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PRECISING OF THE VERTEBRAL BODY GEOMETRY BY USING BÉZIER CURVES*

MÁRIA TJEŠŠOVÁ† AND MÁRIA MINÁROVÁ‡

Abstract. The biomechanical investigation of the human spine requires first of all the proper geometry of the focused object. The paper deals with a geometrical model developing of the human lumbar vertebral body by using Bézier curves - the tool for the curves creation of demanded shape and properties. The background theory and algorithm for the Bézier curves and their compound generation are introduced, the resulting geometry and the corresponding biomechanical computation results are performed.

Key words. human lumbar spine, functional spinal unit, Bézier curve, Berstein polynomial, De-Casteljau algorithm

AMS subject classifications. 74S05, 65D17

1. Introduction. Nowadays, when the widespread diseases of the spine still persist, it is important to improve bit a bit the understanding of the spine reaction (physiological and pathological) to a load. As the lumbar spine is the most afflicted one within the entire human spine, the special medical and biomechanical attention is devoted to it. The research, the part of which is described in the paper, is aimed to the lumbar spine behaviour investigation under the various types of load. The motion segment (functional spinal unit), see Fig.1.1 - two neighbouring lumbar vertebrae and the intervertebral disc between - is taken into the consideration. Especially, the weight bearing part of the vertebra (vertebral body), see the Fig.1.2 and its shape influencing the functionality of the spine is focused herein. Accordingly, as the vertebral body and the neighbouring disc are in body to body fusion, they both match well (have the same horizontal profile) at the location of the contact.

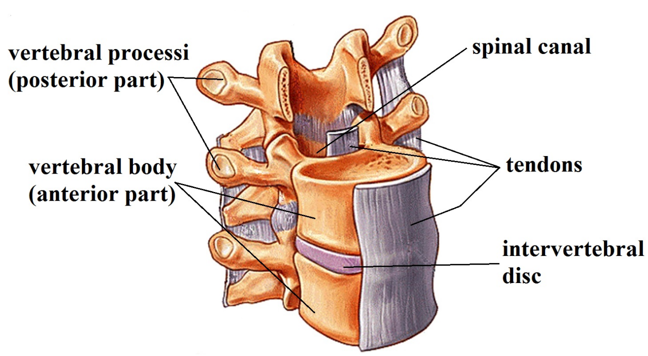

FIGURE 1.1. *Functional spinal unit - motion segment of the human spine*

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†Department of Mathematics and Descriptive Geometry, Slovak University of Technology, Faculty of Civil Engineering, Bratislava 81005, Slovakia (maria.tjessova@stuba.sk).

‡Department of Mathematics and Descriptive Geometry, Slovak University of Technology, Faculty of Civil Engineering, Bratislava 81005, Slovakia (maria.minarova@stuba.sk).

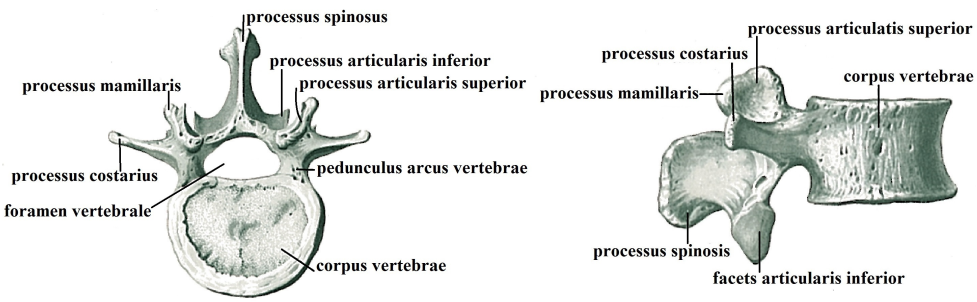

FIGURE 1.2. *Anatomy of lumbar vertebra - frontal view (left) and lateral view (right) [7]*

2. Morphology of the lumbar vertebra and disc. The morphology of the particular vertebrae differs alongside the spine as the functionality of particular vertebrae is different. (The lower position of vertebra, the greater weight bearing capacity is expected.) Like each bone, the vertebra consists of the cortical and cancellous bone tissue. The stiffer and harder cortical bone encloses the softer feeding cancellous bone inside. The composition together with assigned functionality determines the morphology of the particular vertebra.
The anterior part of the lumbar vertebra, the corpus vertebrae (vertebral body), see Fig.1.2, is robust and compact. On the other hand, the posterior part is branched, involving the processi and facets. The two (anterior and posterior) parts are configured in a way that they form a hole between. The functionality of entire spine and its parts determine the shape of the vertebral body and intervertebral discs, as well. [2] The hole between the anterior (frontal) an posterior (back) parts of the vertebrae called foramen, see Fig.1.2, that together with the foramens of all vertebrae alongside the entire spine form the spinal canal which encloses and protects the spinal chord inside the canal. [8]

