Acta Math. Univ. Comenianae
Vol. LXXXIV, 2 (2015), pp. 179–190

OPTION PRICING WITH DYNAMICALLY CORRELATED STOCHASTIC INTEREST RATE

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Abstract. In this work we review several option pricing models with stochastic interest rate and extend these models by incorporating a local time dependent correlation between the underlying and the interest rate. We compare the difference between using a constant and a dynamic correlation by analyzing some numerical benchmarks. Furthermore, we conduct experiments on fitting the pricing model to the market price. Our analysis shows that the option pricing within the Black-Scholes framework can not be improved significantly by incorporating stochastic interest rate even when using a nonlinear correlation term.

1. Introduction

The Black-Scholes model [**2**] defining the fair price of European-style options is one of the most famous models. However, due to the assumption that the stock log-return follows a geometric Brownian motion (with constant volatility), the widening gap between model and market data could exist almost all the time. For this reason, the Black-Scholes model has been generalized to allow stochastic volatility, see e.g. [**5**], [**6**], and hence the pricing performance has been thus improved.

The other strong assumption of a constant interest rate is also not realistic. The first work on incorporating stochastic interest rate into the Black-Scholes model is Merton [**8**]. Afterwards, a couple of works on option pricing under stochastic interest rate was published, e.g. [**1**], [**3**], [**4**] and [**9**]. However, some empirical findings indicated that stochastic interest rates may be not relevant for the pricing and hedging of short term options, see e.g. [**4**] and [**7**]. Besides, the paper [**3**] concluded that allowing interest rates to be stochastic does not necessarily improve the pricing performance any further, even for long-term options, once the model has accounted for stochastically varying volatility.

We have realized that the correlation between interest rates process and underlying process in the works mentioned above has been assumed to be constant. Unfortunately, this assumption is also dubious due to the fact that financial quantities may be correlated in a nonlinear way, even stochastically, see [**10**], [**11**] and [**12**]. Besides, it has been inferred in [**12**] and [**13**] that the Heston model and the

Received September 27, 2014; revised Februar 10, 2015.
2000 *Mathematics Subject Classification.* Primary 91G20, 91G30, 91G80, 35Q91.
Key words and phrases. Option pricing; atochastic interest rate; dynamic correlation.

model of Quanto-option pricing can be better fitted to the market data using a dynamic (i.e. only time-dependent) correlation than using a constant correlation. Thus, it is interesting to ask whether stochastic interest rates could be relevant for the hedging and pricing of options if the correlation between the interest rates and the underlying asset is not considered as a constant.

Motivated by this question, we review and extend some option pricing models with stochastic interest rate by allowing for a nonconstant correlation. Firstly, we compare the option pricing between using a constant and a nonconstant correlation by analyzing some numerical results. Secondly, we conduct an experiment on fitting the pricing models to the market data, in order to check, whether stochastic interest rates are important for option pricing while allowing a nonconstant correlation.

The paper is organized as follows. In the next section, we review and extend two different pricing models with stochastic interest rate and dynamic correlation. Section 3 is devoted to investigate the difference of model calibration between using a constant and a dynamic correlation. Finally, Section 4 concludes this work.

2. Option Pricing with dynamically correlated Stochastic Interest Rate

In this Section, we consider two pricing models with stochastic interest rate. First, we review and extend the Merton model [**8**] of pricing European option where bond price dynamics are allowed. Besides, we study the option pricing model with stochastic interest rate given by Vasicek stochastic differential equation (SDE) in [**9**] and [**7**].

2.1. The Merton model

We use the following SDE to describe the stock price S_t and the bond price P_t dynamics

$$\frac{\mathrm{d}S_t}{S_t} = \mu_S \mathrm{d}t + \sigma_S \mathrm{d}W_t^1, \tag{1}$$

$$\frac{\mathrm{d}P_t}{P_t} = \mu_P \mathrm{d}t + \sigma_P \rho_t \mathrm{d}W_t^1 + \sigma_P \sqrt{1-\rho_t^2} dW_t^2, \tag{2}$$

