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# CHILD - RELATED PENSION BENEFITS: THE CASE OF SLOVAKIA*

IGOR MELICHERČÍK† AND TATIANA JAŠURKOVÁ‡

**Abstract.** Population aging and low birth rates are linked to the problem of unsustainability of ongoing pension systems. As demographic predictions follow unfavorable developments, adjusting such pension systems is inevitable. This paper discusses introducing child-related benefits into pension system models and their advantages and disadvantages. The model with child-related pension benefits dependent on the average wage is examined concerning the effects of the child factor on individual fertility. We estimate the size of the child factor in the current setting of Slovakia's pension system. Finally, the optimal setting of the above pension system model is presented and compared with the presented alternatives. We show that the current setting of the pension system can be brought closer to the optimum by, for example, more generous awarding of personal wage points for raising children.

**Key words.** pension system in Slovakia, sustainability, demographic challenges, parental bonus, child-related benefits

**AMS subject classifications.** 62P20, 91B50, 91B52

**1. Introduction.** The long-term trend of population aging comes with wide-ranging economic implications. The progress and availability of health care for the general population increased the average life expectancy. However, the level of total fertility rate is far below 2.1, which is, according to [3], the level necessary to maintain the population in developed countries. Combined, these factors lead to several socio-economic challenges, one of which is the sustainability of the pension systems.

The pension system's sustainability depends on its stability and the financial balance between contributions and expenditures. The primary financial source of the pension system is individual contributions in the form of regularly paid financial contributions from earned wages. Therefore the stability of such a pension scheme lies within a continuous replacement of the generations. That way, the new contributors can cover the retirees' costs. The upbringing of an offspring thus becomes as essential of a contribution to the pension system as the payment of contributions itself. Both forms of contributions to the pension scheme should then be adequately assessed when calculating an individual's pension benefit. This idea brings us to the so-called child pension models as mentioned in [11]. This type of pension scheme introduces child-related benefits in various ways. It seems to positively affect fertility, considering several empirical studies [8, 9]. According to [7], the child pension scheme should not wholly replace the PAYG pension systems, but their combination may be the way to solve the sustainability problem.

As of January 2023, Slovakia introduced parental pension into the pension benefit scheme, see [10]. Children can contribute 3% of their average monthly wage to increase their parent's pension benefits (the benefits are equally divided between both parents). Such change may be considered as a try to implement child-related pension scheme

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†Department of Applied Mathematics and Statistics, Faculty of Mathematics, Physics and Informatics, Comenius University, Mlynská dolina, 842 48 Bratislava, Slovakia (igor.melichercik@fmph.uniba.sk).

‡DSS VÚB Generali, Slovakia (jasurkova.tatiana@gmail.com).

to compensate parents for the costs of bringing up a child. However, the question of whether it is a meaningful change remains.

The paper is structured as follows: in the first part, two simple models of the pension system with child benefits are presented. We then look at the properties of the average wage benefit model. Finally, we will estimate the child factor for the current setting of Slovakia's first pillar of the pension system and its potential adjustment. The results are compared with the optimal child factor calculated using the presented model.

**2. Two pension models with child factor.** Let us start with a simple pension model with three overlapping generations. For simplicity, we consider a one-sex generation consisting of women. The first period represents childhood when the individual is inactive and dependent on parental care. In the second period, $t$, she works for a wage $w_t$ and contributes $\tau_t w_t$ to the pension system, where $\tau_t$ is the contribution rate[1]. In the third period, she receives a pension. Assuming the pension system, where raising children has a direct impact on the final pension income, we will set the pension benefit in the following form:

$$p_{t+1} = B_{t+1} \frac{w_t}{\overline{w}_t} + \alpha \tau_{t+1} W_{t+1}, \tag{2.1}$$

where $W_{t+1}$ represents the wages of all children of the individual. Thus, the benefit consists of two parts. The first part $B_{t+1} \frac{w_t}{\overline{w}_t}$ is based on the ratio of the individual's wage $w_t$ to the average wage $\overline{w}_t$ of a given generation at time $t$. It reflects the workload of an individual in an economically active period. The second part represents the $\alpha$-part of the offspring's contributions to the pension system. Such a setting reflects the newly defined parental pension in the pension system in Slovakia, where the child-related part of the pension benefit is linked to the wages of the individual's children.

