oscillations with sufficiently big amplitude. Wind resonance occurrence is excluded since value of experimental Strouhal number is less than $\mathrm{Sh} = 0{,}2$ corresponding to wind resonance. Such an experiment is described in details in [1].

Such phenomenon is typical, in particular, for building structures with big aspect ratio: for some angles of incidence there are oscillations of the construction cross-section across wind direction. The similar effect occurs with transmission lines: under some conditions there are wind-generated oscillations of power line wires across the stream with amplitudes up to 10 meters and more. This phenomenon is called 'galloping' and it is extremely undesirable since often involves breakage of constructions and destruction of power transmission towers.

For some airfoils placed in a stream of gas or liquid there is also a phenomenon of 'autorotation' — sharp increase in amplitude of torsional oscillations around of an axis, which can be parallel or perpendicular to a stream. This phenomenon in many cases also is undesirable and dangerous from the point of view of accident-free construction service.

Occurrence of the described phenomena depends on flow features and related with the so-called 'loss of aerodynamic damping' [1]. The analysis of aerodynamic characteristics of various airfoils shows, that such phenomenon is typical both for bluff and streamline airfoils [2].

1.2. Known instability conditions. In experimental research [3] the behavior of biplane model in air stream was investigated provided that the model had one degree of freedom — only torsional oscillations around a long axis of the model were

*Applied Mathematics dep., Bauman Moscow State Technical University, Moscow, 105005, Russia (iliamarchevsky@mail.ru).

possible. It had been noted, that there was an interval of incidences at which the amplitude of these oscillations was sufficiently big. Experiments were processed by H. Glauert [4] who obtained autorotation necessary condition:

$$G(\alpha) = C_{xa} + C'_{ya} < 0. \tag{1}$$

Here C_{xa}, C_{ya} — stationary aerodynamic coefficients of drag force and lift force correspondingly. The prime hereinafter denotes derivative on an angle of incidence α.

Investigating galloping of power line wires, J. Den-Hartog [5] has studied the behavior of a semicircle airfoil with one degree of freedom (oscillations across the stream). He has obtained galloping necessary condition which has the same appearance (1).

Therefore, the inequality (1) is the necessary condition of wide oscillations with one degree of freedom — autorotation and galloping. Glauert–Den-Hartog's condition is confirmed in numerous experiments [1, 6] and it is used, in particular, in construction: when designing the high-altitude constructions subjected to wind loadings, it is necessary to orientate construction cross-section so that angles of incidence in relation to dominating winds would be outside instability interval [7].

V. I. Vanko in [8] investigated some mechanical construction to simulate the behavior of a power line wire under wind. That construction had three degrees of freedom: movements along axes Ox, Oy and rotation around the center-of-mass were considered. It was obtained, that for any wind velocity V_∞ there exists a unique solution of the equations of movement, which corresponds to the equilibrium of the airfoil in the flow.

It was shown using Lyapunov technique of stability investigation, that for airfoil equilibrium:

a) inequality (1) is the instability sufficient condition for the model with one degree of freedom if we wouldn't consider viscous resistance of constraints;

b) the equilibrium instability sufficient condition (for three degrees of freedom) which depends only on aerodynamic coefficients of airfoil was obtained:

$$W(\alpha) = C_{xa}(C_{xa} + C'_{ya}) + C_{ya}(C_{ya} - C'_{xa}) < 0. \tag{2}$$

In this paper airfoil movements with 1, 2 and 3 degrees of freedom is investigated, and stability criteria for its equilibrium are obtained.

Some of instability sufficient conditions depend only on aerodynamic coefficients of airfoil, so it is important to have an effective method, which makes it possible to determine these coefficients with satisfactory accuracy and small computational time. If Mach number is less than 0,4 then the stream could be considered as incompressible, and meshless vortex element method [9, 10, 11, 12] for the flow simulating is very effective. In order to lower computational time, the modification of vortex element method based on Prandtl idea about boundary layer will be used.

2. Airfoil stability criteria. In the present paper movement in the flow of an airfoil with three various viscous-elastic constraints (with three degrees of freedom) is investigated. The design model for the considered case is shown on Fig 1.

Here V_∞ — wind velocity, which supposed to be horizontal; f_x, f_y, f_m — elastic characteristics of constraints; ν_x, ν_y, ν_m — viscous characteristics of constraints; m and J — mass and moment of inertia of the airfoil with respect to the fastening point correspondingly.

