

## COLORING HYPERGRAPHS DEFINED BY STABBED PSEUDO-DISKS AND *ABAB*-FREE HYPERGRAPHS

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ABSTRACT. What is the minimum number of colors that always suffice to color every planar set of points such that any disk that contains enough points contains two points of different colors? It is known that the answer to this question is either three or four. We show that three colors always suffice if the condition must be satisfied only by disks that contain a fixed point. Our result also holds, and is even tight, when instead of disks we consider their topological generalization, namely *pseudo-disks*, with a non-empty intersection. Our solution uses the equivalence that a hypergraph can be realized by stabbed pseudo-disks if and only if it is *ABAB-free*. These hypergraphs are defined in a purely abstract, combinatorial way and our proof that they are 3-chromatic is also combinatorial.

### 1. INTRODUCTION

Given a family of hypergraphs  $\mathcal{H}$  and a positive integer  $c$ , let  $m(\mathcal{H}, c)$  denote the least integer such that the vertices of every hypergraph  $H \in \mathcal{H}$  can be colored with  $c$  colors such that every hyperedge of size at least  $m(\mathcal{H}, c)$  is non-monochromatic (i.e., contains two vertices with different colors). In other words, for every hypergraph  $H \in \mathcal{H}$  the sub-hypergraph of  $H$  that consists of all the hyperedges of size at least  $m(\mathcal{H}, c)$  is  $c$ -colorable. We denote by  $\chi_m(\mathcal{H})$  the least integer  $c$  for which such a finite  $m(\mathcal{H}, c)$  exists.

A family of geometric (or topological) regions  $\mathcal{F}$  and a set of points  $S$  naturally define a hypergraph  $H(S, \mathcal{F})$  whose vertices are the points in  $S$  and whose hyperedge set consists of every subset  $S' \subseteq S$  for which there is a region  $F' \in \mathcal{F}$  such that  $S' = F' \cap S$ . The family of (finite) hypergraphs  $\mathcal{H}(\mathcal{F})$  defined by a family of geometric regions  $\mathcal{F}$  consists of all the hypergraphs  $H(S, \mathcal{F})$  for some (finite) point set  $S$ . We also say that  $\mathcal{F}$  can *realize*  $H(S, \mathcal{F})$ . By a slight abuse of notation we thus write  $m(\mathcal{F}, c)$  and  $\chi_m(\mathcal{F})$  instead of  $m(\mathcal{H}(\mathcal{F}), c)$  and  $\chi_m(\mathcal{H}(\mathcal{F}))$ , respectively.

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Typically, one is interested in determining whether it holds that  $\chi_m(\mathcal{F}) = 2$  or at least  $\chi_m(\mathcal{F}) < \infty$  for a given family of geometric regions  $\mathcal{F}$ . These questions are motivated by problems concerning *cover-decomposability* and *conflict-free* colorings. For more about these connections we refer to the surveys [12, 17]. For example, it is known [3] that  $m(\mathcal{F}_\square, 2) \leq 215$ , where  $\mathcal{F}_\square$  is the family of axis-parallel squares in the plane. In other words, it is possible to color any set of points in the plane with the colors blue and red, such that every axis-parallel square that contains at least 215 points from this set of points contains a blue point and a red point. Since, by definition,  $\chi_m(\mathcal{H}) > 1$ , it follows that  $\chi_m(\mathcal{F}_\square) = 2$ . On the other hand, considering the family of axis-parallel rectangles  $\mathcal{F}_\square$ , it is known [6] that  $\chi_m(\mathcal{F}_\square)$  is infinite.

