Acta Math. Univ. Comenianae
Vol. LXXXIX, 1 (2020), pp. 153–160

SOME COMMON FIXED POINT RESULTS IN PARTIAL $b_v(s)$-METRIC SPACES

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ABSTRACT. In this paper, we give a proof for some common fixed point theorems in framework of partial $b_v(s)$-metric spaces. As corollaries of our result, we get numerous results in other classes of metric spaces.

1. INTRODUCTION

Bakhtin [**12**] and Czerwik [**16**] (see also [**7, 8**]) introduced *b-metric spaces*, modifying the triangle inequality to the following form

$$d(x,z) \le s[d(x,y) + d(y,z)], \tag{1}$$

where $s \ge 1$ is a fixed real number. On the other hand, Branciari [**14**] substituted the triangle inequality by a *polygonal* inequality of the form

$$d(x,z) \le d(x,y_1) + d(y_1,y_2) + \cdots + d(y_v,z) \tag{2}$$

for arbitrary x, z and for all distinct points $y_1, y_2, \ldots, y_v$, each of them different from x and z (in particular, for $v = 2$, the inequality (2) is called *rectangular*). Further, a lot of fixed point results for single and multi-valued mappings were obtained in both kinds of spaces by various authors see ([**4, 5, 6, 9, 10, 11, 13, 15, 18, 22, 24, 25, 33, 35**]).

George et al. [**20**], as well as Roshan et al. [**32**], independently introduced b-rectangular metric spaces by combining inequalities (1) and (2) (in the case $v = 2$). Mitrović and Radenović [**28**] defined the concept of $b_v(s)$-metric spaces for arbitrary positive integer v (see the definition in the next section), thus generalizing all the mentioned types of spaces. They obtained some fixed point results in this new framework. It should be noted that these spaces might not be Hausdorff, that a $b_v(s)$-metric need not be continuous and that a convergent sequence might not be a Cauchy one. Finally, Abdullahi and Kumam [**1**] defined partial $b_v(s)$-metric spaces and some fixed point results have been established.

Received December 2, 2018; revised May 22, 2019.
2010 *Mathematics Subject Classification.* Primary 47H10; Secondary 55M20.
Key words and phrases. fixed points; common fixed points; partial $b_v(s)$-metric space.

In this paper, we give a proof for some common fixed point theorems in framework of partial $b_v(s)$-metric spaces. We use contractive conditions involving rational expressions of Khan type, as well as of Dass-Gupta type, to obtain some common fixed point results in the framework of partial $b_v(s)$-metric spaces. Thus, we obtain generalizations of several known fixed point results from the literature.

2. Partial $\mathbf{b_v(s)}$-metric spaces

Definition 2.1 ([**1**])**.** Let X be a non-empty set, $s \geq 1$ be a real number, $v \in \mathbb{N}$, and let p_{b_v} be a function from $X \times X$ into $[0, \infty)$. Then (X, p_{b_v}) is said to be a partial $b_v(s)$-metric space if for all $x, y, z \in X$ and for all distinct points $y_1, y_2, \dots, y_v \in X$, each of them different from x and z, the following hold:

(Pbv1) $x = y$ if and only if $p_{b_v}(x, x) = p_{b_v}(x, y) = p_{b_v}(y, y)$,

(Pbv2) $p_{b_v}(x, x) \leq p_{b_v}(x, y)$,

(Pbv3) $p_{b_v}(x, y) = p_{b_v}(y, x)$,

(Pbv4) $p_{b_v}(x, z) \leq s[p_{b_v}(x, y_1) + p_{b_v}(y_1, y_2) + \dots + p_{b_v}(y_v, z)] - \sum\limits_{i=1}^{v} p_{b_v}(y_i, y_i)$.

The notions of a convergent sequence, a Cauchy sequence and the completeness of a partial $b_v(s)$-metric space are introduced in the same way as in standard metric spaces (see also [**34**]).

