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CUBIC CAYLEY GRAPHS OF GIRTH AT MOST 6 AND THEIR HAMILTONICITY

E. ABOOMAHIGIR and R. NEDELA

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Abstract. Thomassen's conjecture states that a cubic graph with sufficiently large cyclic connectivity is hamiltonian. Even the following strong conjecture could hold: A cyclically 7-connected cubic graph is hamiltonian, or it is the Coxeter graph. Assuming the conjecture holds true, to prove the hamiltonicity of cubic Cayley graphs it is sufficient to examine cubic Cayley graphs of girth at most 6. Motivated by this, we characterise cubic Cayley graphs of girth at most six and identify few "hard families" of cubic Cayley graphs of small girth for which we are not able to verify whether they are hamiltonian.

1. Introduction

Let S be a set that generates a finite group G such that $1 \notin S$ and $S = S^{-1}$. The *Cayley* graph of S in G , denoted $X = \mathrm{Cay}(G; S)$, is a graph whose vertices are the elements of G and the adjacency relation is defined as follows: $x \in G$ is adjacent to $y \in G$ if and only if $y = xs$ for some $s \in S$. In 1969, Lovász [**15**] asked whether every finite connected vertex-transitive graph contains a Hamilton path, that is, a simple path visiting each vertex exactly once. A cycle passing through all vertices of G is a Hamilton cycle of G. Every examined finite connected vertex-transitive graph admits a Hamilton cycle except the five known counterexamples, these are: the complete graph K_2, the Petersen graph, the Coxeter graph and two graphs derived from the Petersen and Coxeter graphs by replacing each vertex with a triangle. Apart from K_2, all of these are cubic graphs. However, none of these four graphs is a Cayley graph, that is, a vertex-transitive graph with a regular subgroup of automorphisms. This has led to a folklore conjecture that every connected Cayley graph possesses a Hamilton cycle. Thomassen [**8, 21**]

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conjectured that only finitely many connected vertex-transitive graphs without a Hamilton cycle exist, in contrast, Babai in [**6, 7**] conjectured that infinitely many such graphs exist.

Let X be a connected graph. A subset $F \subseteq E(X)$ of edges of X is said to be cycle-separating if $X - F$ is disconnected and at least two of its components contain cycles. We say that X is *cyclically k-connected*, if no set of fewer than k edges is cycle-separating in X. The edge cyclic connectivity $\zeta(X)$ of X is the largest integer k not exceeding the *Betti number* $\beta(X) = |E(X)| - |V(X)| + 1$ of X for which X is cyclically k-edge-connected. If X has no cycle-separating edge cut we set $\zeta(X) = \beta(X)$. Note that for cubic graphs the concepts of edge cyclic connectivity and vertex cyclic connectivity coincide.

All known non-hamiltonian cubic graphs have cyclic-connectivity at most 7. The only known example which is cyclically 7-connected is the Coxeter graph. Thomassen [**22**] has conjectured that if the cyclic connectivity of a cubic graph X is large enough, then X is hamiltonian.

Conjecture 1.1 (Thomassen [**22**]). Cubic graphs with sufficiently large cyclic connectivity are hamiltonian

Since there are infinitely many non-hamiltonian cyclically 6-connected cubic graphs (for instance the Isaacs flower snarks), the strongest version of Thomassen's conjecture reads as follows.

Conjecture 1.2. A cyclically 7-connected cubic graph contains a Hamilton cycle, or it is the Coxeter graph.