An intriguing question is to determine  $\chi_m(\mathcal{F}_\circ)$ , where  $\mathcal{F}_\circ$  is the family of disks in the plane. It follows from the Four Color Theorem and the planarity of Delaunay-triangulations, that any finite set of points can be 4-colored such that no disk containing at least two points is monochromatic, i.e.,  $m(\mathcal{F}_\circ, 4) = 2$ , and thus  $\chi_m(\mathcal{F}_\circ) \leq 4$ . It is also known [13] that  $\chi_m(\mathcal{F}_\circ) > 2$ . Moreover,  $\chi_m(\mathcal{F}) > 2$  even when  $\mathcal{F}$  is the family of *unit* disks [11]. Therefore, it remains an open problem whether  $\chi_m(\mathcal{F}_\circ) = 3$  or  $\chi_m(\mathcal{F}_\circ) = 4$ .

We consider a generalization of disks, namely, *pseudo-disks*. Roughly speaking, a family of regions is a family of pseudo-disks if they behave like disks in the sense that the boundaries of every two regions intersect at most twice. We say that a family of regions is *stabbed* if their intersection is non-empty, that is, there exist a point that *stabs* (i.e., it is contained in) all the regions. We say that a family of regions is *internally stabbed* if the intersection of their interiors is non-empty. Our main result is that coloring with three colors is possible (and sometimes necessary) for families of stabbed pseudo-disks.

**Theorem 1.** *Let  $\mathcal{F}$  be a family of pseudo-disks whose intersection is non-empty and let  $S$  be a finite set of points. Then it is possible to color the points in  $S$  with three colors such that any pseudo-disk in  $\mathcal{F}$  that contains at least two points from  $S$  contains two points of different colors. Moreover, for every integer  $m$  there is a set of points  $S$  and a family of pseudo-disks  $\mathcal{F}$  with a non-empty intersection, such that for every 2-coloring of the vertices of the hypergraph  $H(S, \mathcal{F})$  there is a hyperedge of size at least  $m$  which is monochromatic.*

To summarize with our notation,  $m(\mathcal{F}_\circ, 3) = 2$  and  $\chi_m(\mathcal{F}_\circ) = 3$ , where we denote by  $\mathcal{F}_\circ$  the families of stabbed pseudo-disks.

It is important to note that the above-mentioned construction from [11] of a family  $\mathcal{F}$  of unit disks (or more generally, *translates* of any region with a smooth boundary) such that  $\chi_m(\mathcal{F}) > 2$  is not a family of stabbed pseudo-disks (although it is stabbed by two points, that is, there are two points such that every region contains at least one of them).

From Theorem 1 it is easy to conclude the following.

**Corollary 2.** *Given a finite set of points  $S$  it is possible to color the points of  $S$  with three colors such that any disk that contains the origin and at least two points from  $S$  contains two points with different colors.*

This corollary is already nontrivial for unit disks containing the origin. By a well-known duality concerning translates of regions (see e.g., [12]) we have:

**Corollary 3.** *It is possible to decompose a sufficiently thick covering of any region of radius at most one by finitely many unit disks into three parts such that any two of the three parts cover the whole region.*

We present two proofs for the upper bound  $\chi_m(\mathcal{F}_\odot) \leq 3$  of Theorem 1. The first proof is a direct proof that uses some previous results about the so-called “shrinkability” of a family of pseudo-disks [5, 16] that rely on a highly nontrivial sweeping machinery from [18].<sup>1</sup> For some of these results we provide new and simplified proofs. Our second proof of the upper bound  $\chi_m(\mathcal{F}_\odot) \leq 3$  is completely self-contained and of a more combinatorial flavor. It is based on an equivalence between hypergraphs defined by stabbed pseudo-disks and *ABAB-free* hypergraphs. This equivalence also implies that  $\chi_m(\mathcal{F}_\odot) \geq 3$  following a result from [8]. *ABAB-free* hypergraphs. Let  $l \geq 1$  be a number such that  $2l$  is an integer. We denote by  $(AB)^l$  the alternating sequence of letters  $A$  and  $B$  of length  $2l$ . For example,  $(AB)^{1.5} = ABA$  and  $(AB)^2 = ABAB$ .