with the instantaneous expected returns μ_S, μ_P, the instantaneous variances σ_S^2, σ_P^2 and the two independent Brownian motions W_t^1, W_t^2. Let $P(t,T)$ be the bond price which pays one unit of currency at maturity T (or say $\tau = T - t$ years later from now). Besides, we denote the European option price function by $H(S,P,\tau;K)$ for using the constant correlation $\rho_t = \rho$ between the returns on the stock and on the bond and by $V(S,P,\rho_\tau,\tau;K)$ for using the corresponding dynamic correlation ρ_t, where K is the strike price. Merton [**8**] has shown that $H(S,P,\tau;K)$ must satisfy

$$\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 H}{\partial S^2} + \rho \sigma_S \sigma_P SP \frac{\partial^2 H}{\partial S \partial P} + \frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 H}{\partial P^2} - \frac{\partial H}{\partial \tau} = 0 \tag{3}$$

subject to the boundary conditions

$$\begin{cases} H(0, P, \tau; K) = 0 \\ H(S, 1, 0; K) = \max(0, S - K). \end{cases} \tag{4}$$

(3) is a second-order, linear partial differential equation (PDE) of parabolic type with a singularity at $S = 0$. Furthermore, if we assume that the returns on the stock and on the bond are correlated with an appropriate time-varying function ρ_t (without stochasticity), it is straightforward to deduce

$$\frac{1}{2}\sigma_S^2 S^2 \frac{\partial^2 V}{\partial S^2} + \underline{\rho_t} \sigma_S \sigma_P SP \frac{\partial^2 V}{\partial S \partial P} + \frac{1}{2}\sigma_P^2 P^2 \frac{\partial^2 V}{\partial P^2} - \frac{\partial V}{\partial \tau} = 0, \tag{5}$$

subject to the boundary conditions

$$\begin{cases} V(0, P, \rho_\tau, \tau; K) = 0 \\ V(S, 1, \rho_0, 0; K) = \max(0, S - K). \end{cases} \tag{6}$$

Following the methodologies of [**8**], we define $x = \frac{S}{KP_\tau}$ which can be described with the aid of Itô's lemma as

$$\begin{aligned} \frac{\mathrm{d}x}{x} &= [\mu_S - \mu_P + \sigma_P^2 - \rho_t \sigma_P \sigma_S]\mathrm{d}t + \sigma_S \mathrm{d}W_t^1 \\ &\quad - \sigma_P \rho_t \mathrm{d}W_t^1 - \sigma_P \sqrt{1 - \rho_t^2}\mathrm{d}W_t^2, \end{aligned} \tag{7}$$

from which we obtain the instantaneous variance of the return on x given by

$$\sigma_t^2 := \sigma_P^2 + \sigma_S^2 - 2\rho_t \sigma_P \sigma_S. \tag{8}$$

Next, we define $v = \frac{V}{KP}$ and substitute x and v in (5) to get

$$\frac{1}{2}\sigma_t^2 x^2 \frac{\partial^2 v}{\partial x^2} - \frac{\partial v}{\partial \tau} = 0. \tag{9}$$

Finally, we consider a new time variable $T(\tau) := \int_0^\tau \sigma_t^2 \mathrm{d}t$ and define $y(x, T) := v(x, \tau)$ which can be substituted into (9) to obtain the heat equation

$$\frac{1}{2}x^2 \frac{\partial^2 y}{\partial x^2} - \frac{\partial y}{\partial \tau} = 0, \tag{10}$$

subject to the boundary conditions, $y(0, T) = 0$ and $y(x, 0) = \max(0, x - 1)$. It is well-know the fact that the heat equation (10) can be solved analytically. The the solution of $V(S, P, \rho_\tau, \tau; K)$ can thus be found as

$$V(S, P, \rho_\tau, \tau; K) = S\Phi(d_1) - KP\Phi(d_2), \tag{11}$$

with

$$d_1 := \frac{\ln \frac{S}{K} - \ln P + \frac{1}{2}\int_0^\tau \sigma_s^2 \mathrm{d}s}{\sqrt{\int_0^\tau \sigma_s^2 \mathrm{d}s}}, \qquad d_2 := d_1 - \sqrt{\int_0^\tau \sigma_s^2 \mathrm{d}s},$$

where σ_t is defined in (8) and $\Phi(x)$ denotes the standard normal cumulative distribution function. So far, in order to compute the European call option price we need to know the formula of P_τ and a reasonable local correlation function ρ_t.

