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DIFFERENT WAYS OF USING SECOND PILLAR SAVINGS IN SLOVAKIA*

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Abstract. Recently, first pensions have been paid from the savings corresponding to the second pillar of the pension system in Slovakia. The paper emphasizes that the advantage of the second pillar cannot be assessed solely using the level of pensions paid. Its strength lies in the number of alternatives it offers. Based on calculations, we have analysed benefits of various alternatives in different circumstances. At present, the need for a long-term care in the case of dependency is a common problem. Most pensioners do not have means to cover the associated costs. We present our own model of the long-term care insurance as well as the replacement rate it could provide.

Key words. Pension system in Slovakia, Second pillar, Lifetime annuity, Long-term care insurance

AMS subject classifications. 62P05, 91B30, 91B28

1. Introduction. Since January 2005, pensions in Slovakia are operated by a three-pillar system: the compulsory, Pay-As-You-Go (PAYG) first pillar, the second pillar in the form of the old-age pension saving and the third pillar as a voluntary supplementary pension saving. Only the first pillar is compulsory and the future pensioner can redirect part of the contributions to the saving (second) pillar. In this case, the first pillar pension is reduced accordingly. This reduced pension can be supplemented by benefits from savings of the second pillar.

Several publications addressed the question of whether participation in the second pillar is advantageous. The answer is not uniform and depends on a specific wage profile. For low-income groups, the law only offers the possibility of paying benefits in the form of a lifetime annuity. According to [8], lifetime annuity benefits may not really cover (even in the case of zero fees of insurance companies) the reduction of the first pillar pension. For higher-income savers, the probability of covering the reduction is higher due to the partial solidarity of the first pillar. Many authors (see e.g. [7], [12]) argued that buying a lifetime annuity is not an optimal use of pension savings.

However, Act 43/2004 Coll. provides more possibilities for using pension savings. If the sum of pension benefits paid from other sources reaches a minimum reference amount (currently an average old-age pension), the pensioner may apply for a program withdrawal or a temporary pension. The program withdrawal also includes the possibility to withdraw the entire saved amount at once. This greatly expands the possibilities of using pension savings.

Pension benefits should be set to meet the needs of pensioners. Due to health problems, long-term care is often necessary during retirement. The extent of care depends on the specific disability and can also be a full-time care. Pensioners typically do not have necessary resources to finance it. The main contribution of this paper is a

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model of the use of pension savings for the purchase of a long-term care insurance. We have calculated the resulting replacement rates that can be achieved using realistic savings levels.

The paper is organised as follows. In the next section we discuss the level of pension benefits when buying the lifetime annuities. In the third section we deal with program withdrawal, temporary pensions and yield withdrawal. The fourth section discusses the long-term care insurance. In last section we conclude.

2. Expected level of savings and lifetime annuities. The mandatory part of the pension system in Slovakia has two pillars: the public, compulsory, non-funded (Pay-As-You-Go) first pillar and the private, fully funded second pillar. The contribution rate (for the old-age pensions) is currently set at 18% for the first pillar (in the case a pensioner decides to stay only in the public scheme only) or 13% for the first pillar and 5% for the second pillar (in the case a pensioner decides to participate in both pillars) with future planned increase to 6%.

Adequacy of pension savings can be assessed in several ways. In [6] authors introduced a retirement-years indicator (D_T). It was calculated as the ratio of the sum S_T saved at the time of retirement T and the last yearly wage W_T before retirement: $D_T = S_T/W_T$. This indicator can be easily recalculated to the replacement rate (the ratio of the first pension to W_T (cf. [8]).

2.1. Survival and mortality probabilities. In order to recalculate the savings to the replacement rate, we have used an approach from [13]. Pricing of annuity products has been based on the relevant survival probabilities ${}_tp_x$ representing the probability that an individual at age x survives to at least age $x+t$. Denote by q_x the probability that an individual being at age x dies before age $x+1$. Then ${}_tp_x = \prod_{h=0}^{t-1}(1-q_{x+h})$. In our practical calculations we have used three sets of mortality rates:

- $q_x^{(S)}$ representing the static mortality rates from the Mortality Tables of the SO SR [10] for the total Slovak population in year 2018,
- $q_x^{(H)}$ denoting predicted mortality rates using the Lee-Carter longevity model for the future period 2018-2066; parameters of the model were estimated in R software [11] using the `demography` package [5] and data from The Human Mortality Database [4] for the total Slovak population from 1950 to 2017,
- $q_x^{(L)}$ representing pessimistic longevity (in terms of insurance) mortality rates from the lower bound of the 90% prediction interval for the aforementioned Lee-Carter predictions (see also [13]).