Assuming that Conjecture 1.2 is confirmed in the affirmative, to prove Lovász's conjecture for cubic Cayley graphs, it is sufficient to examine cubic Cayley graphs of $\zeta(X) \leq 6$. In 1995, Nedela and Škoviera [**18**] proved that the cyclic connectivity $\zeta(X)$ of a cubic vertex-transitive graph X equals its girth $g(X)$. Recall that the *girth* of X length of a shortest cycle in X. It follows that it is sufficient to consider cubic Cayley graphs with $g(X) \leq 6$. Since the action of the Cayley group is regular, the existence of girth cycles of length at most 6 implies a relation of length at most 6 in terms of the generators. Since the number of generators is at most 3, there are just finitely many cases for the relation to consider. In what follows we present case-by-case analysis according to the short relation. In most cases we obtain strong restrictions on the structure of the Cayley group, and consequently, on the structure of the Cayley graphs in consideration. Then we examine whether these cubic Cayley graphs are hamiltonian. The analysis splits into two main cases depending on the structure of the generating set S of the Cayley group G:

Type 1. $S = \{a, b, c\}$, where a, b and c are involutions, that is $a^2 = b^2 = c^2 = 1$,
Type 2. $S = \{a, b, b^{-1}\}$, where $a^2 = 1$ is an involution, and the order of b is more than 2.

The main result of the paper is stated in Theorems 3.1 and 4.1.

The reader may find some similarities to our analysis of cubic Cayley graphs of small girth in the book [**11**]. However, the problem considered there is different.

Moreover, the obtained partial results in [**11**] are not stated in the form ready for direct use for our purposes.

2. Preliminaries and examples

Obvious examples of cubic Cayley graphs of small girth are the n-sided prisms and the Mobius ladders. The n-sided prism can be defined as a Cayley graph $\mathrm{Cay}(G; y, x^{\pm 1})$ for $G = C_n \times C_2 = \langle x \rangle \times \langle y \rangle$, where x is of order n and $y \notin \langle x \rangle$ is an involution. The Möbius ladder of even order n is a circulant $\mathrm{Cay}(Z_n; n/2, \pm 1)$. Both families of graphs have girth at most four. In general, a Cayley graph based on an abelian group has girth at most four. A cubic Cayley graph based on an abelian group of girth 3 is either K_4, or the 3-sided prism.

In what follows we list statements establishing hamiltonicity of distinguished sets of cubic Cayley graphs. The following statement is a consequence of a stronger result known under the name Chen-Quimpo theorem.

Theorem 2.1 ([**9**]). *Every connected Cayley graph of an abelian group of order at least three is hamiltonian.*

For the cubic Cayley graphs of girth four, the following two statements will be useful.

Theorem 2.2 ([**20**]). *There is a Hamilton cycle in every cubic Cayley graph of girth four.*

Theorem 2.3 ([**19**]). *Let G be a finite group, generated by three involutions a, b, c. Suppose $ab = ba$. Then the Cayley graph $\mathrm{Cay}(G, \{a, b, c\})$ contains a Hamilton cycle.*

As concerns the existence of a Hamilton cycle in Cayley graphs based on metacyclic groups, we present the following two results.

Theorem 2.4 ([**4**]). *Every connected cubic Cayley graph on a dihedral group is hamiltonian.*

For each integer r such that $r^n \equiv 1 \pmod{m}$, there is a semi-direct product of $\mathbb{Z}_m$ with $\mathbb{Z}_n$, the cyclic groups of orders m and n, respectively. It is the group $G = \langle x, y \mid x^m = y^n = 1, y^{-1}xy = x^r \rangle$.

Corollary 2.5 ([**1**]). *The Cayley graph on every semi-direct product $\mathbb{Z}_m \rtimes \mathbb{Z}_n$ of two cyclic groups of orders m and n (other than $m = 1, n = 1$ or 2; or $m = 2$, $n = 1$) with the standard generating set has a Hamilton cycle.*

An interesting sporadic example of this sort is the generalised Petersen graph $GP(8, 3)$ which is a Cayley graph $\mathrm{Cay}(G; a, b^{\pm 1})$, where $G = \langle a, b \mid a^2 = b^8 = 1, aba = b^3 \rangle$.

Honeycomb graphs. Honeycomb graphs are the cubic graphs which admit a hexagonal embedding in the torus. They can be defined by means of three integer parameters. Alspach and Dean in [**3**] proved that they are Cayley graphs. The existence of a Hamiltonian cycle of the honeycomb graphs was investigated by several authors, partial results can be found in [**5, 4, 2**]. Finally, Yang et al. [**23**] proved that the honeycomb graphs are hamiltonian.

Theorem 2.6 ([**23**])**.** *Honeycomb graphs are hamiltonian.*

All the previous examples of cubic Caley graphs of girth at most 6 are based on solvable groups. However, there are infinitely many examples, where the Cayley groups are unsolvable; they can be even simple non-abelian. For instance, such graphs of type 2 can be constructed as the torsion-free quotients of the triangle groups

$$\Delta(2,m,n) = \langle a, b \mid a^2 = b^m = (ab)^n \rangle.$$

For each $m \in \{3,5,6\}$ and for each $n \geq 6$, there are infinitely many finite torsion-free quotients $\bar{G}$, giving rise to cubic Cayley graphs $\mathrm{Cay}(\bar{G}; \bar{a}, \bar{b}^{\pm 1})$ of girth m, where by $\bar{a}$ and $\bar{b}$ we denote the images of the generators of G in the natural projection $G \to \bar{G}$. This is a consequence of the fact that provided $\frac{1}{m} + \frac{1}{n} \leq \frac{1}{2}$, the group $\Delta(2,m,n)$ is an infinite residually finite group. The examination of Hamilton cycles of large graphs from this family of graphs seems to be notoriously hard to handle.

3. Cayley graphs of type 1

In what follows we assume that $X = \mathrm{Cay}(G,S)$ is a finite cubic Cayley graph, where S consists of three distinct involutions a, b, and c. Clearly, the girth of X is at most the minimum of the orders of ab, bc and ac. without loss of generality we assume that $|ab| \leq |bc| \leq |ac|$.

We first characterise the cubic Cayley graphs of type 1 of girth at most six.

Proposition 3.1. *Let X be a Cayley cubic graph $X = \mathrm{Cay}(G; a, b, c)$, where $a^2 = b^2 = c^2 = 1$ and $|ab| \leq |bc| \leq |ac|$. Let the girth of X be at most six. Then one of the following happen*

- *$g(X) = 3$, $G = C_2 \times C_2$, and $X \cong K_4$,*
- *$g(X) = 4$, and $(ab)^2 = 1$,*
- *$g(X) = 6$, $(abc)^2 = 1$, and X is the honeycomb graph,*
- *$g(X) = 6$ and $(ab)^3 = 1$.*

Proof. Accordingly to the girth of X we distinguish three cases.

Case 1: $g(X) = 3$. Then up to symmetry, either $abc = 1$, or $aba = 1$. The first one implies $a = cb$, and $1 = a^2 = (cb)^2$. Thus $G = \langle b, c \mid b^2 = c^2 = 1, (bc)^2 = 1 \rangle \cong C_2 \times C_2$. The second relation implies $b = 1$, a contradiction.

Case 2: $g(X) = 4$. Then up to symmetry, either $(ab)^2 = 1$, or $b = abc$. However, the second relation implies $a = bcb$, and consequently, $1 = a^2 = (bc)^2$. Hence $G \cong C_2 \times C_2$ and so $g(X) = 3$, a contradiction.

Case 3: $g(X) = 5$. Then up to symmetry, we have $b = (ac)^2$. It follows that G is dihedral of order 8. Moreover, b is the central involution commuting with both a and c. In particular, we have $(ab)^2 = 1$ implying $g(X) = 4$, a contradiction.

Case 4: $g(X) = 6$. Assume just two of the three generators appear in the relation of length 6. Then up to symmetry, $(ab)^3 = 1$.

Therefore we assume that all the three generators appear in the short relation. There are two subcases to consider.

Assume one of the generators, say c, appears just once. Then up to symmetry, c can be expressed as $c = ababa$. It follows that G is dihedral, and since $1 = c^2 = (ab)^4$, the order of G is 8. In particular, $c = ababa = bab$, hence we have a relation of length 4, a contradiction.