We make use of the following lemmas obtained in [**1**].

Lemma 2.2. *Let (X, p_{b_v}) be a partial $b_v(s)$-metric space, $T\colon X \to X$, and let $\{x_n\}$ be a sequence in X defined by $x_0 \in X$ and $x_{n+1} = Tx_n$ such that $x_n \neq x_{n+1}$, $(n \geq 0)$. Suppose that $\lambda \in [0, 1)$ is such that*

$$p_{b_v}(x_{n+1}, x_n) \leq \lambda p_{b_v}(x_n, x_{n-1}) \text{ for all } n \in \mathbb{N}.$$

Then $x_n \neq x_m$ for all distinct $n, m \in \mathbb{N}$.

Lemma 2.3. *Let (X, p_{b_v}) be a partial $b_v(s)$-metric space and let $\{x_n\}$ be a sequence in X such that x_n $(n \geq 0)$ are all different. Suppose that $\lambda \in [0, 1)$ and c_1, c_2 are real nonnegative numbers such that*

$$p_{b_v}(x_m, x_n) \leq \lambda p_{b_v}(x_{m-1}, x_{n-1}) + c_1\lambda^m + c_2\lambda^n \quad \text{for all } m, n \in \mathbb{N}.$$

Then $\{x_n\}$ is a Cauchy sequence.

3. A common fixed point theorem of Jungck type in partial $b_v(s)$-metric spaces

Theorem 3.1. *Let T and I be commuting mappings of a complete partial $b_v(s)$-metric space (X, p_{b_v}) into itself satisfying the inequality*

$$p_{b_v}(Tx, Ty) \leq \lambda p_{b_v}(Ix, Iy) \tag{3}$$

for all $x, y \in X$, where $0 < \lambda < 1$. If the range of I contains the range of T and if I is continuous, then T and I have a unique common fixed point.

Proof. Let $x_0 \in X$ be arbitrary. Then Tx_0 and Ix_0 are well defined. Since $Tx_0 \in I(X)$, there is any $x_1 \in X$ such that $Ix_1 = Tx_0$. In general, if x_n is chosen, then we choose a point x_{n+1} in X such that $Ix_{n+1} = Tx_n$. We show that $\{Ix_n\}$ is a Cauchy sequence. From (3), we have

$$p_{b_v}(Ix_m, Ix_n) = p_{b_v}(Tx_{m-1}, Tx_{n-1}) \leq \lambda p_{b_v}(Ix_{m-1}, Ix_{n-1}).$$

So,

$$p_{b_v}(Ix_m, Ix_n) \leq \lambda p_{b_v}(Ix_{m-1}, Ix_{n-1}) \qquad \text{for all } m, n \in \mathbb{N}. \tag{4}$$

Now, we have the following two cases.

Case 1. If $Ix_n = Ix_{n+1}$ for some $n \geq 0$, then $Tx_n = Ix_n = \omega$. We show that ω is a unique common fixed point of T and I. Indeed, first

$$T\omega = TIx_n = ITx_n = I\omega.$$

Let $p_{b_v}(\omega, T\omega) > 0$. In this , we have

$$\begin{aligned} p_{b_v}(\omega, T\omega) &= p_{b_v}(Tx_n, T\omega) \leq \lambda p_{b_v}(Ix_n, I\omega) = \lambda p_{b_v}(\omega, I\omega) \\ &= \lambda p_{b_v}(\omega, T\omega) < p_{b_v}(\omega, T\omega), \end{aligned}$$

a contradiction. Since the condition (3) implies that $Tx_n = Ix_n = \omega$ is a unique common fixed point of T and I, the proof of Case 1 is completed.

Case 2. If $Ix_n \neq Ix_{n+1}$ for all $n \geq 0$, from Lemma 2.2 and the inequality (4), we obtain $Ix_n \neq Ix_{n+k}$ for all $n \geq 0, k \geq 1$. Thus, from Lemma 2.3, we obtain that $\{Ix_n\}$ is a Cauchy sequence in X.