**Definition 4** ( $(AB)^l$ -free hypergraphs).

1. Two subsets  $A, B$  of an ordered set of elements form an  $(AB)^l$ -sequence if there are  $2l$  elements  $a_1 < b_1 < a_2 < b_2 < \dots$  such that  $\{a_1, a_2, \dots\} \subset A \setminus B$  and  $\{b_1, b_2, \dots\} \subset B \setminus A$ .
2. A hypergraph with an ordered vertex set is  $(AB)^l$ -free if it does not contain two hyperedges  $A$  and  $B$  that form an  $(AB)^l$ -sequence.
3. A hypergraph with an unordered vertex set is  $(AB)^l$ -free if there is an order of its vertices such that the hypergraph with this ordered vertex set is  $(AB)^l$ -free.
4. The family of all  $(AB)^l$ -free hypergraphs is denoted by  $(\mathcal{A}B)^l$ -free.

$(AB)^l$ -free hypergraphs were introduced in [8], where it was shown that *ABA-free* hypergraphs are equivalent to hypergraphs defined by pseudo-halfplanes. It was also proved in [8] that  $\chi_m(\mathcal{A}B\mathcal{A}\text{-free}) = 2$  (along with further strengthenings) and that  $\chi_m(\mathcal{A}B\mathcal{A}B\text{-free}) > 2$ .

**Theorem 5** ([8]). *For every  $m \geq 2$  there exists an *ABAB-free*  $m$ -uniform hypergraph which is not 2-colorable.*

Here we extend these results by showing that  $m(\mathcal{A}B\mathcal{A}B\text{-free}, 3) = 2$  which implies that  $\chi_m(\mathcal{A}B\mathcal{A}B\text{-free}) = 3$ .

**Theorem 6.** *Every *ABAB-free* hypergraph is proper 3-colorable.*

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<sup>1</sup>We would like to note that some of these papers (sometimes implicitly) assume stricter conditions, like no three pseudo-disks should pass through a point. We believe that these conditions could be removed with some extra care, but that would require to repeat the whole argument. Therefore, we do not go into details, especially since we also give a self-contained proof for our main result.

Theorem 1 then follows from Theorems 1 and 5 and an equivalence between  $ABAB$ -free hypergraphs and hypergraphs defined by stabbed pseudo-disks.

**Theorem 7.** *A hypergraph is  $ABAB$ -free if and only if it can be realized by a family of stabbed pseudo-disks.*

As a side question we consider  $(AB)^l$ -free hypergraphs for  $l > 2$  and show using a construction similar to the one from [8] that  $\chi_m(\mathcal{A}BABA\text{-free}) = \infty$ .

**Theorem 8.** *For every  $c \geq 2$  and  $m \geq 2$  there exists an  $ABABA$ -free  $m$ -uniform hypergraph which is not  $c$ -colorable.*

**Further related work.** As we mentioned before,  $\chi_m(\mathcal{F}_\square) = 2$ , where  $\mathcal{F}_\square$  denotes the family of axis-parallel squares. By affine transformations the same result holds for families of homothets of a fixed parallelogram. It is also known that  $\chi_m(\mathcal{F}_\Delta) = 2$ , for each family  $\mathcal{F}_\Delta$  of homothets of a given triangle [7]. There are also good estimates of  $m(\mathcal{F}_\Delta, 2)$ , namely,  $5 \leq m(\mathcal{F}_\Delta, 2) \leq 9$  [9]. Pálvölgyi and Tóth [15] proved that for a family  $\mathcal{F}$  of translates of a given open convex polygon  $\chi_m(\mathcal{F}) = 2$ . Perhaps the most interesting open problem concerning 2-coloring is whether the same bound holds for homothets of a given convex polygon. Pálvölgyi and Keszegh [10] showed that  $\chi_m \leq 3$  in this case. For further results about translates and homothets of convex shapes, see e.g., [12, 7, 3, 10] and the webpage [1].