For the mortality rates applies: $q_x^{(S)} \geq q_x^{(H)} \geq q_x^{(L)}$.

2.2. Svensson yield curve. In accord with [14], we have used the Svensson yield curve as a functional form for the spot interest rates depending on corresponding maturities. The Svensson yield curve is given by

$$R(t) = \beta_0 + \beta_1 \frac{1-\exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} + \beta_2 \left[\frac{1-\exp\left(-\frac{t}{\tau_1}\right)}{\frac{t}{\tau_1}} - \exp\left(-\frac{t}{\tau_1}\right)\right] + \\ + \beta_3 \left[\frac{1-\exp\left(-\frac{t}{\tau_2}\right)}{\frac{t}{\tau_2}} - \exp\left(-\frac{t}{\tau_2}\right)\right], \tag{2.1}$$

where $R(t)$ is a yield from a bond investment with continuous compounding, t is a time to maturity, $t \in (0, T_{max}]$, T_{max} is the maximum time to maturity, $\tau_1, \tau_2, \beta_0, \beta_1, \beta_2, \beta_3$ are parameters of the Svensson yield curve. The discounting factor corresponding to the maturity t is then given by $P(t) = e^{-R(t)t}$. The interpretation of the parameters can be found in [1]: β_0 is the long-term asymptotic value of $R(t)$, β_1 is the spread between the long term and short-term rates, i.e. $\beta_0+\beta_1$ is the short term rate (the rate corresponding to zero maturity). The parameters $\tau_1, \tau_2, \beta_2, \beta_3$ specify the positions, magnitudes and directions of two humps corresponding to the Svensson curve.

2.3. Pension annuity product. Consider a person with a retirement age x having saved the amount S_T. The basic equivalence equation represents the expected present values of all cash-flows related to the yearly annuity payment P_x:

$$S_T = P_x a_x(1+\beta) + P_x\alpha + S_T A^1_{x:\overline{\frac{1}{12}}|} + P_x(MA)^1_{x:\overline{7}|}\,. \tag{2.2}$$

On the left-hand side stands the accumulated sum S_T representing a premium of the product. The value $P_x a_x$ is the expected present value of the whole life yearly paid annuity-immediate P_x, where

$$a_x = \sum_{t=1}^{\omega-x} {}_tp_x P(t)$$

denotes the expected present value of a whole life 1 monetary unit (m. u.), paid at the end of each year under the condition that the person is alive and ω is maximum age to which a person can live to see (regarding used life tables $\omega = 110$). The term $A^1_{x:\overline{\frac{1}{12}}|}$ expresses expenditures of the insurance company associated with a quick withdrawal of finances from investment funds. The expression $P_x(MA)^1_{x:\overline{7}|}$ represents the expected present value of the sum of not yet paid monthly annuities in the amount of 1 m. u. in the case of the beneficiary death during the period of the first seven years of the retirement period. This amount relates to the expenditures described in the Act 43/2004, Article 32 - Annuity, paragraph 2. Finally α and β represent fees charged to the first and following annuity payments. For a detailed explanation of all terms in (2.2) see [13]. Dividing both sides of (2.2) by W_T one has

$$RE_x = \frac{P_x}{W_T} = \frac{D_T\left(1 - A^1_{x:\overline{\frac{1}{12}}|}\right)}{\left((1+\beta)a_x + \alpha + (MA)^1_{x:\overline{7}|}\right)}\,, \tag{2.3}$$

where RE_x is the replacement rate (the ratio of the yearly pension to the last yearly salary before retirement).

Tab. 2.1 contains replacement rates for different levels of savings and retirement ages calculated using Svensson ECB all bonds curve (the parameters were estimated in [3], date 2019-10-24) and fees $\alpha = 50\%$, $\beta = 8\%$. Since in the case of lifetime annuities the insurance companies usually consider pessimistic longevity, we have applied probabilities of death $q_x^{(L)}$ (see Section 2.1). According to [8], at least 17% replacement rate is needed to compensate for the reduction of the first pillar pension. Following recent legislative changes, the total contribution rate for the purpose of reducing the first pillar pension was increased from 18% to 22.75%[1]. Therefore, the

[1]The first pillar pension is reduced by the part $\delta/22.75$ for the period of participation in the second pillar, where δ is the contribution rate (in percentage) to the second pillar. The original reduction ratio was $\delta/18$.