By completeness of $I(X)$, there exists $u \in X$ such that

$$\lim_{n\to\infty} Ix_n = \lim_{n\to\infty} Tx_{n-1} = u.$$

Now, since I is continuous, (3) implies that both I and T are continuous. Since T and I commute, we obtain

$$Iu = I(\lim_{n\to\infty} Tx_n) = \lim_{n\to\infty} ITx_n = \lim_{n\to\infty} TIx_n = T(\lim_{n\to\infty} Ix_n) = Tu.$$

Let $v = Iu = Tu$. We get $Tv = TIu = ITu = Iv$.

If $Tu \neq Tv$, from (3), we obtain

$$p_{b_v}(Tu, Tv) \leq \lambda p_{b_v}(Iu, Iv) = \lambda p_{b_v}(Tu, Tv) < p_{b_v}(Tu, Tv),$$

a contradiction. So we have $Tu = Tv$, and finally, we obtain $Tv = Iv = v$, i.e., v is a common fixed point for T and I. Condition (3) implies that v is the unique common fixed point. □

In case $v = 2$ we obtain the following corollaries.

Corollary 3.2 ([**29**, Theorem 2.1])**.** *Let T and I be commuting mappings of a complete rectangular b-metric space (X, d, s) into itself satisfying the inequality*

$$d(Tx, Ty) \leq \lambda d(Ix, Iy)$$

for all $x, y \in X$, where $0 < \lambda < 1$. If the range of I contains the range of T and if I is continuous, then T and I have a unique common fixed point.

Corollary 3.3 ([**27**, Theorem 2.1])**.** *Let (X, d) be a complete rectangular b-metric space with coefficient $s > 1$ and $T\colon X \to X$ be a mapping satisfying*

$$d(Tx, Ty) \leq \lambda d(x, y)$$

for all $x, y \in X$, where $\lambda \in [0, 1)$. Then T has a unique fixed point.

Also from Theorem 3.1, we obtain the result of Aleksić et al. [**3**].

Corollary 3.4 ([**3**, Theorem 2.1])**.** *Let T and I be commuting mappings of a complete $b_v(s)$-metric space (X, d, s) into itself satisfying the inequality*

$$d(Tx, Ty) \leq \lambda d(Ix, Iy)$$

for all $x, y \in X$, where $0 < \lambda < 1$. If the range of I contains the range of T and if I is continuous, then T and I have a unique common fixed point.

4. A common fixed point theorem of Khan type in partial $b_v(s)$-metric spaces

Rational expressions in contractive conditions were firstly used by Dass and Gupta [**17**], Khan [**26**] (corrected by Fisher [**19**]), and Jaggi [**21**]. Later on, there have been a lot of papers using several variants of such conditions in various contexts (see, e.g., [**2, 30, 31, 32**]).

Let (X, p_{b_v}) be a partial $b_v(s)$-metric space and $I, T\colon X \to X$ be two mappings. We introduce the following function $k\colon X \times X \to [0, 1]$ by

$$k_{xy} = \begin{cases} \dfrac{p_{b_v}(Ix, Ty)}{\max\{p_{b_v}(Ix, Ty), p_{b_v}(Iy, Tx)\}} & \text{if } \max\{p_{b_v}(Ix, Ty), p_{b_v}(Iy, Tx)\} \neq 0, \\ 1/2 & \text{if } \max\{p_{b_v}(Ix, Ty), p_{b_v}(Iy, Tx)\} = 0. \end{cases}$$

Theorem 4.1. *Let T and I be two commuting mappings of a complete partial $b_v(s)$-metric space (X, p_{b_v}) into itself, satisfying the inequality*

$$p_{b_v}(Tx, Ty) \leq \lambda \max\{p_{b_v}(Ix, Iy), k_{xy} p_{b_v}(Ix, Tx) + k_{yx} p_{b_v}(Iy, Ty)\} \tag{5}$$

for all $x, y \in X$, where $\lambda \in [0, 1)$. If the range of I contains the range of T and if I is continuous, then T and I have a unique common fixed point.