3. Bézier curves.

3.1. Modelling of the vertebral body border by using Bézier curves. For the sake of better accuracy of further biomechanical analysis (further intended exploring the mechanical response to the load first of all) it is essential to arrange as precise geometry as possible. At the same time, the geometry should match with proportions of the particular patient anatomy (including anomalies, deformities, diseases, etc.). In our case (the healthy spine is being modelled) the vertebral body shape consists of three Bézier curves. Two of them bound the base of the vertebral body and the third one, the lateral meridian curve, describes the lateral curvature of the vertebra, see Fig.3.1. Herein, each horizontal sections of the vertebral body yields the curve of the same shape as the base border curve is. Regarding the further finite element analysis, it is worthwhile to make the horizontal sections profile at that levels in which the intended nodes of further finite element meshing will be placed. At the same time, the scale factor is determined by the lateral meridian curve. Therefore the vertebra outer shape is designed by shifting and scaling both horizontal curves along the lateral curve.

DEFINITION 3.1. *Let $V_0, V_1, V_2, ..., V_n$ be a sequence of points in a space $E(E^2, E^3)$ (control polygon points). Then a set of points from the space E coordinates of which read the equation:*

$$b^n(t) = \sum_{i=0}^{n} V_i B_{in}(t), \tag{3.1}$$

where

$$B_{in}(t) = \binom{n}{i} t^i (1-t)^{n-i}, i = 1, 2, ..., n, t \in \langle 0, 1 \rangle \tag{3.2}$$

are named Bernstein basis polynomial is called Bézier curve of degree n. [4]

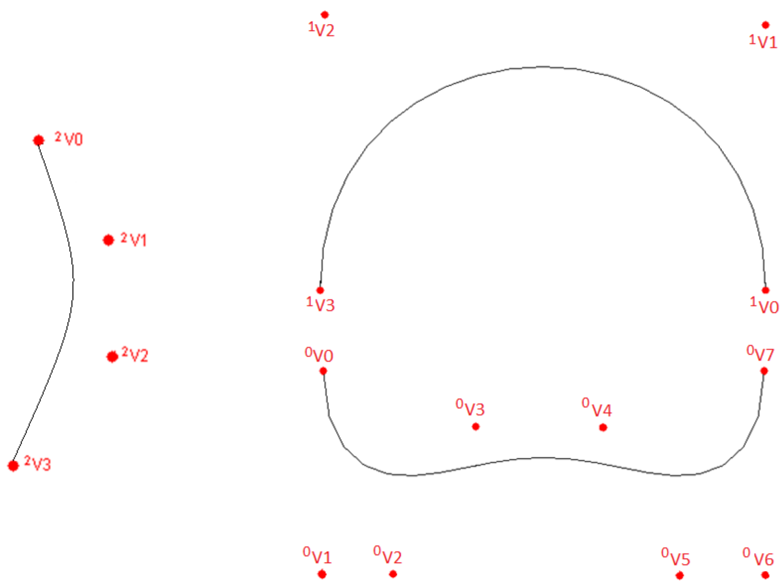

FIGURE 3.1. *Control points and Bézier curves describing the vertebral body shape.*

During the modelling of the vertebral body the following properties of the Bézier curves are utilized:[3, 1]

A. Interpolation of the two outermost points of the control polygon:
$b^n(0) = V_0$, $b^n(1) = V_n$, i.e. Bézier curve passes through the first and the last point of the control polygon.

B. Convex hull:
Bézier curve $b^n(t)$ lies within the convex envelope of its control points $V_0, V_1, ...V_n$ ($B_{in}(t)$ are non-negative polynomials of parameter $t \in \langle 0, 1 \rangle$ and $b^n(t)$ is convex combination of control points V_i).

C. Symmetry:

$$\sum_{i=0}^{n} V_i B_{in}(t) = \sum_{i=0}^{n} V_{n-i} B_{in}(1-t) \tag{3.3}$$

i.e. the curve will not change due to order of control points is switched and parameter t runs from 1 to 0 at the same time.