Assume that each of the generators appears in the relation twice. Then up to symmetry, the relation occurs in one of the following forms: $(abc)^2 = 1$, $abacbc = 1$. Assume the second relation holds. Then $b(ac)b = ac$, and thus the subgroup $\langle ac \rangle$ is normal of index four. In particular, in the Cayley graph X there are exactly two alternating cycles coloured by a and c joined by the perfect matching formed by the edges coloured b. The commuting rules $b(ac) = (ac)b$ and $b(ca) = (ca)b$, show that the Cayley graph is a prism implying $g(X) = 4$, a contradiction. It remains to show that if $(abc)^2 = 1$, then X is a honeycomb graph. Indeed, the relation $(abc)^2 = 1$ gives rise to a hexagonal cycle double cover, such that every edge is traversed by a hexagon in each direction. In particular, say an edge $\{g, ga\}$ is in the two hexagons: $(g, ga, gab, gabc, gabca, gabcab)$ and $(ga, g, gc, gcb, gcba, gcbac, gcbacb)$, see Figure 1. □

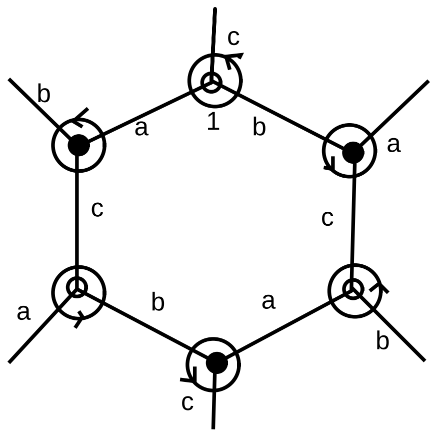

Figure 1. A honeycomb graph on torus from quotients of $G = \langle b, a, c \mid a^2 = b^2 = c^2 = 1, abcabc = 1 \rangle$.

Now we shall examine the existence of Hamilton cycles in the Cayley graphs in Proposition 3.1.

Theorem 3.2. *Let X be a Cayley cubic graph $X = \mathrm{Cay}(G; a, b, c)$, where $a^2 = b^2 = c^2 = 1$ of girth at most six. Then either X is hamiltonian, or $(ab)^3 = 1$.*

Proof. If the girth of X is at most four, then the Hamilton cycle follows from Theorem 2.3. Suppose $g(X) = 6$ and the relation $(abc)^2 = 1$. Then by the main theorem of [**23**] X is hamiltonian. □

For the remaining class of graphs of type 1 satisfying the relation $(ab)^3 = 1$ we have just a partial result proved recently by Nedela and Škoviera.

Theorem 3.3. *Let X be a Cayley cubic graph $X = \mathrm{Cay}(G; a, b, c)$, where $a^2 = b^2 = c^2 = (ab)^3 = (bc)^3 = 1$. Then X admits a Hamilton path, and it is hamiltonian whenever $|G| \equiv 2 \pmod 4$, or if $|ac|$ is even.*

4. Cayley graphs of type 2

In this section we shall deal with the cubic Cayley graphs $X = \mathrm{Cay}(G; a, b)$, where $a^2 = 1$, and $|b| > 2$.

Theorem 4.1. *Let X be a Cayley cubic graph $X = \mathrm{Cay}(G; a, b)$, where $a^2 = 1$. Let the girth $g(X)$ of X be at most 6. Then one of the following cases happens:*

- *$g(X) = 3$, and $b^3 = 1$, or $G \cong C_4$ and $X \cong K_4$,*
- *$g(X) = 4$, $a = b^3$, $G \cong C_6$ and X is $K_{3,3}$,*
- *$g(X) = 4$, $aba = b^{\pm 1}$, G is abelian or dihedral, and X is a prism or a Möbius ladder,*
- *$g(X) = 4$, and $b^4 = 1$,*
- *$g(X) = 5$, and $b^5 = 1$,*
- *$g(X) = 6$, and $G = \langle a, b \mid a^2 = b^8 = 1, aba = b^{\pm 3}\rangle$, X is the generalised Petersen graph $GP(8, 3)$,*
- *$g(X) = 6$, $ab^2a = b^{\pm 2}$ and either $|b|$ is odd and G dihedral, or X is a honeycomb graph,*
- *$g(X) = 6$, and either $(ab)^3 = 1$ or $b^6 = 1$.*

Proof. We distinguish the four cases with respect to the girth of X. Note that a (short) relation is invariant under a cyclic permutation.

Case 1: $g(X) = 3$. Up to symmetry, we have that either $b^3 = 1$, or $a = b^{\pm 2}$. In the second case $1 = a^2 = b^4$, hence $G \cong C_4$ and X is K_4.

Case 2: $g(X) = 4$. Up to symmetry we have the following cases to consider: $b^4 = 1$, $abab^{-1} = 1$, $(ab)^2 = 1$ and $a = b^3$. The case $a = b^3$ forces $G \cong C_6$ and $X \cong K_{3,3}$. If $(ab)^2 = 1$, G is abelian. If $a \in \langle b\rangle$, then X is the Möbius ladder, while if $a \notin \langle b\rangle$, then X is the prism. In the case $aba = b^{-1}$, G is dihedral, and X is a prism.

Case 3: $g(X) = 5$. Up to symmetry, the short relation takes one of the following forms: $b^5 = 1$, $a = b^{\pm 4}$, $abab^2 = 1$, $abab^{-2} = 1$. In the case $a = b^{\pm 4}$, G is cyclic. Assume $abab^{\pm 2} = 1$. In the case $(ab)^2 b = 1$ by changing the generating set to c = ab; b, we see that $b^{-1} = c^2$, therefore $G = \langle c\rangle$ is cyclic, the girth $g(X) = 4$, a contradiction. In the case $abab^{-2} = 1$, we have $ab = b^2 a$. It follows that $ba = ab^2 = (ab)b = b^2 ab = b^4 a$, and therefore $b^3 = 1$, and the girth is 3, a contradiction. We are left with the case $b^5 = 1$.

Case 4: $g(X) = 6$. Up to symmetry, a 6-cycle in X forces one of the following relations to hold: $b^6 = 1$, $a = b^5$, $aba = b^{\pm 3}$, $ab^2 a = b^{\pm 2}$, $(ab)^3 = 1$, $(ab)^2 ab^{-1} = 1$. The last relation gives $b^{-1} = abab^{-1}a$ implying $|b| = 2$, a contradiction. The relation $a = b^5$ implies that G is cyclic and consequently, $g(X) = 4$, a contradiction. If $aba = b^{\pm 3}$, then $b = a^2ba^2 = b^9$, implying $b^8 = 1$. Hence, G is the group of order 16, and X has two b-cycles of length 8 joined by the perfect matching formed by the a-edges. The relation $aba = b^{\pm 3}$ proves that $X = GP(8, 3)$. Assume that $ab^2a = b^{\pm 2}$. If $|b|$ is odd, then $G = \langle a, b\rangle = \langle a, b^2\rangle$, and G is either cyclic, or dihedral. Assume $|b|$ is even. To see that X is a honeycomb graph we form a hexagonal double cycle by directed cycles such that each edge is traversed by a directed hexagon in both directions. Recall that in X, considered as a coloured Cayley graph, there are two kinds of edges, a-edges that correspond to the involutory generator, and b edges which are directed from g to gb. We claim that the hexagonal cycles given by the relation ab^2ab^2 in the first case, or by the relation ab^2ab^{-2} in the second case, give the required hexagonal double cycle cover. In the first case, the two hexagons passing through an a-edge $\{g, ga\}$, $g \in G$, are $(g, ga, gab, gab^2, gab^2a, gab^2ab)$ and $(ga, g, gb, gb^2, gb^2a, gb^2ab)$. For a b-edge, $\{g, gb\}$ the two covering hexagons are: $(g, gb, gb^2, gb^2a, gb^2ab, gb^2ab^2)$ and $(gb, g, gb^{-1}, gb^{-1}a, gb^{-1}ab^{-1}, gb^{-1}ab^{-2})$. Finding two hexagons passing through an edge in the second case is left to the reader. Using Euler's formula one can easily see that the underlying surface is either the torus, or it is Klein's bottle. To exclude the second case we can prove that the surface is orientable as follows. First observe that $\langle b^2\rangle$ is a normal cyclic subgroup of order t, for some $t > 1$. Secondly, the factor group $G/\langle b^2\rangle$ is dihedral. It follows that there exists a least positive m such that $(ab)^m = b^{2r}$, for some r. The relation $ab^2ab^{\pm 2} = 1$ induces an orientation on each of the hexagons, and these local orientations are compatible globally. In particular, the two integers t and r determine both the group and the embedding of X into the torus. □