**Outline.** Due to space limitations most of the proofs are omitted and can be found in the full version of this paper [2]. In Section 2 we prove that every  $ABAB$ -free hypergraph is 3-colorable. We conclude with some remarks and open problems in Section 3.

## 2. COLORING $ABAB$ -FREE HYPERGRAPHS

In this section we prove Theorem 6 which says that every  $ABAB$ -free hypergraph is 3-colorable.

Let  $H$  be an  $ABAB$ -free hypergraph. A pair of vertices of  $H$  is called *unsplittable* if by adding this pair as a hyperedge of size two to  $H$  we get an  $ABAB$ -free hypergraph. For a pair of vertices  $E = \{p, q\}$  we say that a hyperedge  $B$  *splits* this pair if  $E$  and  $B$  form an  $EBEB$ - or  $BEBE$ -sequence.

**Lemma 9.** *Every hyperedge of an  $ABAB$ -free hypergraph contains a pair of vertices that is unsplittable.*

*Proof.* Let  $A$  be a hyperedge of an  $ABAB$ -free hypergraph  $H$ . If  $A$  is of size two, then its vertices form an unsplittable pair, for otherwise there would be a hyperedge  $B$  that splits  $A$  and this would contradict that  $H$  is  $ABAB$ -free.

Thus we may assume that  $A$  is of size at least 3. Consider a left-to-right order of the vertices of  $H$  by which  $H$  is  $ABAB$ -free. We write  $a < b$  if  $a$  and  $b$  are two vertices of  $A$  such that  $a$  is to the left of  $b$ . Denote the vertices of  $A$  according to their order by  $A = \{a_1, a_2, \dots, a_k\}$ . Two such vertices are called *consecutive* if one follows the other in this order. We will prove that one of the consecutive pairs of vertices of  $A$  is an unsplittable pair.

Assume on the contrary that none of the consecutive pairs is unsplittable. A consecutive pair  $E = \{a_i, a_{i+1}\}$  is *left-splittable* (resp., *right-splittable*) if there exists a hyperedge  $B \in H$  such that they together form a  $BEBE$ -sequence (resp.,  $EBEB$ -sequence). By our assumption every consecutive pair is either left-splittable or right-splittable or both. A consecutive pair is called *one-sided splittable* (or simply *one-sided*) if it is not both left-splittable and right-splittable. Notice that the leftmost consecutive pair  $C = \{a_1, a_2\}$  cannot be left-splittable. Indeed, a hyperedge  $B$  left-splitting it would also form a  $BCBC$ -sequence, which is a contradiction. Similarly, the rightmost consecutive pair cannot be right-splittable. Thus the family of one-sided splittable pairs is non-empty.

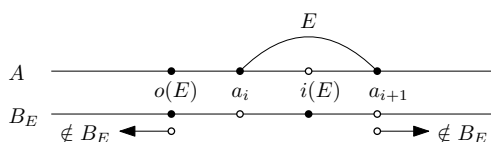


Figure 1.  $E$  is an only left-sided pair with witness hyperedge  $B_E$ .

For each only left-sided pair  $E = \{a_i, a_{i+1}\}$  let  $B_E$  be a hyperedge that together with  $E$  forms a  $B_EEB_EE$ -sequence, see Figure 1. The existence of this sequence implies that  $a_i, a_{i+1} \in E \setminus B_E$  and that there is a vertex  $i(E) \in B_E \setminus A$  among the vertices of  $H$  between  $a_i$  and  $a_{i+1}$  (in the left-to-right order of the vertices of  $H$ ). The leftmost vertex of  $B_E$  is denoted by  $o(E)$ . As  $E$  is left-sided and  $B_E$  is a witness for that, it follows that  $o(E) < a_i$ . Also,  $o(E) \in A \cap B_E$  since if  $o(E) \notin A$  then  $o(E), a_i, i(E), a_{i+1}$  would form a  $B_EAB_EA$ -sequence, a contradiction. Note that there is no vertex in  $B_E$  to the left of  $o(E)$  by definition and there is no vertex in  $B_E$  to the right of  $a_{i+1}$ , for otherwise  $E$  would also be a right-sided pair.