Following the methodologies of [**8**] we assume that the short rate r_t follows a Gauss-Wiener process [1]

$$\mathrm{d}r_t = \mu_r \mathrm{d}t + \sigma_r \rho_t \mathrm{d}W_t^1 + \sigma_r \sqrt{1-\rho_t^2}\mathrm{d}W_t^2. \tag{12}$$

Applying Itô's lemma with $P(\tau; r)$ we obtain

$$\mathrm{d}P = \frac{\partial P}{\partial \tau}\mathrm{d}\tau + \frac{\partial P}{\partial r}\mathrm{d}r_t + \frac{1}{2}\frac{\partial^2 P}{\partial r^2}(\mathrm{d}r_t)^2. \tag{13}$$

Substituting (12) into (13) leads to

$$\begin{aligned}\mathrm{d}P = &\left(-\frac{\partial P}{\partial \tau} + \mu_r \frac{\partial P}{\partial r} + \frac{\sigma_r^2}{2}\frac{\partial^2 P}{\partial r^2}\right)\mathrm{d}t \\ &+ \sigma_r \rho_t \frac{\partial P}{\partial r}\mathrm{d}W_t^1 + \sigma_r\sqrt{1-\rho_t^2}\frac{\partial P}{\partial r}\mathrm{d}W_t^2.\end{aligned} \tag{14}$$

By comparing the coefficients in (14) and (2) we get

$$-\frac{\partial P}{\partial \tau} + \mu_r \frac{\partial P}{\partial r} + \frac{\sigma_r^2}{2}\frac{\partial^2 P}{\partial r^2} = P\mu_P \quad \text{and} \quad \sigma_r \frac{\partial P}{\partial r} = P\sigma_P, \tag{15}$$

which gives

$$\sigma_P = -\tau\sigma_r, \tag{16}$$

and

$$P(\tau; r) = \exp\Big(-r\tau - \frac{\mu_r}{2}\tau^2 + \frac{\sigma_r^2}{6}\tau^3\Big). \tag{17}$$

For ρ_t we employ the *local correlation function* proposed in [**12**], see also [**13**],

$$\rho_t := E\left[\tanh(X_t)\right] \tag{18}$$

for the dynamic correlation function, where X_t is any mean-reverting process with positive and negative values. For a fixed parameter of X_t, the correlation function ρ_t depends only on t. It is obvious that ρ_t takes values only in $(-1, 1)$ for all t and converges for $t \to \infty$. By choosing X_t in (18) to be the *Ornstein-Uhlenbeck process* [**14**]

$$\mathrm{d}X_t = \kappa(\mu - X_t)\mathrm{d}t + \sigma\mathrm{d}W_t, \quad t \geq 0, \tag{19}$$

the closed-form expression for ρ_t has been derived as

$$\rho_t = 1 - \frac{\exp(-A-\frac{B}{2})}{2}\int_{-\infty}^{\infty}\frac{1}{\cosh(\frac{\pi u}{2})}\cdot\exp\left(\mathrm{i}\,u(A+B) + u^2\frac{B}{2}\right)\mathrm{d}u, \tag{20}$$

with

$$A = \exp(-\kappa t)\tanh^{-1}(\rho_0) + \mu(1-\exp(-\kappa t)), \tag{21}$$

$$B = -\frac{\sigma^2}{2\kappa}(1-\exp(-2\kappa t)), \tag{22}$$

where $\kappa \geq 0,\ \sigma \geq 0,\ \mu \in \mathbb{R}$ and $\rho_0 \in (-1, 1)$.

[1]The drawback: A limitation on fitting model to market data could exist due to the linearity of the conditional expectation of r_t, namely $E[r_t|r_s] = (t-s)\mu_r$.

Substituting (16), (17) and (20) into (11), we obtain the European Call-option price with dynamically correlated stochastic interest rate

$$V(S, P, \rho_\tau, \tau; K) = S\Phi(d_1) - KP\Phi(d_2), \tag{23}$$

$$d_1 := \frac{\ln \frac{S}{K} - \ln P + \frac{1}{2}\int_0^\tau \sigma_s^2 \mathrm{d}s}{\sqrt{\int_0^\tau \sigma_s^2 \mathrm{d}s}}, \qquad d_2 := d_1 - \sqrt{\int_0^\tau \sigma_s^2 \mathrm{d}s}$$

and

$$\sigma_t^2 = \tau^2\sigma_r^2 + \sigma_S^2 + 2\rho_t\tau\sigma_r\sigma_S, \tag{24}$$

where P and ρ_t are defined in (17) and (20), respectively. The price of European Put-options are directly available using the Put-Call parity.