Position of the airfoil in space is uniquely defined by three coordinates x, y and ϕ, where x, y — position of the fastening point, ϕ — angle of incidence of the airfoil.

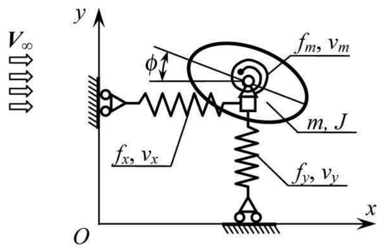

FIG. 1. *The design model.*

Wind influence on the airfoil is considered by drag force, lift force and aerodynamic moment, which could be calculated using following formulas:

$$X_a = \frac{1}{2}C_{xa}\rho V_r^2 S, \quad Y_a = \frac{1}{2}C_{ya}\rho V_r^2 S, \quad M_a = \frac{1}{2}C_{ma}\rho V_r^2 S^2.$$

Here ρ is air density; V_r — relative speed of the wind, $V_{rx} = V_\infty - \frac{dx}{dt}$, $V_{ry} = -\frac{dy}{dt}$; S — typical dimension (chord) of the airfoil. We also assume, that dimensionless aerodynamic coefficients C_{xa}, C_{ya} and C_{ma} for the given airfoil depend only on the incidence and they are continuously differentiable functions.

In this case it is possible to write down system of the differential equations describing movement of the airfoil in a stream:

$$(3)\qquad \begin{cases} m\frac{d^2x}{dt^2} = -f_x \cdot x - \nu_x \cdot \frac{dx}{dt} + \frac{1}{2}\rho S\sqrt{\left(V_\infty - \frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\times \\ \qquad\qquad \times\left[C_{ya}(\alpha)\cdot\frac{dy}{dt} + C_{xa}(\alpha)\cdot\left(V_\infty - \frac{dx}{dt}\right)\right], \\ m\frac{d^2y}{dt^2} = -f_y \cdot y - \nu_y \cdot \frac{dy}{dt} + \frac{1}{2}\rho S\sqrt{\left(V_\infty - \frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\times \\ \qquad\qquad \times\left[C_{ya}(\alpha)\cdot\left(V_\infty - \frac{dx}{dt}\right) - C_{xa}(\alpha)\cdot\frac{dy}{dt}\right], \\ J\frac{d^2\phi}{dt^2} = -f_m \cdot \phi - \nu_m \cdot \frac{d\phi}{dt} + \frac{1}{2}\rho S^2 \cdot \left[\left(V_\infty - \frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2\right]\cdot C_{ma}(\alpha). \end{cases}$$

In system (3) we denoted $\alpha = \phi - \left(\arctan\frac{dy}{dt}\right)\cdot\left(V_\infty - \frac{dx}{dt}\right)^{-1}$. For any wind velocity V_∞ there is at least one equilibrium of the airfoil $x = x_0$, $y = y_0$, $\phi = \phi_0$.

We will investigate stability of airfoil equilibrium using equations of the first approximation. Since coordinates and speeds are considered to be small, it is possible to linearize original nonlinear system (3) near to the equilibrium point $x = x_0$, $y = y_0$, $\phi = \phi_0$ and reduce it to dimensionless type:

$$(4)\qquad \begin{cases} \ddot\xi + \omega_x^2\xi = \frac{1}{2}\epsilon(C_{ya} - C'_{xa})\dot\eta - \epsilon(C_{xa} + \mu_x)\dot\xi + \frac{1}{2}\epsilon C'_{xa}\gamma, \\ \ddot\eta + \omega_y^2\eta = -\frac{1}{2}\epsilon(C_{xa} + C'_{ya} + 2\mu_y)\dot\eta - \epsilon C_{ya}\dot\xi + \frac{1}{2}\epsilon C'_{ya}\gamma, \\ \ddot\gamma + \omega_{ma}^2\gamma = -\frac{1}{2\sigma}\epsilon C'_{ma}\dot\eta - \frac{1}{\sigma}\epsilon C_{ma}\dot\xi + \frac{1}{2\sigma}\epsilon C'_{ma}\gamma - \epsilon\mu_m\dot\gamma. \end{cases}$$