Proof. Let $x_0 \in X$ be arbitrary. Define a sequence $\{x_n\}$ by $Ix_{n+1} = Tx_n$ for all $n \geq 0$. If for some n, $Ix_n = Ix_{n+1}$, then Ix_n is a fixed point of T, so there is nothing to prove. From now on, suppose that $x_n \neq x_{n+1}$ for all $n \geq 0$. From the condition (5), we obtain

$$\begin{aligned} p_{b_v}(x_{n+1}, x_n) \leq \lambda \max\{&p_{b_v}(x_n, x_{n-1}), \\ &k_{x_n x_{n-1}} p_{b_v}(x_n, x_{n+1}) + k_{x_{n-1} x_n} p_{b_v}(x_{n-1}, x_n)\}. \end{aligned} \tag{6}$$

We distinguish two cases:

1. $p_{b_v}(x_{n-1}, x_{n+1}) \neq 0$. In this case, we obtain

$$k_{x_n x_{n-1}} = \frac{p_{b_v}(x_n, x_n)}{\max\{p_{b_v}(x_n, x_n), p_{b_v}(x_{n-1}, x_{n+1})\}},$$

so, $k_{x_n x_{n-1}} = 0$. Now, from (6), we have

$$p_{b_v}(x_{n+1}, x_n) \leq \lambda p_{b_v}(x_n, x_{n-1}) \tag{7}$$

for all $n \in \mathbb{N}$.

2. For some n, $p_{b_v}(x_{n-1}, x_{n+1}) = 0$, we have that $k_{x_n x_{n-1}} = \frac{1}{2}$. It follows from (6), that

$$p_{b_v}(x_{n+1}, x_n) \leq \lambda \max\left\{p_{b_v}(x_n, x_{n-1}), \frac{1}{2}p_{b_v}(x_n, x_{n+1}) + \frac{1}{2}p_{b_v}(x_{n-1}, x_n)\right\}.$$

From the above inequality, we get

$$p_{b_v}(x_{n+1}, x_n) \leq \lambda p_{b_v}(x_n, x_{n-1}) \quad \text{or} \quad p_{b_v}(x_{n+1}, x_n) \leq \frac{\lambda}{2-\lambda} p_{b_v}(x_n, x_{n-1}). \tag{8}$$

Since $\max\{\lambda, \frac{\lambda}{2-\lambda}\} = \lambda$, we conclude that (8) holds for all $n \in \mathbb{N}$. Then from Lemma 2.2, we obtain

$$x_n \neq x_m \qquad \text{for all distinct } n, m \in \mathbb{N}.$$

We note that from the conditions (7) and (8), it follows

$$p_{b_v}(x_{n+1}, x_n) \leq \lambda^n p_{b_v}(x_1, x_0) \tag{9}$$

for all $n \in \mathbb{N}$. Let $m, n \in \mathbb{N}$ be such that $m \neq n-1$ and $m \neq n+1$. Then $\max\{p_{b_v}(x_n, x_{m+1}), p_{b_v}(x_m, x_{n+1})\} \neq 0$, therefore,

$$k_{x_n x_m} = \frac{p_{b_v}(x_n, x_{m+1})}{\max\{p_{b_v}(x_n, x_{m+1}), p_{b_v}(x_m, x_{n+1})\}} \leq 1 \tag{10}$$

and

$$k_{x_m x_n} = \frac{p_{b_v}(x_m, x_{n+1})}{\max\{p_{b_v}(x_n, x_{m+1}), p_{b_v}(x_m, x_{n+1})\}} \leq 1. \tag{11}$$