2.2. Option pricing with Vasicek Interest rate – The Rabinovitch model

Rabinovitch [**9**] investigated the pricing of European option with Vasicek stochastic interest rates and derived a closed form formula. A comparison of pricing formulas of European Call-option with different stochastic interest rate processes can be found in [**7**]. In this section, we consider the pricing of European Call-options with the Vasicek stochastic interest rate and incorporate dynamic correlation.

Again, we consider the following SDEs for the stock price and interest rate dynamics

$$\frac{\mathrm{d}S_t}{S_t} = \mu^S \mathrm{d}t + \sigma_S \mathrm{d}W_t^1, \tag{25}$$

$$\mathrm{d}r_t = \kappa_r(\mu^r - r_t)\mathrm{d}t + \sigma_r\rho_t \mathrm{d}W_t^1 + \sigma_r\sqrt{1-\rho_t^2}\mathrm{d}W_t^2, \tag{26}$$

where W_t^1 and W_t^2 are independent Brownian motions. The pricing formula of European Call-option according to (25) and (26) but with a constant correlation has been given in [**9**], see also [**7**].

Furthermore, if we compare the pricing formula of the Merton model between using constant and dynamic correlation in Section 2.1, we observe that incorporating a dynamic correlation does not change the original pricing formula (with constant correlation) to a large extent, the new pricing formula with dynamic correlation has just the formel which can be obtained directly by fitting in the dynamic correlation function instead of a constant correlation with the original formula.

We can observe that incorporating a dynamic correlation function into the pricing formula with the Vasicek stochastic interest rate provided in [**9**] and [**7**] also in this case. In order to adopt the approach in [**7**] to directly get the pricing formula with dynamic correlation, we need to rewrite (25) and (26) with respect to the

Brownian motions under a risk-neutral probability measure $\mathbb{Q}$ as

$$\frac{\mathrm{d}S_t}{S_t} = \underbrace{(\mu^S - \sigma_S\lambda_S)}_{:=\mu_S}\mathrm{d}t + \sigma_S \mathrm{d}\widetilde{W}_t^1, \tag{27}$$

$$\mathrm{d}r_t = \kappa_r[\underbrace{(\mu^r - \frac{\sigma_r\sqrt{1-\rho_t^2}\lambda_t^r}{\kappa_r})}_{:=\mu_r} - r_t]\mathrm{d}t + \sigma_r\rho_t\mathrm{d}\widetilde{W}_t^1 + \sigma_r\sqrt{1-\rho_t^2}\mathrm{d}\widetilde{W}_t^2, \tag{28}$$

where λ_S and λ_t^r are the market prices of risk. Whilst we assume that the market price of risk λ_t^r to be a constant, this is to say we set $\mu^r - \frac{\sigma_r\sqrt{1-\rho_t^2}\lambda_t^r}{\kappa_r} = \mu_r$. The pricing formula with constant correlation in [**7**] can be thus straightforwardly adopted to find the pricing formula using dynamic correlation. Therefore, we omit the exact derivation and give the pricing formula using dynamic correlation

$$V(S, P, \rho_\tau, \tau; K) = S\Phi(d_1) - KP\Phi(d_2) \tag{29}$$

with

$$d_1 := \frac{\Sigma_{11}^\tau + \Sigma_{12}^\tau - C_\tau}{\sqrt{D_\tau}}, \quad d_2 := d_1 - \sqrt{D_\tau}$$

where

$$C_\tau := \frac{\Sigma_{11}^\tau}{2} - B_\tau + \ln\frac{K}{S}, \qquad D_\tau := \Sigma_{11}^\tau + 2\Sigma_{12}^\tau + \Sigma_{22}^\tau, \quad \Sigma_{11}^\tau := \sigma_S^2\tau,$$