Here $\xi = \frac{x-x_0}{S}$, $\eta = \frac{y-y_0}{S}$, $\gamma = \phi - \phi_0$ — airfoil dimensionless coordinates; derivatives $\dot\xi$, $\dot\eta$, $\dot\phi$ are calculated on dimensionless time $\tau = \frac{V_\infty}{S}t$; $\mu_x = \frac{\nu_x}{\rho SV_\infty}$, $\mu_y = \frac{\nu_y}{\rho SV_\infty}$, $\mu_m = \frac{\nu_m}{\sigma\rho S^3 V_\infty}$ — dimensionless damping coefficients of constraints; σ — the factor defined by form of the airfoil and fastening point position (for example, for circle and square airfoils fastened by the center-of-mass, $\sigma = \frac{1}{8}$ and $\sigma = \frac{1}{6}$ correspondingly);

$\omega_x^2 = \frac{f_x \cdot S}{\rho_0 b V_\infty^2}$, $\omega_y^2 = \frac{f_y \cdot S}{\rho_0 b V_\infty^2}$, $\omega_m^2 = \frac{f_m}{\sigma \rho_0 S b V_\infty^2}$ — frequencies of free oscillations in system without damping; b — the second typical dimension of the airfoil chosen so that $\Sigma = S \cdot b$ — the airfoil area; ρ_0 — density of the airfoil material.

The parameter $\epsilon = \frac{\rho}{\rho_0} \cdot \frac{S}{b}$ is small, $\epsilon \ll 1$, if the airfoil has two close dimensions S and b and density of its material is much bigger than density of the air. For example, for typical steel-aluminium wires of power lines $\epsilon = 10^{-3} \ldots 10^{-4}$. Airfoils of this kind we call heavy and bluff. In this paper we will investigate the stability of equilibriums of such airfoils, and we consider high degrees of ϵ as negligible in comparison with low degrees of ϵ where it possible and correctly.

It's necessary to note, that in system (4) values of aerodynamic coefficients and their derivatives are calculated for the airfoil at the equilibrium (when $\gamma = 0$), i.e. they are constants. If we prove asymptotic stability or instability for zero solution of system (4), equilibrium point $x = x_0$, $y = y_0$, $\phi = \phi_0$ of system (3) also will be asymptotically stable or unstable correspondingly. This follows directly from Lyapunov's theorem on connection between stability of nonlinear and linearized systems.

In this paper we will investigate stability of zero solution of system (4), and also consider the special cases corresponding to the system on Fig. 1 with one and two degrees of freedom. In all cases we will consider system with viscous-elastic constraints (with factors $\epsilon \ll \mu_x, \mu_y, \mu_m \ll \epsilon^{-1}$) as well as with perfectly elastic constraints (when $\mu_x = \mu_y = \mu_m = 0$). We will also consider separately cases of equal and different frequencies ω_x and ω_y. Under these assumptions the right parts of equations in system (4) are proportional to small parameter ϵ. Zero solution at every case is asymptotically stable if and only if all minors of Gurvit's matrix corresponding to the characteristic polynome of system (4) are positive. In view of unhandiness of calculations, we will show only results in this paper. For systems with one, two and three degrees of freedom results of equilibrium point stability investigation are presented in Table 1.

TABLE 1

Stability criteria for airfoil movements with 1, 2 and 3 degrees of freedom for systems under perfectly elastic constraints and systems under viscous-elastic constraints.

Generalized coordinates		*Stability criteria for systems under constraints:* perfectly elastic	viscous-elastic
Movements with 1 degree of freedom			
x		Equilibrium point is always asymptotically stable	
y		$G > 0$	$G_\mu > 0$
ϕ		$F > 0$	$F > 0$
Movements with 2 degrees of freedom			
$x - \phi$		$\begin{cases} P_x > 0 \\ F > 0 \end{cases}$	$F > 0$
$y - \phi$		$\begin{cases} G > 0 \\ P_y > 0 \\ F > 0 \end{cases}$	$\begin{cases} G_\mu > 0 \\ F > 0 \end{cases}$
$x - y$	$\omega_x = \omega_y$	$\begin{cases} M > 0 \\ W > 0 \end{cases}$	$\begin{cases} M_\mu > 0 \\ W_\mu > 0 \end{cases}$
	$\omega_x \neq \omega_y$	$G > 0$	$G_\mu > 0$
Movements with 3 degrees of freedom			
$x - y - \phi$	$\omega_x = \omega_y$	$\begin{cases} M > 0 \\ P > 0 \\ W > 0 \\ F > 0 \end{cases}$	$\begin{cases} M_\mu > 0 \\ W_\mu > 0 \\ F > 0 \end{cases}$
	$\omega_x \neq \omega_y$	$\begin{cases} G > 0 \\ P > 0 \\ F > 0 \end{cases}$	$\begin{cases} G_\mu > 0 \\ F > 0 \end{cases}$