Let $m, n \in \mathbb{N}$ be such that $|m-n| \neq 1$ (if $|m-n| = 1$, (9) is used). Then from (5), (9), (10), and (11), we obtain

$$\begin{aligned}
p_{b_v}(x_m, x_n) &\leq \lambda \max\{p_{b_v}(x_{m-1}, x_{n-1}), k_{x_{m-1}x_{n-1}} p_{b_v}(x_{m-1}, x_m) \\
&\qquad + k_{x_{n-1}x_{m-1}} p_{b_v}(x_{n-1}, x_n)\} \\
&\leq \lambda \max\{p_{b_v}(x_{m-1}, x_{n-1}), \lambda^{m-1} p_{b_v}(x_0, x_1) + \lambda^{n-1} p_{b_v}(x_0, x_1)\} \\
&\leq \lambda p_{b_v}(x_{m-1}, x_{n-1}) + (\lambda^m + \lambda^n) p_{b_v}(x_0, x_1).
\end{aligned}$$

Now, from Lemma 2.3, (by putting $c_1 = c_2 = p_{b_v}(x_0, x_1)$), we obtain that $\{x_n\}$ is a Cauchy sequence in X. By the completeness of (X, p_{b_v}), there exists $x^* \in X$ such that

$$\lim_{n\to\infty} x_n = x^*.$$

We prove that x^* is the unique fixed point of T.

Namely, for any $n \in \mathbb{N}$, we have

$$\begin{aligned}
p_{b_v}(x^*, Tx^*) \leq s[&p_{b_v}(x^*, x_{n+1}) + p_{b_v}(x_{n+1}, x_{n+2}) + p_{b_v}(x_{n+2}, x_{n+3}) + \dots \\
&+ p_{b_v}(x_{n+v-2}, x_{n+v-1}) + p_{b_v}(x_{n+v-1}, x_{n+v}) + p_{b_v}(x_{n+v}, Tx^*)].
\end{aligned}$$

Let us consider the following two cases:

1. $\liminf_{n\to\infty} p_{b_v}(x_n, Tx^*) = 0$. In this case, there exists a subsequence $\{x_{n_k}\}_{k\geq 0}$ of $\{x_n\}$ having the property that $\lim_{k\to\infty} p_{b_v}(x_{n_k}, Tx^*) = 0$. Then

$$\begin{aligned}p_{b_v}(x^*, Tx^*) \leq s[&p_{b_v}(x^*, x_{n_k-v+1}) + p_{b_v}(x_{n_k-v+1}, x_{n_k-v+2}) + \dots\\ &+ p_{b_v}(x_{n_k-2}, x_{n_k-1}) + p_{b_v}(x_{n_k-1}, x_{n_k}) + p_{b_v}(x_{n_k}, Tx^*)].\end{aligned}$$

Since $\lim_{k\to\infty} p_{b_v}(x^*, x_{n_k-v+1}) = 0$ and $\lim_{k\to\infty} p_{b_v}(x_{n_k+i}, x_{n_k+i+1}) = 0$, from the above inequality, we get $p_{b_v}(Tx^*, x^*) = 0$, i.e., $Tx^* = x^*$.

2. $\liminf_{n\to\infty} d(x_n, Tx^*) = c > 0$. Then there exists a subsequence $\{x_{n_k}\}_{k\geq 0}$ of $\{x_n\}$ having the property that $\lim_{k\to\infty} d(x_{n_k}, Tx^*) = c$ and $x_{n_k} \neq Tx^*$ for all $k \in \mathbb{N}$. We have

$$\begin{aligned}p_{b_v}(x^*, Tx^*) \leq s[&p_{b_v}(x^*, x_{n_k-v+2}) + p_{b_v}(x_{n_k-v+2}, x_{n_k-v+3}) + \dots\\ &+ p_{b_v}(x_{n_k-1}, x_{n_k}) + p_{b_v}(x_{n_k}, x_{n_k+1}) + p_{b_v}(x_{n_k+1}, Tx^*)].\end{aligned}$$