$$\Sigma_{22}^\tau := \frac{\sigma_r^2}{\kappa_r^2}\left[\tau - \frac{3 + \mathrm{e}^{-\kappa_r\tau}(\mathrm{e}^{-\kappa_r\tau} - 4)}{2\kappa_r}\right], \quad \Sigma_{12}^\tau := \frac{\sigma_r\sigma_S}{\kappa_r}\int^\tau \rho_s(1 - \mathrm{e}^{(s-\tau)\kappa_r})\mathrm{d}s$$

and

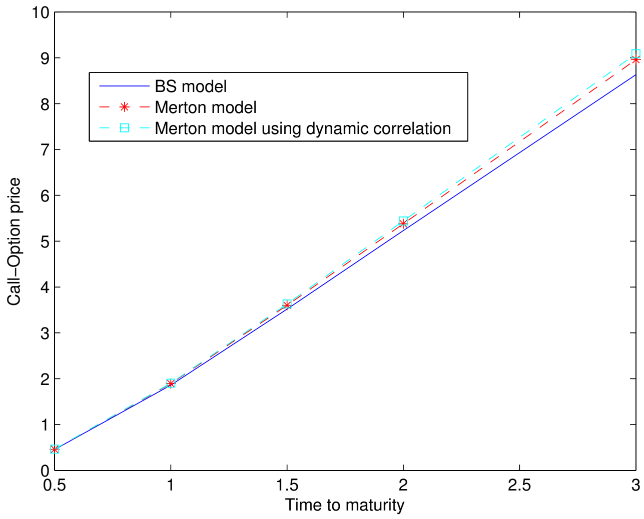
$$B_\tau := \frac{1}{\kappa_r}\left[\kappa_r\mu_r\tau - (r - \mu_r)(\mathrm{e}^{-\kappa_r\tau} - 1)\right], \qquad P_\tau := \mathrm{e}^{\frac{1}{2}\Sigma_{22}^\tau - B_\tau}, \tag{30}$$

ρ_t has been defined in (20).

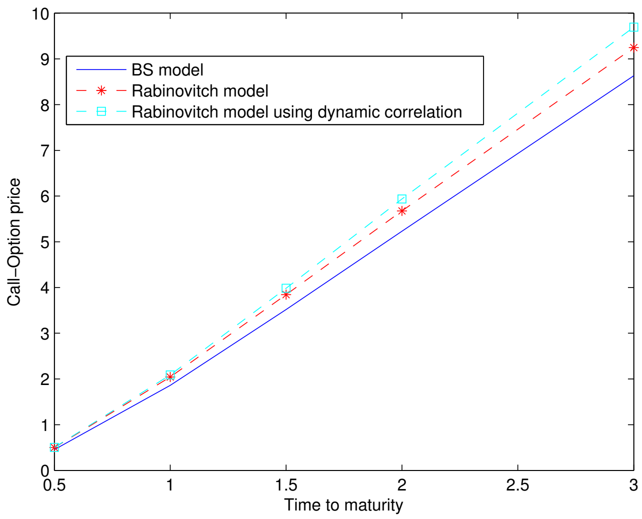
2.3. Numerical Results

In this section, we compare numerically the option prices between using constant and dynamic correlations. in both models above. We choose the parameters $S = 80$, $K = 100$, $\sigma_S = 0.2$, constant correlation: $\rho_c = 0.2$, parameters of dynamic correlation function: $\rho_0 = 0.2$, $\kappa_\rho = 2$, $\mu_\rho = 0.5$, $\sigma_\rho = 0.2$, constant interest rate for the Black-Scholes model: $r_c = 0.05$, stochastic rate for the Merton model: $r_0 = 0.05$, $\mu_r = 0.001$, $\sigma_r = 0.1$ and for the Rabinovitch model: $r_0 = 0.05$, $\kappa_r = 2$, $\mu_r = 0.001$, $\sigma_r = 0.1$.

We compute the prices of the European Call-option using the Black-Scholes model, using the Merton model and the Rabinovitch model with constant and dynamic correlation for the different maturities $T = 0.5, 1, 1.5, 2, 2.5, 3$ years and display them in Figure 1. We can easily see the difference between the Black-Scholes model and the model using stochastic interest rate. However, as mentioned in the introduction, some empirical findings showed us that stochastic interest rates (with constant correlation) may not be important for the option pricing. From Figure 1



(a) Merton model



(b) Rabinovitch model

Figure 1. