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ON ALMOST KENMOTSU MANIFOLDS WITH GENERALIZED NULLITY DISTRIBUTION

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Abstract. The object of the present paper is to classify generalized $(k,\mu)'$-almost Kenmotsu manifolds satisfying certain semisymmetry conditions. We prove that Weyl semisymmetric and h'-semisymmetric almost Kenmotsu manifolds with generalized $(k,\mu)'$-nullity distribution are both locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4)\times\mathbb{R}^n$. Also we characterize Weyl Ricci semisymmetric almost Kenmotsu manifolds with generalized $(k,\mu)'$-nullity distribution.

1. Introduction

Geometry of Kenmotsu manifolds was originated by Kenmotsu [**8**] and became an interesting area of research in differential geometry. As a generalization of Kenmotsu manifolds, the notion of almost Kenmotsu manifolds was first introduced by Janssens and Vanhecke [**7**]. In recent years, some results regarding such manifolds were given in ([**4**], [**5**], [**13**], [**14**], [**15**]). Almost Kenmotsu manifolds satisfying the (k,μ) and $(k,\mu)'$-nullity conditions were introduced by Dileo and Pastore [**4**], where both k and μ are constants. In 2011, Pastore and Saltarelli in [**10**] extended the above nullity conditions to the corresponding generalized nullity conditions for which both k and μ are smooth functions. Recently some results on generalized (k,μ) and $(k,\mu)'$-almost Kenmotsu manifolds satisfying some curvature conditions were obtained by Wang et al. ([**12**],[**16**]).

A Riemannian manifold M^{2n+1} is called locally symmetric if its curvature tensor R is parallel, that is, $\nabla R = 0$, where ∇ is the Levi-Civita connection. It was introduced by Shirokov in [**9**]. The notion of semisymmetric manifolds, a proper generalization of locally symmetric manifolds worked out by Catan in 1927, is defined by $R(X,Y)\cdot R = 0$, where $R(X,Y)$ acts on R as a derivation. A complete intrinsic classification of these manifolds was given by Szabo [**11**]. A Riemannian manifold is said to be Weyl semisymmetric if $R(X,Y)\cdot C = 0$, where C is the

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Weyl curvature tensor of type $(1,3)$ and defined by [**18**],

$$
\begin{aligned}
C(X,Y)Z = R(X,Y)Z &- \frac{1}{(2n-1)}[S(Y,Z)X - S(X,Z)Y \\
&+ g(Y,Z)QX - g(X,Z)QY] \\
&+ \frac{r}{2n(2n-1)}[g(Y,Z)X - g(X,Z)Y],
\end{aligned} \tag{1}
$$

where X, Y, Z are any vector fields, S is the Ricci tensor of type $(0,2)$, and Q is the Ricci operator defined by $S(X,Y) = g(QX,Y)$. Semisymmetry implies Weyl semisymmetry, but the converse is not true in general. Recently in [**16**], Wang et al. proved that semisymmetric almost Kenmotsu manifold with generalized $(k,\mu)'$-nullity distribution is locally isometric to either the hyperbolic space $\mathbb{H}^{2n+1}(-1)$ or the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.

On the other hand, an almost Kenmotsu manifinold with generalized (k,μ)-nullity distribution is said to be h'-semisymmetric if the curvature tensor satisfies $(R(X,Y) \cdot h')Z = 0$ for all smooth vector fields X, Y, Z.

In [**17**], Yildiz and De studied ϕ-semisymmetric and h-semisymmetric (k,μ)-contact manifolds. In [**6**], De et al. studied ϕ-concircularly semisymmetric and h-concircularly semisymmetric (k,μ)-contact manifolds, and they proved that in both cases the manifolds become η-Einstein.

Motivated by the above studies, in this paper, we study Weyl semisymmetric almost Kenmotsu manifinolds with generalized $(k,\mu)'$-nullity distribution. Besides this, we study h'-semisymmetric almost Kenmotsu manifolds and the curvature condition Weyl Ricci semisymmetric $(C \cdot S = 0)$ almost Kenmotsu manifolds with generalized $(k,\mu)'$-nullity distribution. Our results give some complete classification of such manifolds with some semisymmetry conditions and generalize some corresponding results obtained by Wang et al. Precisely, we state that on any almost Kenmotsu manifold with generalized $(k,\mu)'$ of dimension ≥ 5, the following two assertions:

(1) Weyl semisymmetric almost Kenmotsu manifinolds with generalized $(k,\mu)'$-nullity distribution,
(2) h'-semisymmetric almost Kenmotsu manifinolds with generalized $(k,\mu)'$-nullity distribution, are both locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.