Table 2.1

Replacement rates of whole-life annuity payments for various levels of savings and current initial ages of the pensioner using mortality rates $q_x^{(L)}$.

D_T/Age	62	65	70	75	80
1.5	0.064718	0.070183	0.082237	0.099687	0.125017
2.0	0.086290	0.093578	0.109649	0.132916	0.166690
2.5	0.107863	0.116972	0.137061	0.166146	0.208362
3.0	0.129435	0.140367	0.164474	0.199375	0.250034
3.5	0.151008	0.163761	0.191886	0.232604	0.291707
4.0	0.172581	0.187155	0.219298	0.265833	0.333379

required compensation from the savings of the second pillar is only about 13.5% $\sim 18/22.75 \times 17\%$. Using the results from Tab. 2.1 one can conclude, that achieving such a replacement rate requires the level of savings 3-3.5 of yearly salaries. In [8] authors reported realistic level of savings D_T in the second pillar after 40 years of saving between 1.5 and 4 yearly salaries. Compensation for the reduction of the first pillar by savings from the second pillar is therefore questionable. On the other hand, [7] and [12] argued that immediate purchase of an annuity after retirement is suboptimal.

3. Other legal possibilities of using pension savings.

3.1. Temporary pension and programmed withdrawal. According to current legislation, a saver is entitled to a program withdrawal or a temporary pension if the sum of pension benefits paid from other sources reaches a minimum reference amount (currently an average old-age pension). A temporary pension is an insurance product paid by insurance companies. The length of the contract can be 5, 7 or 10 years. Unlike the second-pillar lifetime pension, the temporary pension does not include the insurer's obligation to pay 7 years of pension benefits. In the event of the death of the beneficiary, the payment of benefits shall cease. In addition, a temporary pension does not insure longevity. The product can be valued using formula (2.3) while omitting the expression valuating the mandatory 7-years benefits. In order to highlight the disadvantage of the temporary pension, we have calculated the results of the annuity payments using zero fees α and β. The replacement rates have been calculated for the level of savings[2] $D_T = 3$ and mortality rates $q_x^{(L)}$. Note that according to Tab. 3.1, there is no big difference (for initial ages of 62-65 years) between the temporary pension and the withdrawal of the full amount (which is a legal form of the programmed withdrawal) with gradual spending without any institutional assistance. One can observe a substantial difference only for higher ages. On the other hand, in this case, the risk of death before the end of the planned period is substantial. To conclude, a temporary pension is probably not a good alternative for using pension savings. Main reasons are low interest rates and low probabilities of death.

In contrast, the programmed withdrawal is not an insurance product. When using this method of payment, the savings remain in the pension management company (PMC), with which the pensioner concludes a retirement benefit plan by programmed withdrawal. Under this agreement, the PMC will pay a pension from a personal account under pre-agreed terms. The beneficiary determines the monthly amount and the length of the retirement benefits. In the event of death, the remaining funds are subject to inheritance. It is worth to note that by using the programmed withdrawal, one can avoid the annuity fees. An interesting set of dynamic and static strategies

[2] It is worth to note, that the replacement rates are linear with respect to the level of savings D_T.

TABLE 3.1

Replacement rates of temporary pension payments using mortality rates $q_x^{(L)}$ for various ages of the pensioner and durations of pensions. The level of savings $D_T = 3$ and zero fees $\alpha = \beta = 0\%$.

Age/Duration	5	7	10
62	0.617842	0.447818	0.322131
65	0.620755	0.450924	0.325617
70	0.628771	0.459251	0.334950
75	0.642634	0.474752	0.354056
80	0.677258	0.512612	0.397505

of programmed withdrawal can be found in [12]. The authors also calculated the expected value of the bequest corresponding to the respective strategies. Most of the strategies presented avoided ruin and can be considered as a more effective alternative comparing to lifetime annuities. In [2] authors discussed a combination of a programmed withdrawal and buying a lifetime annuity. They considered the programmed withdrawal with monthly payments one would receive when buying an annuity at retirement. While monthly benefits were paid, the remaining savings were invested. After 10 years, the lifetime annuity was purchased from the rest of the savings. The authors reported, that the resulting monthly lifetime benefit was with probability 93 percent higher than that resulting from the lifetime annuity purchased at retirement. A thorough analysis of strategies using deferred lifetime annuities taking into account stochastic interest rates and mortality rates can be found in [7].