In order to balance the contributions and benefits of the system, one has:

$$B_{t+1} = (1 - \alpha)\, \tau_{t+1} \overline{W}_{t+1}.$$

Pension benefit (2.1) is then reformulated as

$$p_{t+1} = \tau_{t+1} \overline{W}_{t+1} \left[ (1 - \alpha) \frac{w_t}{\overline{w}_t} + \alpha \frac{W_{t+1}}{\overline{W}_{t+1}} \right], \tag{2.2}$$

Hence the increase of child factor $\alpha$ reduces the pension benefit claim if $\frac{W_{t+1}}{\overline{W}_{t+1}} < \frac{w_t}{\overline{w}_t}$, i.e. when the total wage of the children in relation to its average value is lower than the ratio of the parent's salary to its generational average. This is necessarily fulfilled if the individual has no child, if the child has died, or when her children are unemployed or have gone abroad and therefore do not report any income.

To suppress these negative consequences, the pension benefit formula is modified as follows

$$p_{t+1} = B_{t+1} \frac{w_t}{\overline{w}_t} + \alpha \tau_{t+1} n_{t+1} \overline{w}_{t+1}, \tag{2.3}$$

where $n_{t+1}$ is the number of individual's children. The first part of the pension benefit formula remained the same as before. On the other hand, the second component containing the child factor $\alpha$ implies the entitlement to a pension benefit solely depending

[1] Throughout the paper, time periods are indicated by subscripts.

on the individual's fertility $n_{t+1}$ and the contribution $\tau_{t+1}\overline{w}_{t+1}$ from the average wage of overall offspring generation. Therefore, the child-related part of the pension benefit is not totally individually based but uses the average wage. In order for the pension system to be balanced, one can calculate

$$B_{t+1} = \tau_{t+1}\overline{W}_{t+1} - \alpha\tau_{t+1}\overline{n}_{t+1}\overline{w}_{t+1} = (1-\alpha)\,\tau_{t+1}\overline{n}_{t+1}\overline{w}_{t+1}.$$

Pension benefit (2.3) can be then expressed as

$$p_{t+1} = \tau_{t+1}\overline{w}_{t+1}\left[(1-\alpha)\,\overline{n}_{t+1}\frac{w_t}{\overline{w}_t} + \alpha n_{t+1}\right]. \tag{2.4}$$

The growth of the child factor is thus beneficial if $\frac{n_{t+1}}{\overline{n}_{t+1}} > \frac{w_t}{\overline{w}_t}$. In comparison to pension benefit (2.2), the latter defined pension benefit (2.4) contains the parental allowance based on individual fertility. However, its final value depends on the average wage of the offspring generation. It, therefore, does not have the shortcomings of the model where the pension benefit depends on the wages of one's children. Likewise, the average wage-scaled pension benefit is close to the setting of the pension system of the Slovakia, where during parental leave, the parent is allocated a constant 0.6 personal wage point (the ratio of the individual's and average salary) as the compensation benefit. In January 2023, a parental pension was added to this compensation, which can be modeled using equation (2.2).

**3. Fenge-Meier model.** In this section, we follow up on the results of [6]. The model framework consists of three periods of overlapping generations, assuming the identity of all individuals in a given generation. In the first period of life, the individual depends on parental care.

Following the transition to an economically active period $t$, a childless individual earns wage $\tilde{w}_t$. The wage $\tilde{w}_t$ is reduced by pension contribution rate $\tau_t$. If the individual in the second period decides to raise $n_{t+1}$ children, the wage is further reduced by the factor $1 - f(n_{t+1})$, where $f(n_{t+1})$ is the loss function representing the lost wage due to the raising $n_{t+1}$ children, that satisfies $f(0) = 0$, $f'(n_{t+1}) > 0$ and $f''(n_{t+1}) \geq 0$. In the first period, individual's disposable income is divided between consumption $c_t$ and personal savings $s_t$.

In the last period of life, the individual no longer works and receives the pension benefit

$$p^*_{t+1} = \tau_{t+1}\tilde{w}_{t+1}\left[1 - f(\overline{n}_{t+2})\right]\left[(1-\alpha)\,\overline{n}_{t+1}\frac{1-f(n_{t+1})}{1-f(\overline{n}_{t+1})} + \alpha n_{t+1}\right]. \tag{3.1}$$

Therefore in the old age period, the consumption $z_{t+1}$ of an individual consists of the pension benefit $p^*_{t+1}$ and the individual savings multiplied by the interest rate factor $R_{t+1}$.

Supposing that the individual maximizes her lifetime utility $U$, the pension model in [6] then considers the optimization problem

$$\begin{aligned} \max_{s_t,\, n_{t+1}} \quad & U(c_t, z_{t+1}, n_{t+1}) \\ \text{s.t.} \quad & c_t + s_t = (1-\tau_t)\left[1 - f(n_{t+1})\right]\tilde{w}_t, \\ & z_{t+1} = R_{t+1}s_t + p^*_{t+1}, \end{aligned} \tag{3.2}$$

where $\tilde{w}_t$ represents the wage of a childless individual. The lifetime utility function $U(c_t, z_{t+1}, n_{t+1})$ is considered to be continuous, strictly increasing, and concave in each of the underlying arguments. Moreover, we assume the lifetime utility function to exhibit additive separability (i.e. $U(c_t, z_{t+1}, n_{t+1})$ can be written in the form $U_1(c_t) + U_2(z_{t+1}) + U_3(n_{t+1})$).

Let us assume a homogeneous wage for the childless, i.e. $\tilde{w}_t = \overline{\tilde{w}}_t$ and the resulting individual wage $w_t = \tilde{w}_t\,[1 - f(n_{t+1})]$. Denote by $\overline{f(n_{t+1})}$, $\overline{f(n_{t+2})}$ the average values of $f(n_{t+1})$ and $f(n_{t+2})$ respectively. Then the pension benefit (2.4) can be rewritten as

$$p_{t+1} = \tau_{t+1}\tilde{w}_{t+1}\left[1 - \overline{f(n_{t+2})}\right]\left[(1-\alpha)\,\overline{n}_{t+1}\frac{1 - f(n_{t+1})}{1 - \overline{f(n_{t+1})}} + \alpha n_{t+1}\right].$$

According to the Jensen's inequality for the convex function, $f(\overline{n}_{t+1}) \le \overline{f(n_{t+1})}$ holds true for any loss function $f(n_{t+1})$. Assuming a homogeneous population with $n_t = \overline{n}_t$ or linear form of the loss function, we obtain equality between $f(\overline{n}_{t+1})$ and $\overline{f(n_{t+1})}$. In such a case, the average wage-related pension benefit (2.4) and the pension benefit (3.1) coincide. In the following, we present some important results of the analysis of the optimization problem (3.2) from [6].

First, we consider a homogeneous population with $n_t = \overline{n}_t$. Analysis of the necessary optimality conditions leads to the following effect of the child factor on individual fertility: $\frac{\partial n_{t+1}}{\partial \alpha} > 0$. As expected, an increase in the child factor $\alpha$ leads to a structural change in the pension system with a greater emphasis on the part dependent on individual fertility. This ultimately leads to fertility growth.