2. Almost Kenmotsu manifolds

A differentiable $(2n+1)$-dimensional manifold M is said to have a (ϕ,ξ,η)-structure or an almost contact structure if it admits a $(1,1)$ tensor field ϕ, a characteristic vector field ξ, and a 1-form η

$$\phi^2 = -I + \eta \otimes \xi, \ \eta(\xi) = 1, \tag{2}$$

satisfying ([**2**],[**3**]), where I denotes the identity endomorphism. Here also $\phi\xi = 0$ and $\eta \circ \phi = 0$, both can be derived easily from (2).

If a manifold M with a (ϕ, ξ, η)-structure admits a Reimannian metric g such that

$$g(\phi X, \phi Y) = g(X, Y) - \eta(X)\eta(Y)$$

for any vector fields X, Y of T_pM^{2n+1}, then M is said to have an almost contact structure (ϕ, ξ, η, g). The fundamental 2-form Φ on an almost contact metric manifold is defined by $\Phi(X, Y) = g(X, \Phi Y)$ for any X, Y of T_pM^{2n+1}. The condition for an almost contact metric manifold being normal is equivalent to the vanishing of the $(1, 2)$-type torsion tensor N_ϕ, defined by $N_\phi = [\phi, \phi] + 2d\eta \otimes \xi$, where $[\phi, \phi]$ is the Nijenhuis torsion of ϕ. Recently in ([**4**],[**5**],[**12**]), almost contact metric manifold, such that η is closed and $d\Phi = 2\eta \wedge \Phi$, are studied. They are called almost Kenmotsu manifolds. Obviously, a normal almost Kenmotsu manifold is a Kenmotsu manifold. Also Kenmotsu manifolds can be characterized by $(\nabla_X \phi)Y = g(\phi X, Y)\xi - \eta(Y)\phi X$ for any vector fields X,Y. It is well known [**8**] that a Kenmotsu manifold M^{2n+1} is locally a warped product $I \times_f N^{2n}$, where N^{2n} is a Kähler manifold, I is an open interval with coordinate t, and the warping function f, defined by $f = ce^t$ for a positive constant c. Let us denote the distribution orthogonal to ξ by $\mathcal{D}$ and defined by $\mathcal{D} = \mathrm{Ker}(\eta) = \mathrm{Im}(\phi)$. In an almost Kenmotsu manifold, since η is closed, $\mathcal{D}$ is an intregrable distribution. Let M^{2n+1} be an almost Kenmotsu manifold. By $h = \frac{1}{2}\pounds_\xi \phi$, we denote $l = R(\cdot, \xi)\xi$ on M^{2n+1}. The tensor fields l and h are symmetric operators and satisfy the following relations [**4**]:

$$h\xi = 0,\ l\xi = 0, \quad \mathrm{tr}(h) = 0, \quad \mathrm{tr}(h\phi) = 0, \quad h\phi + \phi h = 0, \tag{3}$$

$$\nabla_X \xi = -\phi^2 X - \phi h X (\Rightarrow \nabla_\xi \xi = 0), \tag{4}$$

$$\phi l \phi - l = 2(h^2 - \phi^2), \tag{5}$$

$$R(X, Y)\xi = \eta(X)(Y - \phi h Y) - \eta(Y)(X - \phi h X) + (\nabla_Y \phi h)X - (\nabla_X \phi h)Y \tag{6}$$

for any vector fields X, Y. The $(1, 1)$-type symmetric tensor field $h' = h \circ \phi$ is anticommuting with ϕ and $h'\xi = 0$. Also it is clear that ([**1**], [**4**], [**16**])

$$h = 0 \Leftrightarrow h' = 0, \qquad h'^2 = (k + 1)\phi^2 (\Leftrightarrow h^2 = (k + 1)\phi^2). \tag{7}$$