3.2. Investment return withdrawal and later purchase of annuity. The legislation does not require the purchase of a lifetime annuity from the second pillar savings even in the case of receiving a pension from the first pillar. There is no reason to rush to buy an annuity while the pensioner's income is sufficient. Such a situation occurs, e.g., when he/she is working after retirement age. In such a case, it may be advantageous to postpone the purchase of an annuity, or not to buy annuity at all, and to use the savings later for a more reasonable purpose or to leave them as a bequest. Paragraph 46i of Act 43/2004 Coll. gives a possibility to the saver to apply for the return on investment payment if he/she has reached the retirement age, while not being the recipient of the retirement pension or early retirement pension by programmed withdrawal. Note that the return on investment is not a retirement pension. Therefore, savings remain the property of the saver. Having the savings D_T (in W_T units) and taking into account the annual investment return (after deducting reasonable PMC costs) r_s, the saver can receive the $D_T r_s$ replacement rate each year. For example, if one considers realistic values $D_T = 3$ and $r_s = 5\%$, then a replacement rate of 15% can be expected. This is an interesting amount, considering that, unlike the purchase of a lifetime annuity, savings are still the property of the saver.

Suppose the saver decides to postpone the purchase of the annuity by m years. If he/she does not collect the return on investment, then D_T increases with factor $(1 + r_s)^m$. For $r_s = 5\%$ the value of the increase factor is 1.63. In addition, if the pensioner continues to work, additional contributions increase the savings. In the case of a later purchase of annuity, a lower expected life expectancy affects the monthly benefit positively. The replacement rates for the lifetime annuities can be calculated using formula (2.3). The values for corresponding D_T and different ages of annuitization can be found in Tab. 2.1.

4. Long-term care insurance. At present, the need for a long-term care in the case of dependency is a common problem. Most pensioners do not have means to

Table 4.1
Shares requiring long-term care in the population of Belgium (2007). Source: [15].

Age cohort	% of LTC	Age cohort	% of LTC
50-54	4.7465	70-74	15.1312
55-59	5.8612	75-79	20.5790
60-64	8.3343	80-84	36.6185
65-69	8.5914	85+	72.2630

Table 4.2
Shares requiring long-term care - our estimates.

x	γ_x	x	γ_x	x	γ_x	x	γ_x	x	γ_x
62	0.083346	72	0.130976	82	0.412192	92	0.820447	102	0.986828
63	0.083974	73	0.148912	83	0.449982	93	0.849176	103	0.989226
64	0.084602	74	0.169144	84	0.500467	94	0.869314	104	0.991964
65	0.085229	75	0.191308	85	0.554848	95	0.896845	105	0.992040
66	0.085857	76	0.215056	86	0.604945	96	0.928758	106	0.993453
67	0.086485	77	0.240068	87	0.647689	97	0.944044	107	0.994203
68	0.088826	78	0.266057	88	0.689024	98	0.962694	108	0.995286
69	0.094435	79	0.295770	89	0.723902	99	0.975702	109	0.996704
70	0.103436	80	0.339986	90	0.762286	100	0.982063	110	0.997455
71	0.115704	81	0.377514	91	0.791143	101	0.984773		

cover the associated costs. In this section we present our model of the long-term care insurance as well as the replacement rate it could provide. We have supposed that a person can be in one of the following three states:

1. Healthy
2. Dependent (needing a long-term care)
3. Dead

Furthermore, we have assumed that a healthy person can become dependent or die and a dependent person cannot become healthy.

Since the realistic data of number of persons requiring the long-term care in Slovakia are not available, we have used the data from Belgium [15]. The shares requiring long-term care in Belgium are in Tab. 4.1. In the first step, we have interpolated these data using weighted averages, where as weights we have chosen population sizes at ages 65, 66, ... 105 years in the total Slovak population (see Mortality Tables of the SO SR 2018 [10]). In the second step, we have fitted the interpolated values by a polynomial-exponential function, while values $\gamma_{106}, \gamma_{107}, \ldots, \gamma_{110}$ were extrapolated using the fitted curve. The resulting estimates of shares γ_x requiring the long term care for various age cohorts x can be found in Tab. 4.2.