D. Affinity:
Bézier curves are invariant as far as the affine transformation concerned, i.e. every affine transformation T holds:

$$T(\sum_{i=0}^{n} V_i B_{in}(t)) = \sum_{i=0}^{n} T(V_i) B_{in}(t) \tag{3.4}$$

which means that affine image of Bézier curve specified by control points $V_0, V_1, ..., V_n$ will be the same as the Bézier curve specified by the affine images of the control points of the original control polygon.

3.2. Calculating and developing of the Bézier curves - De Casteljau's algorithm (CA) [4]. The curve at point t can be represented by the recurrence relation:

$$B_{in}(t) = (1-t)B_{in-1}(t) + tB_{i-1n-1}(t). \tag{3.5}$$

The algorithm for n control points $V_0, V_1, ...V_n$ can be sketched as follows:
Input: control points $V_0, V_1, ...V_n$ and parameter $t \in <0,1>$

0) $V_i^0(t) = V_i \quad i = 0, 1, ..., n$
1) $V_i^1(t) = (1-t)V_i^0(t) + tV_{i+1}^0(t) \quad i = 0, 1, ..., n-1$
   $\vdots$
r) $V_i^r(t) = (1-t)V_i^{r-1}(t) + tV_{i+1}^{r-1}(t) \quad r = \{1, 2, ..., n\} \quad i = 0, 1, ..., n-r$
   $\vdots$
n-1) $V_0^{n-1}(t) = (1-t)V_0^{n-2}(t) + tV_1^{n-2}(t), \quad V_1^{n-1} = (1-t)V_1^{n-2}(t) + tV_2^{n-2}(t)$
n) $V_0^n(t) = (1-t)V_0^{n-1}(t) + tV_1^{n-1}(t)$

Output: point of the curve $b^n(t) = V_0^n(t)$

Algorithm for calculation of a point $b^n(t)$ for fixed $t \in <0,1>$ of the future curve can be written as follows:

- load the points $V_0, V_1, V_2, ..., V_n$, put $V_0^0 = V_0, V_0^1 = V_1, ..., V_0^n = V_n$
- $t = 0$
- choose $\triangle t$
- $b^0 = V_0$
- repeat until $t < 1$:
  - ⋄ $t = t + \triangle t$
  - ⋄ repeat for $r = 1, ..., n$
    - ∗ repeat for $i = 0, ..., n-r$
      - · $V_i^r = (1-t)V_i^{r-1} + tV_{i+1}^{r-1}$
  - ⋄ draw segment $b^0 V_0^n$
  - ⋄ $b^0 = V_0^n$

Let us note that during the creation of the Bézier curve of degree n by using CA in fixed point $t \in \langle 0, 1\rangle$:

a) on the each side of the segment $V_i V_{i+1}$ of its control polygon point $V_i^1(t) = (1-t)V_i + tV_{i+1}$ is created.
b) for each $r \in 1, 2, ..., n$ on each edge $V_i^{r-1}(t)V_{i+1}^{r-1}(t)$ the point $V_i^r(t) = (1-t)V_i^{r-1}(t) + tV_{i+1}^{r-1}(t)$ is created.

This process of creation of points is called linear interpolation.

3.3. Continuity and smoothness of Bézier curves and their compositions. For the inscription (to the control polygon) of the curves with more complicated shape (with a larger number of control points) there is a need of utilization of

Bézier curves of a higher degree. The mentioned approach is often too cumbersome and the better way to solve the problem conveniently is to to combine a couple of Bézier curves in a junction of segments of the simple curvatures satisfactory fulfilling the smooth criteria.
Let ${}^0S$ be a segment of Bézier curve with m control points ${}^0V_0, {}^0V_1, ..., {}^0V_m$:
${}^0S$: ${}^0b(t) = \sum_{i=0}^{m} {}^0V_i B_{im}(t), t \in \langle 0, 1\rangle$
and we are attempting to connect the curve ${}^0S$ with another curve
${}^1S$: ${}^1b(t) = \sum_{i=0}^{n} {}^1V_i B_{in}(t), t \in \langle 0, 1\rangle$
with n control points ${}^1V_0, {}^1V_1, ..., {}^1V_n$
and realize the connection "sufficiently smoothly".
We use two types of geometrical connection of Bézier curves of different order:

a) 0^{th} order connection G^0: ${}^0b(1) = {}^1b(0) \Leftrightarrow {}^0V_m = {}^1V_0$
b) 1^{st} order connection G^1: ${}^1b'(0) = \beta 1 {}^0b'(0)$, $\beta 1 > 0 \Leftrightarrow {}^1V_1 = {}^0V_m + \beta 1({}^0V_n - {}^0V_{n-1})$, point ${}^1V_1$ is point of the segment ${}^0V_{m-1}\,{}^0V_m$.