In Table 1 we have denoted:

$$
\begin{aligned}
&G = C_{xa} + C'_{ya}, \quad G_\mu = G + 2\mu_y,\\
&M = G + 2C_{xa}, \quad M_\mu = G_\mu + 2(C_{xa} + \mu_x),\\
&W = C_{xa}(C_{xa} + C'_{ya}) + C_{ya}(C_{ya} - C'_{xa}),\\
&W_\mu = (C_{xa} + \mu_x)(C_{xa} + C'_{ya} + 2\mu_y) + C_{ya}(C_{ya} - C'_{xa}),\\
&P_x = \frac{C'_{xa}C_{ma}}{\omega_x^2 - \omega_m^2}, \quad P_y = \frac{C'_{ya}C'_{ma}}{\omega_y^2 - \omega_m^2}, \quad P = P_y + 2P_x,\\
&F = \frac{2f_m}{\rho S^2} - V_\infty^2 C'_{ma}.
\end{aligned}
$$

For each case, the inequality or system of inequalities at which airfoil equilibrium point is asymptotically stable is obtained. At the same time, if even one inequality changes sense on opposite, from Gurvits's criterion and Lyapunov's theorem follows, that equilibrium point becomes unstable. In this sense we can say, that we have obtained necessary and sufficient conditions (criteria) of airfoil stability in a stream.

Within the bounds of the considered model, analytical expressions for necessary and sufficient stability conditions of equilibrium of heavy bluff airfoils in the flow are obtained. These conditions generalize known conditions [4, 6, 8], in particular, Glauert–Den-Hartog's and Vanko's instability conditions.

If the airfoil has rotational degree of freedom condition $F > 0$ is the necessary condition of stability, so the equilibrium point is unstable when $F < 0$, or

$$V_\infty > \sqrt{\frac{2f_m}{\rho S^2 C'_{ma}}} = V_{cr}. \tag{5}$$

Formula for V_{cr} in (5) is similar to formula for critical speed of flutter, which is well-known.

In case of viscous-elastic constraints all stability conditions (with the exception of condition $F > 0$) depend only on airfoil aerodynamic coefficients C_{xa} and C_{ya} and damping coefficients μ_x and μ_y.

In case of perfectly elastic constraints there are instability sufficient conditions $G < 0$, $M < 0$ and $W < 0$, which depend only on aerodynamic coefficients of the airfoil C_{xa} and C_{ya}. These conditions are invariant to a choice of the airfoil fastening point, mass and the moment of inertia of the airfoil, rigidity of constraints. Therefore, change of these mechanical parameters of construction does not influence on character of stability of its equilibrium point.

The results of stability investigation, especially invariant instability sufficient conditions, could be used for equilibrium stability analysis of heavy bluff airfoils in the flow.

3. Vortex element method for airfoil aerodynamic coefficients determination. In order to simulate the flow around the airfoil we use vortex element method, which is very similar to method [10, 12]. But following L. Prandtl, we consider that it is possible to split the flow into two domains: inviscid flow far from the airfoil and boundary layer near it. So we consider vortex elements moving along fluid lines, and viscosity influence we consider only as the vorticity generation factor. This simplification of the vortex element method allows to lower computational time, but the accuracy of calculated values of lift force and drag force remains high.

Calculated results are close to experimental data for flows with Reynolds number about $10^4 \ldots 10^5$. Pressure in the flow and aerodynamic coefficients therefore could be calculated using Bernulli and Cauchy-Lagrange integral analog [12].

Results of flow simulation around the circular airfoil are shown on Fig. 2. Calculated nonstationary aerodynamical coefficients time dependencies are shown on Fig. 3. Experimental values for its mean values (stationary aerodynamic coefficients) are $C_{xa} \approx 1{,}2$, $C_{ya} \approx 0$.