Theorem 4.2. *Let* $X = \mathrm{Cay}(G; a, b^{\pm 1})$*,* $a^2 = 1$ *and* $|b| > 2$*, be a cubic Cayley graph of girth at most six. Then either* X *is hamiltonian, or one of the following relations hold:* $b^3 = 1$*,* $b^5 = 1$*,* $b^6 = 1$*,* $(ab)^3 = 1$*.*

Proof. For the exceptional graphs mentioned in Theorem 4.1 the Hamilton cycle can be easily checked directly. If the girth is at most four then the statement follows from Theorem 2.2. If X is the honeycomb graph, then it follows from Theorem 2.6. Now the statement follows from Theorem 4.1. □

For the remaining four difficult cases we have an almost complete result in the case $(ab)^3 = 1$ proved in several papers by Glover, Marušič, Kutnar, etc. We summarise it as follows.

Theorem 4.3 ([**12, 13, 14**])**.** *Let* $k \geq 3$ *be an integer and let* $G = \langle a, b \mid a^2 = 1, b^k = 1, (ab)^3 = 1, \ldots\rangle$ *be a finite group. Then the Cayley graph* $X = \mathrm{Cay}(G, \{a, b, b^{-1}\})$ *has a Hamilton cycle except* $|G| \equiv 0 \pmod 4$ *and* $k \equiv 2 \pmod 4$*. In the latter case* X *has a Hamilton path.*

For the case $b^3 = 1$ we have the following statement.

Proposition 4.4. *If* Conjecture 1.2 *is true, then the Cayley graphs* $\mathrm{Cay}(G; a, b)$*, where* $a^2 = b^3 = 1$*, are hamiltonian.*

Proof. In the Cayley graph X the b-edges form a triangular 2-factor F. Form a new cubic graph Y by contracting each triangle of F to a vertex. Clearly X is hamiltonian if and only if Y is hamiltonian. Moreover, G acts regularly on the arcs of Y. If the girth of Y is at least 7, then the Hamilton cycle of Y follows from Conjecture 1.2. Suppose $g(Y) \leq 6$. Arc-transitive cubic graphs of small girth are analysed in [**10**]. The following facts are well known. If $g(Y) \leq 5$ and Y is an arc-transitive cubic graph, then Y is one of the following graphs: K_4, $K_{3,3}$, the cube Q_3, the Petersen graph and the dodecahedron. Between them the Petersen graph does not admit a group of automorphisms acting regularly on the arcs. The other four are known to be hamiltonian. If $g(Y) = 6$, then by a result of Miller [**17**], either Y is the generalised Petersen graph $GP(8, 3)$, or it is a honeycomb graph. Both $GP(8, 3)$ and the honeycomb graphs are known to be hamiltonian. □

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E. Aboomahigir, Department of Mathematics, Faculty of Mathematical Sciences, Tarbiat Modares University, P.O. Box 14115-137, Tehran, Iran, *current address*: NTIS, University of West Bohemia, Technická 8, 301 00 Plzeň 3, Czech Republic,
e-mail: `e.aboomahigir@modares.ac.ir`

R. Nedela, Department of Mathematics and NTIS, University of West Bohemia, Technická 8, 301 00 Plzeň 3, Czech Republic,
e-mail: `nedela@savbb.sk`