3. ξ BELONGS TO THE GENERALIZED $(k, \mu)'$-NULLITY DISTRIBUTION

This section is devoted to study almost Kenmotsu manifolds with ξ belonging to the generalized $(k, \mu)'$-nullity distribution. Let $M^{2n+1}(\phi, \xi, \eta, g)$ be an almost Kenmotsu manifold with ξ belonging to the generalized $(k, \mu)'$-nullity distribution, then according to Pastore and Saltarelli [**10**], we have

$$R(X, Y)\xi = k[\eta(Y)X - \eta(X)Y] + \mu[\eta(Y)h'X - \eta(X)h'Y], \tag{8}$$

where k, μ are smooth functions on M^{2n+1} and $h' = h \circ \phi$. Let $X \in \mathcal{D}$ be the eigen vector of h' corresponding to the eigen value λ. Then from (7), it is clear that $\lambda^2 = -(k + 1)$. Therefore, $k \leq -1$ and $\lambda = \pm\sqrt{-k - 1}$. By $[\lambda]'$ and $[-\lambda]'$ we denote the corresponding eigenspaces related to the non-zero eigen value λ and $-\lambda$ of h', respectively. Before presenting our main theorems, we recall some results:

Lemma 3.1 ([**10**, Lemma 5.1]). *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be an almost Kenmotsu manifold with ξ belonging to the generalized $(k, \mu)'$-nullity distribution and $h' \neq 0$. Then, for any $X, Y, Z \in \mathcal{D}$, one has*

$$R(X,Y)h'Z - h'R(X,Y)Z = (k+2)[g(Y,Z)h'X - g(X,Z)h'Y + g(h'X,Z)Y - g(h'Y,Z)X].$$

Lemma 3.2 ([**10**, Theorem 5.1]). *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be a generalized $(k, \mu)'$-almost Kenmotsu manifold such that $h' \neq 0$. Then for any $X_\lambda, Y_\lambda, Z_\lambda \in [\lambda]'$ and $X_{-\lambda}, Y_{-\lambda}, Z_{-\lambda} \in [-\lambda]'$, the Riemannian curvature tensor satisfies*

$$\begin{aligned}
R(X_\lambda, Y_\lambda)Z_{-\lambda} &= 0,\\
R(X_{-\lambda}, Y_{-\lambda})Z_\lambda &= 0,\\
R(X_\lambda, Y_{-\lambda})Z_\lambda &= (k+2)g(X_\lambda, Z_\lambda)Y_{-\lambda},\\
R(X_\lambda, Y_{-\lambda})Z_{-\lambda} &= -(k+2)g(Y_{-\lambda}, Z_{-\lambda})X_\lambda,\\
R(X_\lambda, Y_\lambda)Z_\lambda &= (k-2\lambda)[g(Y_\lambda, Z_\lambda)X_\lambda - g(X_\lambda, Z_\lambda)Y_\lambda],\\
R(X_{-\lambda}, Y_{-\lambda})Z_{-\lambda} &= (k+2\lambda)[g(Y_{-\lambda}, Z_{-\lambda})X_{-\lambda} - g(X_{-\lambda}, Z_{-\lambda})Y_{-\lambda}].
\end{aligned}$$

Lemma 3.3 ([**10**, Proposition 3.2]). *Let $(M^{2n+1}, \phi, \xi, \eta, g)$ be a generalized $(k, \mu)'$-almost Kenmotsu manifold such that $h' \neq 0$. Then*

$$\xi(\lambda) = -\lambda(\mu+2), \qquad \xi(k) = -2(k+1)(\mu+2).$$

Moreover, in both cases, if $2n+1 \geq 5$, then for any $X \in \mathcal{D}$

$$X(\lambda) = 0, \qquad X(k) = 0, \qquad X(\mu) = 0.$$

If $n > 1$, then the Ricci operator Q of M^{2n+1} is given by [**12**],

$$Q = -2nid + 2n(k+1)\eta \otimes \xi + [\mu - 2(n-1)]h'. \tag{9}$$

Moreover, the scalar curvature of M^{2n+1} is $2n(k-2n)$.