This leads to the question about the optimal value of the child factor $\alpha$. By analyzing the indirect utility function

$$\begin{aligned} U\Bigg\{ \quad & (1-\tau_t)\,[1 - f(n_{t+1}(\alpha))]\,w_t - s_t(\alpha), R_{t+1}s_t(\alpha) \\ & +\tau_{t+1}w_{t+1}\,[1 - f(\overline{n}_{t+2}(\alpha))]\left[(1-\alpha)\,\overline{n}_{t+1}(\alpha)\frac{1 - f(n_{t+1}(\alpha))}{1 - f(\overline{n}_{t+1}(\alpha))} + \alpha n_{t+1}(\alpha)\right], n_{t+1}(\alpha)\Bigg\} \end{aligned}$$

authors in [6] conclude that setting the $\alpha = 1$ is not optimal and does not maximize the utility of the individuals. On the other hand, in a steady state equilibrium with stationary sequence $\{\tau_t, \alpha_t, w_t, R_t\}$ and $n = n_{t+1} = \overline{n}_{t+1} = \overline{n}_{t+2}$, if a given pension system does not involve any fertility-related pensions, then the introduction of the child factor leads to an increase in overall utility. At the same time, if there is an internal solution, the optimal value of the child factor meets the following condition

$$\alpha^* = \frac{1 - f(n(\alpha^*))}{1 - f(n(\alpha^*)) + n(\alpha^*)f'(n(\alpha^*))}. \tag{3.3}$$

Since the left-hand side of the previous expression (3.3) is increasing in $\alpha^*$ while the right-hand side decreases in $\alpha^*$, the optimal $\alpha^*$ is unique.

**4. Child factor in Slovakia.** The Slovak pension system is based on two pillars. The first pillar is Pay-As-You-Go and defined benefit, the second pillar is savings, defined contribution. Pension system participants can use only the first pillar, or both. In the second case, the pension from the first pillar is reduced proportionally. For simplicity, we assume the existence of only one pillar throughout this article. Even in such a setting, we can make relevant conclusions regarding the child factor in the pension system in Slovakia.

A parental pension has been implemented in Slovakia since January 2023. However, even before that, this pension system had some form of parental bonus. In Slovakia, the monthly pension benefit is calculated using the following formula

$$D = APWP \times CP \times CPV. \tag{4.1}$$

The variable $CPV$ corresponds to the current pension value. Next, the $CP$ stands for the length of the pension contribution period. This also includes the period of parental leave. Finally, the variable $APWP$ represents the average personal wage point, which is calculated as follows

$$APWP = \frac{\text{sum of the personal wage points in the reference period}}{\text{number of years of the pension insurance period}}.$$

Personal wage points ($PWP$) are further given as a ratio of the personal wage assessment base and the general assessment base in a given year.

To calculate the implied child factor in the Slovak pension system, we assume the pension benefit is consistent with the pension benefit formula (3.1). Therefore the assumption about the homogeneous wage $\tilde{w}_t$ is used. Compared to the actual setting of the pension system in Slovakia, which is the combination of the pension system models presented in the first section, this is a simplification. Still, the results can give us an insight into how the modification of the pension system can impact the child factor, fertility, and consumption. We calculate the implied values of the child factor for several settings of the pension system in Slovakia.

**4.1. Case 1.** To compensate for the lost wage due to a child's upbringing, the Slovak pension system includes an artificial assignment of a constant personal wage point, namely 0.6 per year of parental leave. The bonus may be claimed till the child reaches the age of six.

Let a person has $n_{t+1}$ children, spending the maximum possible period of six years on parental leave with each child. Let us also assume that these periods do not overlap. Then the personal wage points of this person are equal to 1 while active working life and 0.6 while on parental leave. The average personal wage point then satisfies

$$APWP = \frac{\sum_{i=1}^{40-6n_{t+1}} 1 + \sum_{i=1}^{6n_{t+1}} 0.6}{40} = \frac{40 - 6n_{t+1} + 3.6n_{t+1}}{40} = \frac{40 - 2.4n_{t+1}}{40},$$

and the pension benefit equals to

$$D = APWP \times CP \times CPV = (40 - 2.4n_{t+1}) \times CPV. \tag{4.2}$$

Comparing (4.2) with the Fenge-Meier equivalent of the formula for the pension benefit (3.1) gives

$$\begin{aligned} &M \times (40 - 2.4n_{t+1}) \times CPV = \\ &\qquad \tau_{t+1}\tilde{w}_{t+1}\left[1 - f(\overline{n}_{t+2})\right]\left[(1-\alpha)\,\overline{n}_{t+1}\frac{1 - f(n_{t+1})}{1 - f(\overline{n}_{t+1})} + \alpha n_{t+1}\right]. \end{aligned} \tag{4.3}$$