Let us denote the probabilities of remaining in the corresponding states, respectively transitions between states as follows:

- p_x^{ii} - the probability that an individual being at age x in state $i \in \{1,2\}$ remains in this state at least to age $x+1$,
- ${}_mp_x^{ii}$ - the probability that an individual being at age x in state $i \in \{1,2\}$ remains in this state at least to age $x+m$,
- p_x^{ij} - the probability that an individual being at age x in state $i \in \{1,2\}$ transits to state $j \in \{2,3\}$, $j > i$ before age $x+1$.

The following equations apply to the mentioned probabilities:

$$p_x^{11} + p_x^{12} + p_x^{13} = 1 \tag{4.1}$$

$$p_x^{22} + p_x^{23} = 1\,. \tag{4.2}$$

By shifting individuals of age x and balancing the number of dependent ones we have

$$(1-\gamma_x)p_x^{12} - \gamma_x p_x^{23} = \gamma_{x+1}\,(1-\overline{q}_x) - \gamma_x\,, \tag{4.3}$$

$$\overline{q}_x = (1-\gamma_x)p_x^{13} + \gamma_x p_x^{23}\,. \tag{4.4}$$

Equations (4.1)-(4.4) are not sufficient to determine all the necessary probabilities p_x^{ij}. Missing equations can be replaced by defining the relationship between the mortality probabilities p_x^{13} and p_x^{23}. One has more options for this definition, e.g.

1. $p_x^{13} = p_x^{23} = q_x^{(H)}$, the case of optimistic (in terms of insurance) longevity,
2. $p_x^{13} = p_x^{23} = q_x^{(L)}$, the case of pessimistic (in terms of insurance) longevity,
3. $p_x^{13} = q_x^{(L)}$ with additional equation $q_x^{(S)} = (1-\gamma_x)p_x^{13} + \gamma_x p_x^{23}$ setting the whole-population mortality rate at the higher level; in this case the dependents (persons dependent on the long-term care) have lower life expectancy comparing to the healthy population,
4. $p_x^{13} = q_x^{(L)}$ and $p_x^{23} = \kappa \dfrac{q_x^{(S)} - (1-\gamma_x)\,p_x^{13}}{\gamma_x}$, where κ is a constant factor such that the dependents have the half life expectancy comparing to the healthy population,
5. $p_x^{13} = q_x^{(S)}$ and $p_x^{23} = \lambda \dfrac{q_x^{(S)} - (1-\gamma_x)\,p_x^{13}}{\gamma_x} = \lambda p_x^{13}$, where $\lambda > 1$ is a constant factor such that the dependents have the half life expectancy comparing to the healthy population.

Combining equations (4.1)-(4.4) with any of these options one can determine all the necessary probabilities p_x^{ij}, $i,j \in \{1,2,3\}$, $i \le j$.

Denote by Z_x a random variable representing the time (in years) in which a healthy person of age x switches to the state 2 (long-term care dependency). It is obvious, that $\Pr(Z_x = 1) = p_x^{12}$. The probabilities for further times $k > 1$, where $k \in \mathbb{N}$, can be calculated as

$$\Pr(Z_x = k) = p_x^{11} \times p_{x+1}^{11} \times \cdots \times p_{x+k-2}^{11} \times p_{x+k-1}^{12}\,. \tag{4.5}$$

Suppose that a person of age x becomes dependent after m years. The expected present value of long-term care lifetime benefits L_x is $L_x \times {}_{m|}a_x^L$, where

$${}_{m|}a_x^L = P(m+1) + \sum_{t=m+1}^{\omega-x-1} P(t+1) \times {}_{t-m}p_{x+m}^{22}\,. \tag{4.6}$$

The probabilities ${}_kp_y^{22}$ that the person of age y remains in the state 2 at least next $k \in \mathbb{N}$ years are calculated as follows:

$${}_kp_y^{22} = p_y^{22} \times p_{y+1}^{22} \times p_{y+2}^{22} \times \cdots \times p_{y+k-1}^{22}\,. \tag{4.7}$$