From (8), it follows that

$$R(\xi, X)Y = k[g(X,Y)\xi - \eta(Y)X] + \mu[g(h'X,Y)\xi - \eta(Y)h'X]. \tag{10}$$

Contracting X in (9), we have

$$S(Y, \xi) = 2nk\eta(Y). \tag{11}$$

4. Weyl semisymmetric almost Kenmotsu manifolds with ξ belonging to the generalized $(k, \mu)'$-nullity distribution

In this section, we characterize Weyl semisymmetric almost Kenmotsu manifolds with ξ belonging to the generalized $(k, \mu)'$-nullity distribution.

Using (8), (10), and (11) in (1), we have

$$C(X,Y)\xi = \alpha[\eta(Y)h'X - \eta(X)h'Y], \tag{12}$$

where

$$\alpha = \mu - \frac{\mu - 2(n-1)}{(2n-1)}.$$

Using (10), (11), and (11) in (1), yields

$$C(\xi, X)Y = \alpha[g(h'X, Y)\xi - \eta(Y)h'X]. \tag{13}$$

Now we prove the following

Theorem 4.1. *Let M^{2n+1} be an Weyl semisymmetric almost Kenmotsu manifold of dimension ≥ 5 with ξ belonging to the generalized $(k,\mu)'$-nullity distribution and $h' \neq 0$. Then the manifold is locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.*

Proof. Suppose $(R(X,Y) \cdot C)(U,V)W = 0$ for all smooth vector fields X, Y, U, V, W.

This implies

$$\begin{aligned} &R(X,Y)C(U,V)W - C(R(X,Y)U,V)W \\ &-C(U,R(X,Y)V)W - C(U,V)R(X,Y)W = 0. \end{aligned} \tag{14}$$

Putting $X = U = \xi$ in (14), yields

$$\begin{aligned} &R(\xi,Y)C(\xi,V)W - C(R(\xi,Y)\xi,V)W \\ &-C(\xi,R(\xi,Y)V)W - C(\xi,V)R(\xi,Y)W = 0. \end{aligned} \tag{15}$$

With the help of (10) and (13), we obtain

$$\begin{aligned} R(\xi,Y)C(\xi,V)W = {} & k\alpha[g(h'V,W)\eta(Y)\xi - g(h'V,W)Y - g(Y,h'V)\eta(W)\xi] \\ & - \mu\alpha[g(h'Y,h'V)\eta(W)\xi + g(h'V,W)h'Y]. \end{aligned} \tag{16}$$

Similarly, using (10) and (13), it follows that

$$\begin{aligned} C(R(\xi,Y)\xi,V)W = {} & k\alpha[g(h'V,W)\eta(Y)\xi - \eta(W)\eta(Y)h'V] \\ & - kC(Y,V)W - \mu C(h'Y,V)W. \end{aligned} \tag{17}$$

Further using (10) and (13), we have

$$\begin{aligned} C(\xi,R(\xi,Y)V)W = {} & -k\alpha[g(h'Y,W)\eta(V)\xi - \eta(W)\eta(V)h'Y] \\ & + \mu\alpha(k+1)[g(Y,W)\eta(V)\xi - \eta(W)\eta(V)h'Y]. \end{aligned} \tag{18}$$

Again using (10), (12), and (13), we get

$$\begin{aligned} C(\xi,V)R(\xi,Y)W = {} & -\alpha[kg(Y,W)h'V + \mu g(h'Y,W)h'V] + \mu g(h'V,h'Y)\eta(W)\xi \\ & - k\alpha[g(h'V,Y)\eta(W)\xi - \eta(Y)\eta(W)h'V]. \end{aligned} \tag{19}$$

Finally, substituting (16)-(19) in (15), yields

$$\begin{aligned} &kC(Y,V)W + \mu C(h'Y,V)W + \alpha[-kg(h'V,W)Y - \mu g(h'V,W)h'Y \\ &+kg(h'Y,W)\eta(V)\xi - k\eta(V)\eta(W)h'Y - \mu(k+1)g(Y,W)\eta(V)\xi \\ &+\mu(k+1)\eta(V)\eta(W)Y + kg(Y,W)h'V + \mu g(h'Y,W)h'V] = 0. \end{aligned} \tag{20}$$

Replacing Y by $h'Y$ and using the fact $h'^2 = (k+1)\phi^2$ in the above equation, we obtain

$$\begin{aligned} kC(h'Y,V)W + \mu(k+1)C(Y,V)W + \alpha[-kg(h'V,W)h'Y \\ +\mu(k+1)g(h'V,W)Y - k(k+1)g(Y,W)\eta(V)\xi \\ +k(k+1)\eta(V)\eta(W)Y - \mu(k+1)g(h'Y,W)\eta(V)\xi \\ +\mu(k+1)\eta(V)\eta(W)h'Y + kg(h'Y,W)h'V - \mu g(Y,W)h'V] = 0. \end{aligned} \tag{21}$$

Multiplying (20) by k and (21) by μ, then subtracting (21) from (20), implies

$$\begin{aligned} [C(Y,V)W + \alpha\{g(h'Y,W)\eta(V)\xi - \eta(V)\eta(W)h'Y \\ +g(Y,W)h'V - g(h'V,W)Y\}] = 0. \end{aligned} \tag{22}$$

Therefore, we infer either

$$k^2 + \mu^2(k+1) = 0$$

or

$$\begin{aligned} C(Y,V)W + \alpha\{g(h'Y,W)\eta(V)\xi - \eta(V)\eta(W)h'Y \\ +g(Y,W)h'V - g(h'V,W)Y\}] = 0. \end{aligned} \tag{23}$$

Case I:

$$\begin{aligned} C(Y,V)W + \alpha\{g(h'Y,W)\eta(V)\xi - \eta(V)\eta(W)h'Y \\ +g(Y,W)h'V - g(h'V,W)Y\}] = 0. \end{aligned} \tag{24}$$

Letting $Y, V, W \in \mathcal{D}(\lambda')$ and then using Lemma 3.2 in (1), it follows from (24) that

$$2n(k+1) - 2\lambda(\mu+1) - (2n-1)\lambda\alpha = 0. \tag{25}$$

Now letting $Y, V, W \in \mathcal{D}(-\lambda')$ and again using Lemma 3.2 in (1), it follows from (24) that

$$2n(k+1) + 2\lambda(\mu+1) + (2n-1)\lambda\alpha = 0. \tag{26}$$

Adding (25) and (26), we have

$$k = -1.$$

If $k = -1$, then from $h'^2 = (k+1)\phi^2$, we have $h' = 0$, which is a contradiction.
Case II:

$$k^2 + \mu^2(k+1) = 0. \tag{27}$$

Then using Lemma 3.3, we have $\mu = -2$ since $k \neq -1$. Using $\mu = -2$ in (27), implies $k = -2$. In this context, according to [**4**, Corollary 4.2], we know that M^{2n+1} is locally symmetric, that is, $\nabla R = 0$. Moreover, taking into account relation (8), we obtain $R(X,Y)\xi = 0$ for any $X, Y \in \mathcal{D}$. Hence according to [**5**, Theorem 6], it is proved that M^{2n+1} is locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$. □

Since conformally symmetric manifolds ($\nabla C = 0$) imply $R \cdot C = 0$, therefore, from the above theorem, we can state the following

Corollary 4.2. *A conformally symmetric almost Kenmotsu manifold of dimension ≥ 5 with ξ belonging to the generalized $(k,\mu)'$-nullity distribution and $h' \neq 0$, is locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.*

Again, since $R \cdot R = 0$ implies $R \cdot C = 0$, we obtain the following

Corollary 4.3. *A semisymmetric almost Kenmotsu manifold of dimension ≥ 5 with ξ belonging to the generalized $(k,\mu)'$-nullity distribution and $h' \neq 0$, is locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.*

Remark. The above corollaries were proved by Wang et al. in [**16**].

5. Weyl Ricci semisymmetric almost Kenmotsu manifolds with ξ belonging to the generalized $(k,\mu)'$-nullity distribution

Definition 5.1. An almost Kenmotsu manifold with ξ belonging to the generalized $(k,\mu)'$-nullity distribution is said to be Weyl Ricci semisymmetric if $C \cdot S = 0$.

In this section, we characterize Weyl Ricci semisymmetric almost Kenmotsu manifolds with ξ belonging to the generalized $(k,\mu)'$-nullity distribution.