Parameter $M$ stands for the number of months of pension benefits payments. If 40 years represent the maximum number of working years, then 40 years correspond to one unit of working time. When raising $n_{t+1}$ offspring, the individual will be left with

$\frac{40-6n_{t+1}}{40}$ units of work, which corresponds to the value of $f(n_{t+1}) = 6n_{t+1}/40$. In order for the equality (4.3) to be satisfied, one has:

$$CPV = \frac{1}{40M}\tau_{t+1}\tilde{w}_{t+1}(1-\alpha)\,\overline{n}_{t+1}\frac{40-6\overline{n}_{t+2}}{40-6\overline{n}_{t+1}}, \tag{4.4}$$

$$\alpha = \frac{3.6\overline{n}_{t+1}}{40-2.4\overline{n}_{t+1}}. \tag{4.5}$$

At the level of the total fertility rate of 1.59 (corresponds to the value for the Slovak Republic in 2020 according to [5]), i.e., when the individual fertility rate is approximated as $1.59/2 = 0.795$, then the value of the child factor in the given pension system is 7.51%.

Now let us assume a stationary state with time-invariant individual fertility, i.e., $\overline{n}_{t+1} = \overline{n}_{t+2} = n$. Equation (4.4) then takes the form

$$CPV = \frac{1}{40M}\tau_{t+1}\tilde{w}_{t+1}\left(1-\frac{3.6n}{40-2.4n}\right)n.$$

Setting the $M = 240$, $\tau_{t+1} = 22.75\%$, $n = 1.59/2$ and $\tilde{w}_{t+1} = 40 \times 12 \times 1296€$ (1296€ corresponds to the Slovakian average monthly gross wage in September 2022, [2]) gives the current pension value of 10.84€. This value is implied by the balance of the pension system budget. If we refer to the current pension value set to 15.13€ in 2022 in Slovakia according to [12], the stationary state value obtained from our model is significantly lower. The reason is low fertility and a non-zero child factor. This can indicate the unsustainability of the Slovak pension scheme. We introduce two modifications to the Slovak pension system by explicitly involving the child factor in the pension benefit.

**4.2. Case 2.** Let the policy change so that an additional personal pension point is equally redistributed between the parents until the child is 25. The average personal wage point of a person with an individual fertility level $n_{t+1}$ then satisfies

$$APWP = \frac{\sum_{i=1}^{40-6n_{t+1}} 1 + 25n_{t+1}}{40} = \frac{40+19n_{t+1}}{40}.$$

Following the previous procedure the child factor is now

$$\alpha = \frac{25\overline{n}_{t+1}}{40+19\overline{n}_{t+1}}. \tag{4.6}$$

For the assumed individual fertility 1.59/2, the child factor is $\alpha = 39.49\%$. This modification rapidly increased the implied child factor compared to the original case. The subsequent current pension value in the stationary fertility state is 7.09€ implying that introducing such a child factor reduces pension value even more.

**4.3. Case 3.** According to [10], the currently introduced parental pension in the Slovak republic pension scheme is 3% of $G_{t+1}$, which is one-twelfth of the total assessment base (gross wage) of the child for the calendar year two years preceding the relevant calendar year $t+1$ from which the pension contribution was paid. The parental bonus is evenly distributed between both parents. Similar to previous cases, we compare this pension benefit (Case 1 + parental pension) with Fenge-Meier pension formula (3.1) for an individual with $n_{t+1}$ children:

$$M\left[(40-2.4n_{t+1})CPV + 0.03n_{t+1}G_{t+1}\right] = \\ \tau_{t+1}\tilde{w}_{t+1}\left[1-f(\overline{n}_{t+2})\right]\left[(1-\alpha)\,\overline{n}_{t+1}\frac{1-f(n_{t+1})}{1-f(\overline{n}_{t+1})} + \alpha n_{t+1}\right].$$