Similarly, for each only right-sided pair  $E = \{a_i, a_{i+1}\}$  take a witness hyperedge  $B_E$  with which it forms an  $EB_EEB_E$ -sequence. Thus  $a_i, a_{i+1} \in A \setminus B_E$  and there is a vertex  $i(E) \in B_E \setminus A$  among the vertices of  $H$  between  $a_i$  and  $a_{i+1}$ . In this case denote by  $o(E)$  the rightmost vertex of  $B_E$ . Therefore,  $a_{i+1} < o(E)$  and, as before, we have that  $o(E) \in A \cap B_E$ .

Among all one-sided pairs of  $A$  let  $E = \{a_i, a_{i+1}\}$  be the pair with the least number of vertices of  $H$  between  $i(E)$  and  $o(E)$ . Without loss of generality we may assume that  $E$  is only right-sided.

As  $o(E) \in A$  and  $a_{i+1} < o(E)$ ,  $o(E) = a_{j+1}$  for some  $j > i$ . Consider the pair  $F = \{a_j, a_{j+1}\}$  (note that  $a_j$  may coincide with  $a_{i+1}$ ). We claim that  $F$  cannot be a right-sided pair. Indeed, assume to the contrary that there exists a hyperedge  $C$  and two vertices  $c_1, c_2 \in C \setminus F$  such that  $a_j < c_1 < a_{j+1} < c_2$  (and therefore  $a_j, c_1, a_{j+1}, c_2$  form an  $FCFC$ -sequence), see Figure 2a. Since  $o(E) = a_{j+1} < c_2$  and  $o(E)$  is the rightmost vertex of  $B_E$ , we also have  $c_2 \notin B_E$ . Also,  $i(E) \notin C$ , otherwise  $i(E), a_j, c_1, a_{j+1}$  would form a  $CACA$ -sequence, a contradiction. Similarly,  $c_1 \notin B_E$ , otherwise  $a_i, i(E), a_{i+1}, o(E)$  would form an

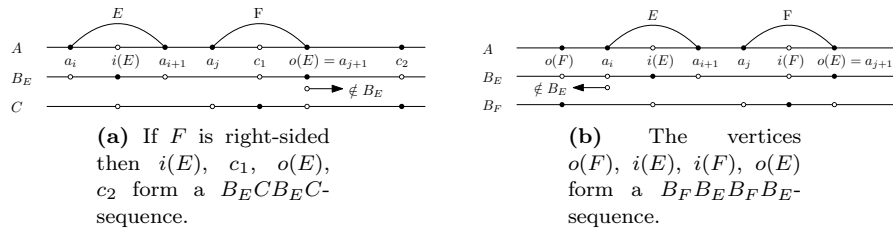


Figure 2. Illustrations for the proof of Lemma 9.

$AB_E AB_E$ -sequence. However, then  $i(E), c_1, a_{j+1}, c_2$  form a  $B_E C B_E C$ -sequence, which is again a contradiction.

Therefore,  $F$  is an only left-sided pair and thus  $o(F) < a_j$ . See Figure 2b. Furthermore,  $o(F) \leq a_i$  for otherwise there would be less vertices of  $H$  between  $o(F)$  and  $i(F)$  than there are between  $o(E)$  and  $i(E)$ , contradicting our choice of  $E$ . We have that  $o(F) \in B_F \cap A$  for otherwise  $o(F), a_j, i(F), a_{j+1}$  would be a  $B_F A B_F A$ -sequence. Furthermore,  $o(F) \notin B_E$  since  $o(F) \leq a_i$  and no vertex of  $B_E$  is left of  $a_i$ . Similarly,  $i(E) \notin B_F$  as otherwise  $i(E), a_j, i(F), o(E)$  would form a  $B_F A B_F A$ -sequence. Finally,  $i(F) \notin B_E$  for otherwise  $a_i, i(E), a_{i+1}, i(F)$  would form an  $AB_E AB_E$ -sequence.