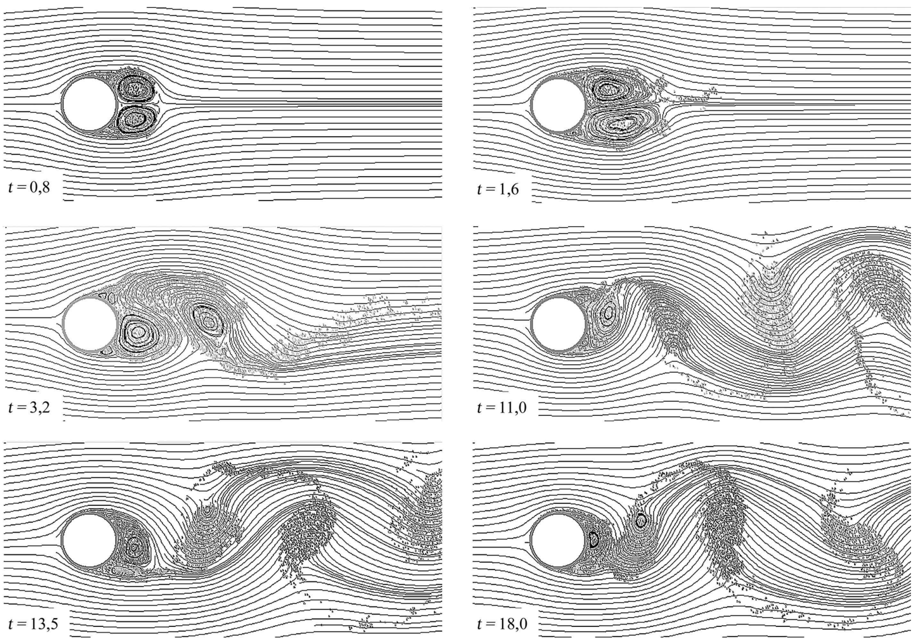

FIG. 2. *Flow around the circular airfoil at* $t = 0{,}8$, $t = 1{,}6$, $t = 3{,}2$, $t = 11{,}0$, $t = 13{,}5$, $t = 18{,}0$ *(t – dimensionless time).*

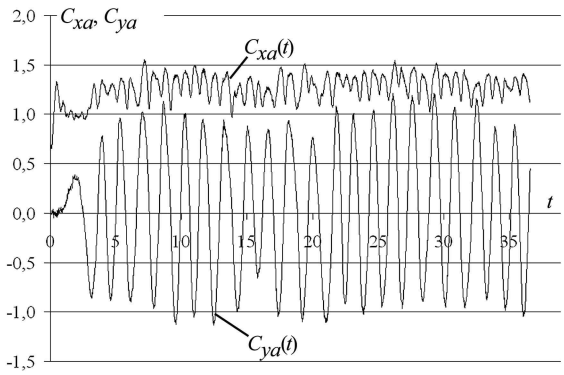

FIG. 3. *Nonstationary aerodynamic coefficients for the circular airfoil. Stationary values (experimental) are* $C_{xa} \approx 1{,}2$, $C_{ya} \approx 0$.

Some results of flow simulation around the semicircular airfoil for different angles of incidence are shown of Fig. 4.

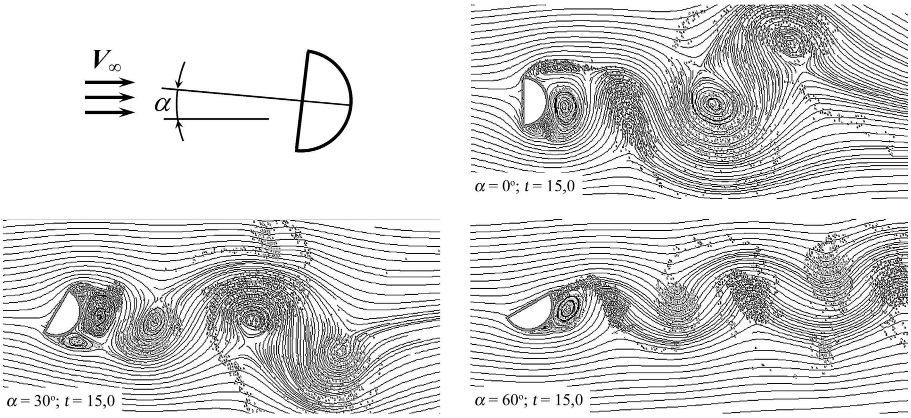

FIG. 4. *Flow around the semicircular airfoil at $\alpha = 0^{\circ}$, $\alpha = 30^{\circ}$, $\alpha = 60^{\circ}$ for $t = 15{,}0$ (α – angle of incidence, t – dimensionless time).*

On Fig. 5 calculated and experimental stationary aerodynamic coefficients dependencies on angle of incidence are shown for the semicircular airfoil.

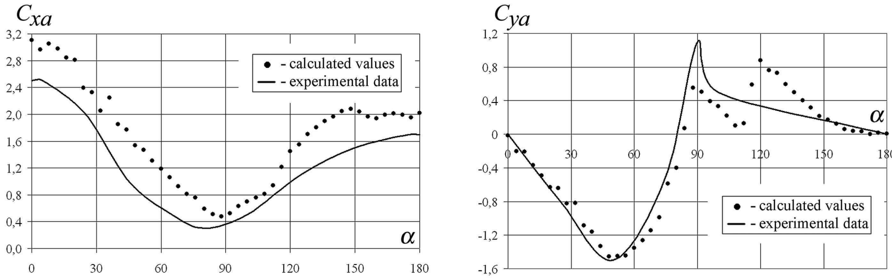

FIG. 5. *Stationary aerodynamic coefficients dependencies on angle of incidence for the semicircular airfoil. $C_{xa}(\alpha)$ – drag coefficient, $C_{ya}(\alpha)$ – lift coefficient (α – angle of incidence).*

Modified vortex element method could be used not only for flow simulation around bluff airfoils, but also for flow simulation around streamline airfoils. For example, TSAGI RII-18 wing airfoil was investigated, and results of stationary coefficients $C_{xa}(\alpha)$ and $C_{ya}(\alpha)$ calculation in comparison with experimental data are shown on Fig. 6.

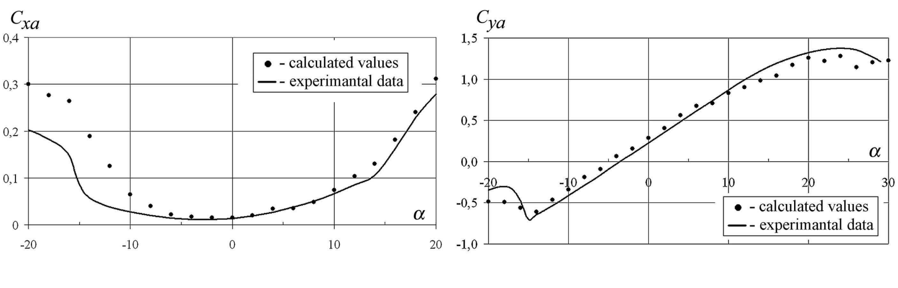

FIG. 6. *Stationary aerodynamic coefficients dependencies on angle of incidence for TSAGI RII-18 wing airfoil. $C_{xa}(\alpha)$ – drag coefficient, $C_{ya}(\alpha)$ – lift coefficient (α – angle of incidence).*

These results prove that modified vortex element method could be used as approximate engineering approach for flow simulation and aerodynamic coefficients calculation.

4. Numerical analytic airfoil stability investigation. The numerical analytic method could be effectively used for stability investigation of an airfoil in the flow. The basis of this method is joint usage of numerical method for flow simulation and airfoil aerodynamic coefficients calculation with analytic instability sufficient conditions, which are invariant and depend only on drag force and lift force coefficients. Main steps of the numerical analytic method are the following.

1) Simulation of the flow around the airfoil at different angles of incidence and its stationary aerodynamic coefficients of drag force and lift force calculation using modified vortex element method.

2) Approximation of obtained values with smooth functions. Chebyshev polynomials and least square method are very useful for solving of this problem.

3) Derivation of $C_{xa}(\alpha)$ and $C_{ya}(\alpha)$ dependencies and calculation $G(\alpha)$ and $W(\alpha)$ functions values. Airfoil equilibrium is unstable at those angles of incidence, for which $G(\alpha) < 0$ or $W(\alpha) < 0$.