Now we prove the following

Theorem 5.2. *Let M^{2n+1} be an almost Kenmotsu manifold of dimension ≥ 5 with ξ belonging to the generalized $(k,\mu)'$-nullity distribution and $h' \neq 0$ satisfying the curvature condition $C \cdot S = 0$. Then the manifold is an η-Einstein manifold.*

Proof. Suppose $(C(X,Y) \cdot S)(U,V) = 0$ for all smooth vector fields X, Y, U, V. That implies

$$S(C(X,Y)U,V) + S(U, C(X,Y)V) = 0. \tag{28}$$

Putting $X = U = \xi$ in (28), yields

$$S(C(\xi,Y)\xi,V) + S(\xi, C(\xi,Y)V) = 0. \tag{29}$$

Using (12) and (13) in (29), we obtain

$$\alpha[S(h'Y,V) + 2nkg(h'Y,V)] = 0. \tag{30}$$

Putting $Y = h'Y$ in (30), and using the fact $h'^2 = (k+1)\phi^2$ yield

$$(k+1)[S(\phi^2 Y,V) + 2nk\alpha g(\phi^2 Y,V)] = 0 \tag{31}$$

since $\alpha \neq 0$.

Using (3) in (30), we have

$$(k+1)[S(Y,V) + 2nkg(Y,V) - 4nk\eta(Y)\eta(V)] = 0. \tag{32}$$

Therefore, either $k+1 = 0$ or $S(Y,V) + 2nkg(Y,V) - 4nk\eta(Y)\eta(V) = 0$. If $(k+1) = 0$, then from $h'^2 = (k+1)\phi^2$, we have $h' = 0$, which is a contradiction. Hence $S(Y,V) = -2nkg(Y,V) + 4nk\eta(Y)\eta(V)$, which implies that the manifold is an η-Einstein manifold. □

6. h'-semisymmetric almost Kenmotsu manifolds with ξ belonging to the generalized $(k,\mu)'$-nullity distribution

This section is devoted to study h'-semisymmetric almost Kenmotsu manifolds with ξ belonging to the generalized $(k,\mu)'$-nullity distribution.

We prove the following

Theorem 6.1. *Let M^{2n+1} be an h'-semisymmetric almost Kenmotsu manifold of dimension ≥ 5 with ξ belonging to the generalized $(k,\mu)'$-nullity distribution and $h' \neq 0$. Then the manifold is locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$.*

Proof. Suppose M^{2n+1} is h'-semisymmetric. Then $(R(X,Y)\cdot h')Z = 0$ implies

$$R(X,Y)h'Z - h'R(X,Y)Z = 0 \tag{33}$$

for all smooth vector fields X, Y, Z. Then from Lemma 3.1, we have

$$(k+2)[g(Y,Z)h'X - g(X,Z)h'Y + g(h'X,Z)Y - g(h'Y,Z)X] = 0. \tag{34}$$

Putting $Y = Z = \xi$ in (34), implies

$$(k+2)h'X = 0. \tag{35}$$

Replacing X by $h'X$ in (35) and using the fact $h'^2 = (k+1)\phi^2$ we have

$$(k+1)(k+2)[-X + \eta(X)\xi] = 0. \tag{36}$$

Taking inner product with W in (36), then putting $X = W = e_i$, and taking summation over i, $1 \leq i \leq (2n+1)$, where $\{e_1, e_2, e_3, \ldots, e_{2n+1}\}$ is a local orthonormal basis of the tangent space at a point of the manifold M, we obtain

$$(k+1)(k+2) = 0.$$

Therefore, either $(k+1) = 0$ or $(k+2) = 0$. If $(k+1) = 0$, then from $h'^2 = (k+1)\phi^2$, we have $h' = 0$, which is a contradiction. Hence $k = -2$. Making use of this from Lemma 3.3 we have $\mu = -2$. In this context, according to [**4**, Corollary 4.2], we know that M^{2n+1} is locally symmetric, that is, $\nabla R = 0$. Moreover, taking into account relation (3.1), we obtain $R(X,Y)\xi = 0$ for any $X, Y \in \mathcal{D}$. Hence according to [[**5**], Theorem 6], it is proved that M^{2n+1} is locally isometric to the Riemannian product $\mathbb{H}^{n+1}(-4) \times \mathbb{R}^n$. □

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