Substituting for the loss function $f(n_{t+1}) = \frac{6n_{t+1}}{40}$ and using the expression for the $CPV$, which is the same as in (4.4), we have the formula for the child factor

$$\alpha = \frac{1.2MG_{t+1} + 3.6\tau_{t+1}\tilde{w}_{t+1}\overline{n}_{t+1}\frac{40-6\overline{n}_{t+2}}{40-6\overline{n}_{t+1}}}{\tau_{t+1}\tilde{w}_{t+1}\left(40 - 6\overline{n}_{t+2}\right) + 3.6\tau_{t+1}\tilde{w}_{t+1}\overline{n}_{t+1}\frac{40-6\overline{n}_{t+2}}{40-6\overline{n}_{t+1}}}. \tag{4.7}$$

In the fertility stationary state, i.e. $\overline{n}_{t+2} = \overline{n}_{t+1} = n$, the expression (4.7) is simplified to

$$\alpha = \frac{\frac{1.2MG_{t+1}}{\tau_{t+1}\tilde{w}_{t+1}} + 3.6n}{40 - 2.4n}.$$

Let $M = 240$ months. For simplicity, let's further assume that the monthly gross wage, with an expected career of 40 years, can be approximated as

$$G_{t+1} = \frac{\tilde{w}_{t+1}(1 - 6n/40)}{40 \times 12}.$$

Then the child factor can be expressed as

$$\alpha = \frac{\frac{0.6(40-6n)}{40\tau_{t+1}} + 3.6n}{40 - 2.4n}. \tag{4.8}$$

With the overall value of the pension contribution rate set to 22.75% and for $n = 1.59/2$, the child factor $\alpha$ is 13.61%. The underlying $CPV$ in the stationary fertility state is 10.12€.

**4.4. Optimal setting of the pension system.** Let the utility function be in the form of weighted sum of logarithms of its arguments, i.e.,

$$U(c_t, z_{t+1}, n_{t+1}) = \ln c_t + \delta \ln z_{t+1} + \gamma \ln n_{t+1},$$

where $\delta$ is a discount factor representing a time preference of the consumption. According to [1], the parameter $\gamma$ states an individual's preferences or motivation to have a child.

Inspired by the form of the loss function from the previous part, where $f(n_{t+1}) = \frac{6}{40}n_{t+1}$ was considered, we now choose $f(n_{t+1}) = an_{t+1}$, where $a \in (0,1)$. The coefficient $a$ represents the cost of bringing up one child. In such a case, where the loss function is linear, we have $f(\overline{n}_{t+1}) = \overline{f(n_{t+1})}$. The following first-order conditions determine the optimal decisions:

$$\begin{aligned}\frac{\partial U}{\partial s_t} &= -\frac{1}{c_t} + \frac{\delta R_{t+1}}{z_{t+1}} \\ &= \frac{-1}{(1-\tau_t)\left[1 - f(n_{t+1})\right]\tilde{w}_t - s_t} \\ &+ \frac{\delta R_{t+1}}{R_{t+1}s_t + \tau_{t+1}\tilde{w}_{t+1}\left[1 - f(\overline{n}_{t+2})\right]\left[(1-\alpha)\,\overline{n}_{t+1}\frac{1-f(n_{t+1})}{1-f(\overline{n}_{t+1})} + \alpha n_{t+1}\right]} = 0,\end{aligned} \tag{4.9}$$