There is experimental data for rhombic and square airfoil [8], which allows to verify this numerical analytic airfoil stability investigation method.

4.1. Rhombic airfoil stability investigation. We will investigate stability in the flow of rhombic airfoil with diagonal ratio 1 : 0,75. Because of airfoil symmetry it is sufficient to investigate it only for angles of incidence $0° \leq \alpha \leq 90°$.

Using modified vortex element method, 46 calculations of flow around the airfoil were made with angle of incidence increment 2°. Some results of flow simulation for different angles of incidence are shown on Fig. 7

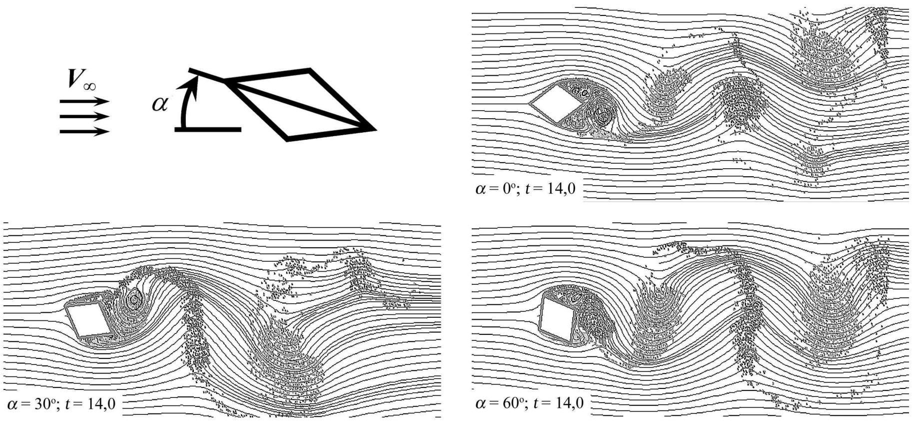

Fig. 7. *Flow around the rhombic airfoil at $\alpha = 0°$, $\alpha = 30°$, $\alpha = 60°$ for $t = 14,0$ (α – angle of incidence, t – dimensionless time).*

Obtained values of stationary aerodynamic coefficients are marked on Fig. 8 by black circles. Smooth curves on Fig. 8 are the plots of approximating functions $C_{xa}(\alpha)$ and $C_{ya}(\alpha)$, which are linear combinations of 16 Chebyshev polynomials with coefficients, calculated using least square method. Derivations $C'_{xa}(\alpha)$ and $C'_{ya}(\alpha)$ could be found and values of functions $G(\alpha)$ and $W(\alpha)$ (see Table 1) also could be calculated. Their plots are also shown on Fig 8.

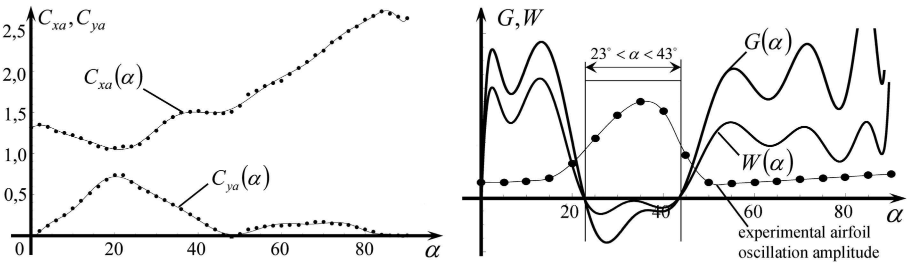

FIG. 8. *Stationary aerodynamic coefficients dependencies on angle of incidence for the rhombic airfoil (left graph). Functions $G(\alpha)$ and $W(\alpha)$ plots for the rhombic airfoil and experimental airfoil oscillation amplitude dependence on angle of incidence (right graph). $C_{xa}(\alpha)$ – drag coefficient, $C_{ya}(\alpha)$ – lift coefficient (α – angle of incidence).*

We can see from Fig. 8 that the region of angles of incidence $23^\circ < \alpha < 43^\circ$, where functions $G(\alpha)$ and $W(\alpha)$ are negative, corresponds to the region, where oscillation amplitude of this airfoil is sufficiently big. Experimental research have been made in wind tunnel [8].