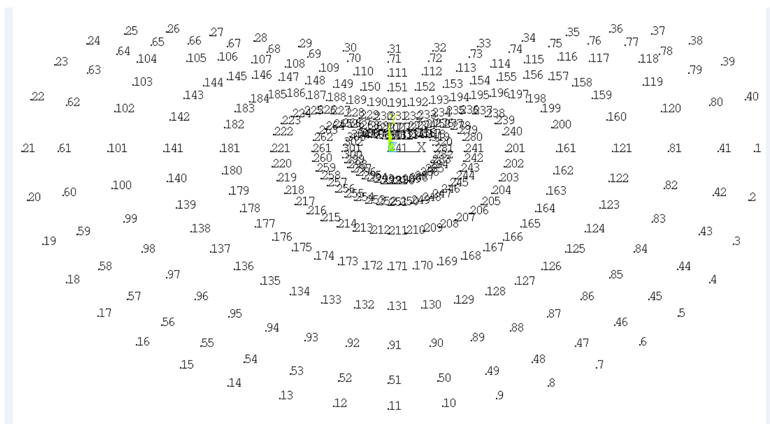

FIGURE 3.2. *The nodes of the meshed base of the vertebra*

In the case of designing the geometry of a vertebra we join the two coplanar curves with aim of forming the base of vertebra. We use the 1^{st} order connection G^1 here. The connection is realized within the control points ${}^0V_0 = {}^1V_3$ and ${}^0V_7 = {}^1V_1$. Then, in 3D space we merge the lateral curve with just accomplished vertebral base by using the 0^{th} order connection (the lateral curve lies in the plane perpendicular to the base plane). The connection G^0 is realized in the control points ${}^0V_0 = {}^1V_0 = {}^2V_0$.

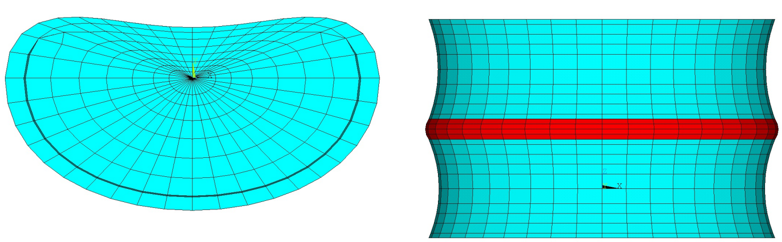

FIGURE 3.3. *Meshed base of the vertebra - the top view (left), frontal view of the motion segment meshing (right)*

4. Illustration of the FEM calculation based on the geometry developed by using Bézier curves. The biomechanical computations are executed on the geometry created by using De-Casteljau's algorithm with combination of the generation of the batch file for FEM modelling (the configuration of the nodes and elements). Within the direct geometry modelling the nodes and elements are created by scaling the border line nodes firstly (shrinking the boundary nodes inwards within the base plane), and shifting together with scaling of the entire base nodes set alongside the lateral governing curve afterwards, see Fig.3.2. When meshed, then material properties are assigned, constraints and loads are applied and the solution is executed. [5, 6] We provide some illustrative results bellow, see Fig. 4.1 - 4.3

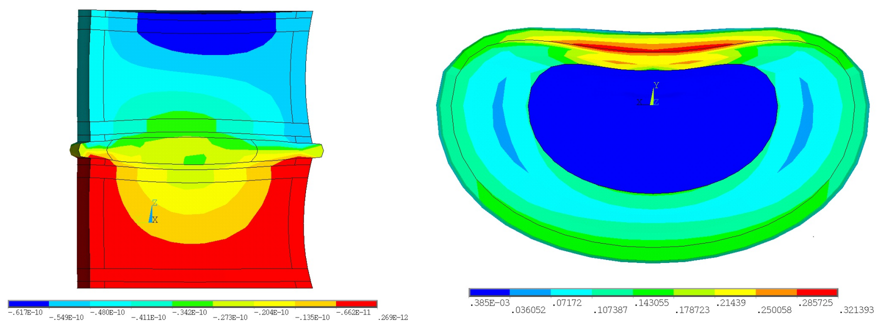

FIGURE 4.1. *Deflection isolines on the motion segment under the uniform distributed load (left) and the stress isolines on the intervertebral disc (right)*

The theoretical biomechanical investigation often reveals the weak places of the focused object. On the Fig.4.1 there is such a place located around the sunken border of the intervertebral discs with significantly higher stress values - this place neighbouring with the foramen, i.e. with spinal cord. As the medical practice shows, this is the place where the most often the disc rupture happens and resulting bulb pushes to the spinal cord causing the pain. Right in such a kind of exploring the shape of the biological object is essential.