$$\begin{aligned}\frac{\partial U}{\partial n_{t+1}} &= \frac{-(1-\tau_t)\,f'(n_{t+1})\tilde{w}_t}{(1-\tau_t)\left[1 - f(n_{t+1})\right]\tilde{w}_t - s_t} \\ &+ \frac{\delta\tau_{t+1}\tilde{w}_{t+1}\left[1 - f(\overline{n}_{t+2})\right]\left[\alpha - (1-\alpha)\,\overline{n}_{t+1}\frac{f'(n_{t+1})}{1-f(\overline{n}_{t+1})}\right]}{R_{t+1}s_t + \tau_{t+1}\tilde{w}_{t+1}\left[1 - f(\overline{n}_{t+2})\right]\left[(1-\alpha)\,\overline{n}_{t+1}\frac{1-f(n_{t+1})}{1-f(\overline{n}_{t+1})} + \alpha n_{t+1}\right]} + \frac{\gamma}{n_{t+1}} = 0.\end{aligned} \tag{4.10}$$

Consider the equilibrium state where the variables $\tau_t$, $R_t$, $\tilde{w}_t$, $s_t$, and $n_t$ are constants over time. In this case, equations (4.9) and (4.10) can be written as

$$\frac{\partial U}{\partial s} = \frac{-1}{(1-\tau)(1-an)\tilde{w} - s} + \frac{\delta R}{Rs + \tau\tilde{w}(1-an)n} = 0, \tag{4.11}$$

$$\frac{\partial U}{\partial n} = \frac{-(1-\tau)a\tilde{w}}{(1-\tau)(1-an)\tilde{w} - s} + \frac{\delta\tau\tilde{w}(\alpha - an)}{Rs + \tau\tilde{w}(1-an)n} + \frac{\gamma}{n} = 0. \tag{4.12}$$

Using equations (4.9)-(4.10), we have calculated the optimal values of $n^*_{t+1}$, $s^*_t$, the implied values of $p^*_{t+1}$, $z^*_{t+1}$ and the total utility $U$ for Cases 1-3 (see above) of the setting of the Slovak pension system. The results can be found in Table 4.1. Subsequently, we have substituted the formulas (4.5)-(4.6) and (4.8) into the equation (4.12), and by solving the equations (4.11)-(4.12), we have calculated the optimal values of $n^*$, $s^*$, the implied values of $p^*$, $z^*$, the equilibrium total utility $U = U_e$ and the equilibrium value of $\alpha = \alpha_e$ for the equilibrium states corresponding to Cases 1-3. The results are in Table 4.2.

Model calibration is as follows. The interest rate factor $R$ was set to 2, which according to [4] corresponds to the return of equal contributions over a lifetime working career with moderate risk aversion. At the same time, the wages of individuals were normalized to one unit, i.e., $\tilde{w}_t = \tilde{w}_{t+1} = \tilde{w} = 1$. Therefore the optimal values of the individual savings correspond to the percentage of their wage. Since according to the equation (4.9), we have: $z_{t+1} = \delta R c_t$, we chose $\delta = 1/4$, because we assume about half the period of receiving the pension compared to the length of the working career, and therefore $z_{t+1} = c_t/2$. Since the Slovak pension system was set according to Case 1 for many years, we set the value $\gamma = 0.1581$, at which in this case the implied value $\alpha$ merges with the equilibrium value $\alpha_e$. As in Cases 1-3, we set the values $a = 6/40 = 0.15$ and $\bar{n}_{t+1} = 1.59/2$.

| Case | $\alpha$ | $n^*_{t+1}$ | $s^*_t$ | $p^*_{t+1}$ | $z^*_{t+1}$ | $U$ |
|---|---|---|---|---|---|---|
| 1 | 7.51% | 0.7951 | 0.0724 | 0.1593 | 0.3040 | -0.8315 |
| 2 | 39.49% | 1.0854 | 0.0586 | 0.1768 | 0.2941 | -0.8239 |
| 3 | 13.61% | 0.8381 | 0.0713 | 0.1595 | 0.3020 | -0.8312 |