The expected present value of long-term care benefits is then

$$\mathrm{E}\left[{}_{Z_x|}a_x^L\right] = \sum_{m=1}^{\omega-x-1} {}_{m|}a_x^L \times \Pr(Z_x = m)\,. \tag{4.8}$$

The basic equivalence equation representing the expected present values of all cash flows related to basic yearly benefits L_x is

$$S_T = L_x \mathrm{E}\left[{}_{Z_x|}a_x^L\right](1+\beta) + L_x\alpha\,. \tag{4.9}$$

The replacement rate of long-term care benefits RL_x can be then calculated as follows:

$$RL_x = \frac{L_x}{W_T} = \frac{D_T}{\mathrm{E}\left[{}_{Z_x|}a_x^L\right](1+\beta)+\alpha}\,. \qquad (4.10)$$

We have calculated replacement rates for three options of the relationship between the mortality probabilities p_x^{13} and p_x^{23} (see above). In our calculations we have considered the fees $\alpha = 10\%$ and $\beta = 9\%$. Comparing to the case of the lifetime annuity, the fee α is lower (50% for the lifetime annuity). This reflects the fact that the expected life expectancy in the state 2 is typically lower than the life expectancy when calculating a lifetime annuity and a high fee corresponding to the first pension could significantly lower the benefits. The decrease of the fee α is compensated with higher fee β ($\beta = 8\%$ for the lifetime annuity). In our calculations we have considered the level of savings $D_T = 3$ yearly salaries.

Option 2, where $p_x^{13} = p_x^{23} = q_x^{(L)}$, can be used to estimate the upper limit of the long-term care (LTC) insurance price. The replacement rates of the LTC benefits and the expected value of 1 m. u. lifetime benefits (paid in the case of necessary LTC) can be found in Tab. 4.3 (columns RR and $\mathrm{E}[{}_{Z_x|}a_x^L]$ respectively). Comparing to the replacement rates for the lifetime annuities (Tab. 2.1) one can observe significantly higher values. For example, for the age of 62, the replacement rate for LTC benefits is about 6-times higher than in the case of lifetime annuity. This value is influenced by two factors. As the age increases, the probability of transition to the state 2 increases, causing the insurance price to rise. On the other hand, life expectancy decreases with increasing age, which has the opposite effect on the insurance price. According to values $\mathrm{E}[{}_{Z_x|}a_x^L]$ in Tab. 4.3 for lower ages, the first factor prevails, for higher ages the decisive factor is the decrease of the life expectancy. In the last two columns of Tab. 4.3 we present the expected value of 1 m. u. lifetime benefits in the case of LTC benefits paid from the age according to the first column and expected lifetime in the state 2 for a person with the age according to the first column (${}_{0|}a_x^L$ and EL in 2 respectively). Compared to $\mathrm{E}[{}_{Z_x|}a_x^L]$, the ${}_{0|}a_x^L$ values are significantly higher, which is related to the uncertainty of a healthy person's transition to the state 2.

Option 5, in which applies $p_x^{13} = q_x^{(S)}$ and $p_x^{23} = \lambda p_x^{13}$, is appropriate to estimate the lower limit of the LTC insurance price. The corresponding values are in Tab. 4.4. Comparing to Option 2, one can observe significantly higher replacement rates and lower expected lifetimes in years in the state 2.

Option 3, where $p_x^{13} = q_x^{(L)}$ with additional equation $q_x^{(S)} = (1-\gamma_x)p_x^{13} + \gamma_x p_x^{23}$, respects the fact, that people needing LTC have lower life expectancy comparing to healthy ones (see e.g. [9]). Moreover, the mortality rate for the whole population is the realistic value $q_x^{(S)}$. The results corresponding to Option 3 are presented in Tab. 4.5. For example, the replacement rate corresponding to the age of 62 is nearly 8-times higher comparing to lifetime annuities (Tab. 2.1). The life expectancy of healthy people (it can be seen in the last column of Tab. 4.3) is significantly higher than that of people in the need of LTC (see the last column of Tab. 4.5).

One can observe, that results in Tab. 4.3-4.5 vary significantly, which shows a big impact of the p_x^{13} and p_x^{23} choices. Precise adjustment would require relevant data on mortality of the dependents. Since we do not yet have these data, we present only three options, two of which represent the lower and upper limit of the insurance price.