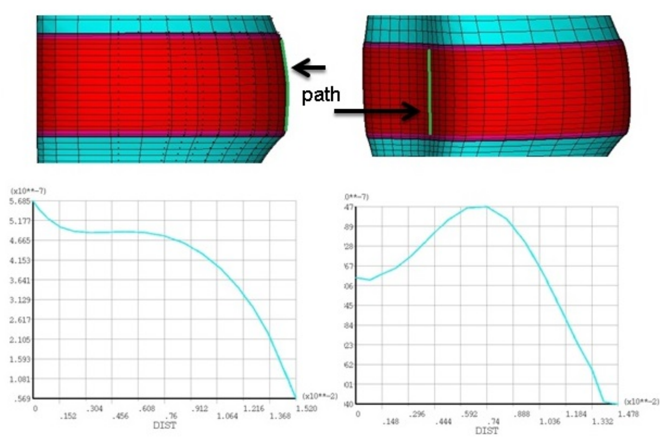

FIGURE 4.2. *Displacement values alongthe paths on the intervertebral disc*

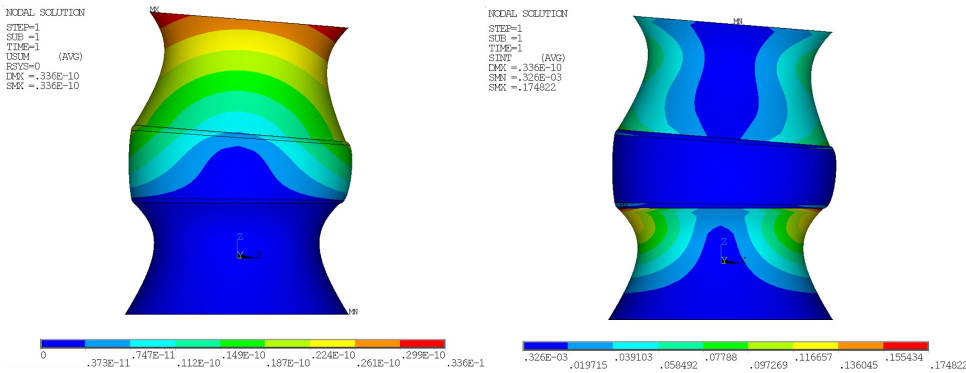

Figure 4.3. *Motion segment under the oblique load: displacement isolines (left), stress isolines (right)*

5. Conclusion. The biomechanical investigation of the human spine undoubtedly contributes to better understanding the behaviour of it and can help to find out the reasons of deformities, pain and diseases; to predict problems or stipulate the appropriate therapy. For the sake of more precise theoretical biomechanical treatment, and with the aim of enabling the comparison of the theoretical results with experimental data in vitro, it is essential to build up the sufficiently precise geometrical model involving the proportions matching the individual anatomy of the particular patient. Bézier curves approach is one of tools.

REFERENCES

[1] W. Cheney, D.Kincaid, *Numerical Mathematics and Computing*, fifth edition, Thomsom Learning (2004).
[2] W. Herzog, *Clinical biomechanics of spinal manipulation*, Churcill livingstone (2000).
[3] *http://www.ams.org/samplings/feature-column/fcarc-bezier*.
[4] *http://fractal.dam.fmph.uniba.sk/ kg/kudlickova/*
[5] M. Kurutz, *Finite Element Analysis*, David Moratal (Ed.) (2001).
[6] M. Minárová. j. Sumec, M. Tješšová, *Behavior of the Intervertebral Disc within the Motion Segment L3-L4 of the Human Spine under Various Types of Physiological Load.* Transactions os the VB - TU Ostrava, Civil Engineering series, vol. 11, No.2 (2012).
[7] D. sinelnikov, *Atlas of human anatomy*, Avicenum/Mir, Prague/Moscow (1980).
[8] J. Sumec, M.Sokol, J. Jíra, *Computer simulation of lumbar spine function*, Proc. IAESTED conference Biomechanics, pp. 109-113 (2003).
[9] M. Tješšová, *Geometrical model of the human lumbar spine and its biomechanical response*, Diploma thesis SUT Bratislava, (2014)