Table 4.1

*Optimal values of $n^*_{t+1}$, $s^*_t$, the implied values of $p^*_{t+1}$, $z^*_{t+1}$ and the total utility $U$ for Cases 1-3 of the setting of the Slovak pension system.*

| Case | $\gamma$ | $\alpha_e$ | $n^*$ | $s^*$ | $p^*$ | $z^*$ | $U_e$ |
|---|---|---|---|---|---|---|---|
| 1 | 0.1581 | 7.51% | 0.7951 | 0.0723 | 0.1593 | 0.3040 | -0.8315 |
| 2 | 0.1581 | 46.31% | 1.1435 | 0.0418 | 0.2155 | 0.2991 | -0.7944 |
| 3 | 0.1581 | 14.03% | 0.8399 | 0.0682 | 0.1670 | 0.3035 | -0.8250 |
| Optimum | 0.1581 | 75.88% | 1.6083 | 0.0062 | 0.2776 | 0.2900 | -0.7792 |
| 2 | 0.3 | 66.47% | 2.1496 | -0.0279 | 0.3313 | 0.2756 | -0.6881 |
| Optimum | 0.3 | 67.48% | 2.1683 | -0.0289 | 0.3328 | 0.2750 | -0.6881 |

Table 4.2

*Optimal values of $n^*$, $s^*$, implied values of $p^*$, $z^*$, $U_e$ and $\alpha = \alpha_e$ for equilibrium states.*

The results show that the equilibrium and implied values of $\alpha$ are not very different from each other. The value of consumption in retirement is close to 0.3 in all cases. The optimal number of children varies more significantly. As the child factor $\alpha$ increases, the optimal number of children and, of course, the pension benefit $p$ increases, while savings $s$ decrease. With approximately the same consumption in retirement, the increasing number of children is thus financed by a reduction in savings. As the child factor increases, lifetime utility also increases. The highest is in Case 2, when the number of children is the highest linked to the pension benefit.

According to the presented model, the equation (3.3) for optimal equilibrium child factor $\alpha^*$ can be simplified to the form $\alpha^* = 1 - an(\alpha^*)$. Inserting $\alpha = 1 - an$ into (4.11) - (4.12) one can calculate the values of $n^*$, $s^*$, the implied values of $p^*$, $z^*$, $U_e$ and the equilibrium value of $\alpha = \alpha_e$ for the optimal equilibrium state. One can observe (see Table 4.2) that $\alpha_e = 75.88\%$ is optimal for $\gamma = 0.1581$. Table 4.2 shows Case 2 is closest to this with the lifetime utility $U_e$ close to the optimal one. From January 1, 2023, the setting of the Slovak pension system is similar to Case 3. It can be seen that, compared to Case 1, fertility in the equilibrium state has increased slightly, but it still does not have a sufficient level. An acceptable value can be observed in Case 2. The current setting of the parental bonus in Slovakia can therefore be brought closer to the optimum, for example, by a more generous distribution of personal wage points for raising children. According to the presented model, we thereby create conditions for increasing fertility. The results from Table 4.2 confirm that for equilibrium states, the optimal number of children is increasing as a function of the child factor. This is consistent with the theoretical conclusions of Section 3.

We have also calculated the corresponding values for $\gamma = 0.3$. Results in Table 4.2 show that for $\gamma = 0.3$, Case 2 is close to the optimum. However, it should be added that in this case, unrealistically high number of children in the equilibrium state associated with negative savings values $s^*$ (which means borrowing instead of saving) emerge. The value $\gamma = 0.3$ is therefore not realistic.

**5. Conclusions.** We have presented two models of the pension system with a child factor. In the first model, pension benefits were tied to the wages of the descendants of the pension system participant. Such a setting is risky because of possible problems of descendants (unemployment, resettlement, etc.). In the second model, benefits were tied to the number of offspring and the average salary of all participants, removing the first setup's disadvantages.

The model with benefits tied to average wages has been rewritten into the formulation of the model of [6]. Our results have proved that (in the equilibrium state) as the child factor increases, so does fertility.

We have calculated the implied size of the child factor for three settings of the pension system in Slovakia. Based on the Fenge-Meier model, we have calculated the value of the optimal child factor. The system can be brought closer to the optimum by a more generous distribution of personal wage points for raising children.

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