It is probably unrealistic to renounce all savings in favour of purchasing the LTC insurance. The authors in [9] presented an interesting idea of purchasing a combined product of a lifetime annuity and LTC insurance. They argued that a cohort buying

TABLE 4.3

Replacement rates of long-term care benefits ($D_T = 3$) and expected lifetimes in years in the state 2 for different ages and mortality settings according to Option 2.

Age	RR	$\mathrm{E}\left[{}_{Z_x|}a_x^L\right]$	${}_{0|}a_x^L$	EL in 2
62	0.73779	3.63869	21.40572	22.83
65	0.68185	3.94477	19.75377	20.70
70	0.62185	4.33421	16.86189	17.18
75	0.66703	4.03448	13.86460	13.75
80	0.79275	3.38011	10.85318	10.48

TABLE 4.4

Replacement rates of long-term care benefits ($D_T = 3$) and expected lifetimes in years in the state 2 for different ages and mortality settings according to Option 5.

Age	RR	$\mathrm{E}\left[{}_{Z_x|}a_x^L\right]$	${}_{0|}a_x^L$	EL in 2
62	1.71761	1.51065	10.14125	9.75
65	1.64340	1.58301	9.09080	8.64
70	1.56713	1.66452	7.41489	6.90
75	1.74809	1.48271	5.79240	5.25
80	2.17250	1.17513	4.33380	3.80

such a combination has a higher mortality rate than a cohort buying a lifetime annuity only. Therefore, setting the mortality rates according to Option 3 is appropriate for calculating the price of the combined product. In Tab. 4.6, lifetime annuity replacement rates calculated with mortality $q_x^{(S)}$ can be found. Compared to Tab. 2.1, where replacement rates are calculated with mortality rates $q_x^{(L)}$, the values from Tab. 4.6 are significantly higher. The saved amount D_T (in our calculations we use $D_T = 3$) can be divided between the purchase of the lifetime annuity and the LTC insurance. For example, a 1:1 split of the saved amount offers a replacement rate for the lifetime annuity of 7.38% and an additional 101.24%/2 = 50.62% if needed the LTC. Other options can be calculated using the linearity of replacement rates with respect to D_T.

5. Conclusions. We have discussed several alternatives of using second pillar savings. Lifetime annuities have a problem to compensate the shortening of the first pillar. Later purchase of the annuity results in significantly higher pension benefits. A program withdrawal combined with investment of unpaid savings ([2], [7]) may be advantageous.

An interesting possibility of using savings is the investment return withdrawal. This option can also be combined with a later purchase of the annuity. Its advantage is that unpaid savings are still the property of the saver. The disadvantage is that in some years zero benefits might be paid.

The immediate purchase of a temporary pension seems disadvantageous. More advantageous benefits can only be obtained after later purchase. On the other hand, this carries a high risk of death before the contract expiration.

The main contribution of the paper is the valuation of the long-term care insurance. When buying it immediately after retirement, high replacement rates can be achieved. The resulting pension benefits can guarantee a reasonable level of long-term care. However, this option implies the renouncing of savings in favour of purchasing the insurance. An interesting option is to purchase a combined product consisting of the life annuity and the LTC insurance. For a cohort that opts for this alternative, the cost of the lifetime annuity may be lower.

Table 4.5

Replacement rates of long-term care benefits ($D_T = 3$) and expected lifetimes in years in the state 2 for different ages and mortality settings according to Option 3.

Age	RR	$\mathrm{E}\left[{}_{Z_x\vert}a_x^L\right]$	${}_{0\vert}a_x^L$	EL in 2
62	1.01243	2.62677	13.31445	13.25
65	0.94404	2.82370	11.27614	11.06
70	0.88993	3.00096	9.52241	9.15
75	0.95058	2.80363	7.97953	7.51
80	1.13325	2.33692	6.56656	6.05

Table 4.6

Replacement rates of whole-life annuity payments for various levels of savings and current initial ages of the pensioner using mortality rates $q_x^{(S)}$.

D_T/Age	62	65	70	75	80
1.5	0.073769	0.081779	0.099300	0.124926	0.160448
2.0	0.098359	0.109039	0.132400	0.166568	0.213930
2.5	0.122948	0.136299	0.165500	0.208210	0.267413
3.0	0.147538	0.163558	0.198599	0.249852	0.320895
3.5	0.172127	0.190818	0.231699	0.291495	0.374378
4.0	0.196717	0.218078	0.264799	0.333137	0.427860

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