4.2. Square airfoil stability investigation. Now we will investigate square airfoil stability in the flow in much the same way. Because of its symmetry, it is necessary to simulate the flow around it only for $0^\circ \le \alpha \le 45^\circ$. Some results are shown on Fig. 9

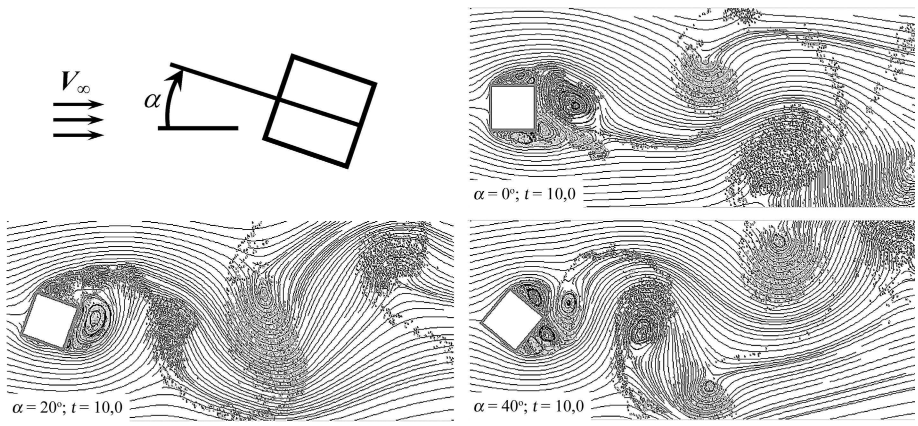

FIG. 9. *Flow around the square airfoil at $\alpha = 0^\circ$, $\alpha = 20^\circ$, $\alpha = 40^\circ$ for $t = 10{,}0$ (α – angle of incidence, t – dimensionless time).*

Obtained values of stationary aerodynamic coefficients are marked on Fig. 10 by black circles. Smooth curves on Fig. 10 are the plots of approximating functions $C_{xa}(\alpha)$ and $C_{ya}(\alpha)$, which are linear combinations of 7 Chebyshev polynomials with coefficients, calculated using least square method. Plots of functions $G(\alpha)$ and $W(\alpha)$ are also shown on Fig 10.

We can see from Fig. 10 that the region of angles of incidence $0^\circ < \alpha < 15^\circ$, where functions $G(\alpha)$ and $W(\alpha)$ are negative, corresponds to the region, where oscillation amplitude of this airfoil is sufficiently big.

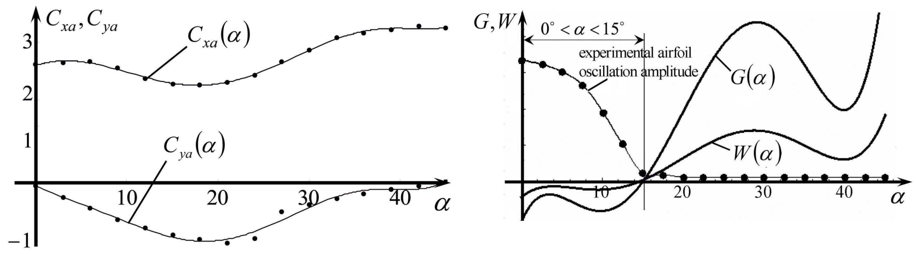

FIG. 10. *Stationary aerodynamic coefficients dependencies on angle of incidence for the square airfoil (left graph). Functions $G(\alpha)$ and $W(\alpha)$ plots for the square airfoil and experimental airfoil oscillation amplitude dependence on angle of incidence (right graph). $C_{xa}(\alpha)$ – drag coefficient, $C_{ya}(\alpha)$ – lift coefficient (α – angle of incidence).*

5. Conclusion. The numeric analytic method of stability investigation of an airfoil in the flow is developed. This method allows to find intervals of angles of incidence corresponding to instable equilibrium points of the airfoil in the flow. Analytic instability sufficient conditions depend only on stationary aerodynamic coefficients of drag force and lift force. These coefficients could be calculated using modified vortex element method, which allows to simulate the flow around the airfoil.

Good agreement between regions of instability, found using numeric analytic method, and obtained in experiment for rhombic and square airfoils proves that this numeric analytic method could be effectively used for stability investigation of an airfoil in the flow.

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