From (5), we obtain

$$p_{b_v}(x_{n_k+1}, Tx^*) = \lambda \max\{p_{b_v}(x_{n_k}, x^*), k_{x_{n_k}x^*} p_{b_v}(x_{n_k}, x_{n_k+1}) + k_{x^*x_{n_k}} p_{b_v}(x^*, Tx^*).$$

Since

$$k_{x_{n_k}x^*} = \frac{p_{b_v}(x_{n_k}, Tx^*)}{\max\{p_{b_v}(x_{n_k}, Tx^*), p_{b_v}(x^*, x_{n_k+1})\}} \to 1 \quad \text{as } k \to \infty,$$

and

$$k_{x^*x_{n_k}} = \frac{p_{b_v}(x^*, x_{n_k+1})}{\max\{p_{b_v}(x^*, x_{n_k+1}), p_{b_v}(x_{n_k}, Tx^*)\}} \to 0 \quad \text{as } k \to \infty,$$

we have $p_{b_v}(x^*, Tx^*) = 0$, and so $Tx^* = x^*$.

In order to prove uniqueness, let y^* be another fixed point of T. Then from (5),

$$\begin{aligned}p_{b_v}(x^*, y^*) &= p_{b_v}(Tx^*, Ty^*)\\ &\leq \lambda \max\{p_{b_v}(x^*, y^*), k_{x^*y^*} p_{b_v}(x^*, Tx^*) + k_{y^*x^*} p_{b_v}(y^*, Ty^*)\}\\ &\leq \lambda p_{b_v}(x^*, y^*) < p_{b_v}(x^*, y^*),\end{aligned}$$

which is a contradiction. Therefore, we must have $p_{b_v}(x^*, y^*) = 0$, i.e., $x^* = y^*$. □

Example 1. Consider $X = \{0, 1, 2, 3\}$. Take the partial $b_v(s)$-metric $p_{b_v}\colon X \times X \to [0, \infty)$ as

$$p_{b_v}(x, y) = \begin{cases} 0 & \text{if } x = y = 0, 2,\\ 2 & \text{if } x, y \in \{0, 1\}, x \neq y,\\ \frac{4}{3} & \text{if } x, y \in \{1, 2\}, x \neq y,\\ 1 & \text{otherwise.}\end{cases}$$

Clearly, (X, p_{b_v}) is a complete partial $b_v(s)$-metric space (with $v = 2$ and $s = \frac{4}{3}$). Let $T, I\colon X \to X$ be defined by

$$T0 = I0 = T1 = I1 = T2 = 0, \quad T3 = I2 = 2, \quad \text{and} \quad I3 = 1.$$

Choose $\lambda = \frac{3}{4}$. It is easy to show that

$$p_{b_v}(Tx, Ty) \leq \lambda p_{b_v}(Ix, Iy)$$

for all $x, y \in X$, that is, (3) holds. All hypotheses of Theorem 3.1 are satisfied, so T and I have a unique common fixed point, which is $u = 0$.

Example 2. Let $X = [0, \infty)$. Consider the partial $b_v(s)$-metric given by

$$p_{b_v}(x, y) = \max\{x, y\} + |x - y|^2.$$

It is obvious that (X, p_{b_v}) is a complete partial $b_v(s)$-metric space ($v = 1$ and $s = 2$). Choose $Tx = 3x$ and $Ix = 5x$. Take $\lambda = \frac{3}{5}$. Clearly, (3) is satisfied for all $x, y \in X$. Also all hypotheses of Theorem 3.1 hold and $u = 0$ is the unique common fixed point of T and I.

Remark 4.2. 1. It is clear that Theorems 3.1 and 4.1 generalize Banach contraction principle in $b_v(s)$-metric spaces (see [**28**, Theorem 2.1]).

2. Also, Theorem 4.1 generalizes the result of Piri et al. (see [**31**, Theorem 2.1]).

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