Thus, the vertices  $o(F), i(E), i(F), o(E)$  form a  $B_F B_E B_F B_E$ -sequence, leading to the final contradiction.  $\square$

*Proof of Theorem 6.* Let  $H$  be an  $ABAB$ -free hypergraph. We call a hyperedge of size at least 3 *unhit* if it does not contain as a subset a hyperedge of size 2. Starting from  $H$  we create a series of hypergraphs as follows. If the current hypergraph contains an unhit hyperedge, then by Lemma 9 this hyperedge contains an unsplittable pair which we add as a new hyperedge and obtain the next hypergraph in our series. Since  $H$  has a finite number of hyperedges and every hypergraph has one less unhit hyperedge than its preceding hypergraph, we get a finite series of hypergraphs. Let  $H'$  be the last hypergraph in this series.

Let  $G$  be the graph that is induced by the hyperedges of  $H'$  of size two. Note that every hyperedge of  $H'$  contains at least one edge of  $G$ . Therefore, a proper coloring of  $G$  is a proper coloring of  $H$ . The graph  $G$  also has the  $ABAB$ -free property. Consider the following drawing of  $G$ . Its vertices are represented by distinct points on a horizontal line according to their  $ABAB$ -free order and its edges are drawn as circular arcs above the line. Since  $G$  is  $ABAB$ -free its drawing does not contain crossing edges. Furthermore, this drawing of  $G$  is outerplanar. Since every outerplanar graph is 3-colorable, this completes the proof.  $\square$

As mentioned in the introduction, using Theorem 7 this also proves the upper bound of Theorem 1.

## 3. DISCUSSION

In the paper we show that pseudo-disk hypergraphs are equivalent to  $ABAB$ -free hypergraphs, and they are properly 3-colorable. Similar questions can be studied about dual- $ABAB$ -free hypergraphs as well, which is equivalent to the so-called cover-decomposition problem for stabbed pseudo-disks. Another version is to forbid  $ABABA$ -sequences cyclically (instead of linearly); such 3-uniform hypergraphs have a nice geometric representation, as *convex geometric 3-hypergraphs* without *strongly crossing* edges, see Suk [19]. It is also a natural question to ask whether strongly crossing convex geometric (non-uniform) hypergraphs can be always 3-colored.

We would also like to remark that having VC-dimension at most  $2l - 1$  is a weaker assumption than being  $(AB)^l$ -free. For any  $c$  and  $m$  there are  $m$ -uniform hypergraphs of VC-dimension 2 that are not  $c$ -colorable; the main construction from both [13] and [14] can be generalized from 2-colors to  $c$ -colors as  $m$ -uniform hypergraphs of VC-dimension 2.

An interesting connection to Radon-partitions is the following. Given three points in  $\mathbb{R}^1$ , they have a unique Radon-partition into two sets,  $A$  and  $B$ , whose convex hulls intersect; the points must follow each other in the order  $A, B, A$ , so this cannot happen for  $ABA$ -free families. Given four points in  $\mathbb{R}^2$ , there are two possible Radon-partitions; the first is when three points of  $A$  contain the only point of  $B$  inside their convex hull, while the second is when there are two points in each of  $A$  and  $B$  such that their connecting segments intersect. Note that none of these configurations are possible for points from the symmetric difference of convex pseudo-disks, i.e., if  $A$  and  $B$  are convex pseudo-disks, then we cannot pick points from  $A \setminus B$  and  $B \setminus A$  that form a Radon-partition. We wonder whether this has some higher dimensional generalizations, or is just a coincidence.

The most natural problem left open is whether  $\chi_m(\mathcal{F}_\circ) = 3$  or  $\chi_m(\mathcal{F}_\circ) = 4$ .

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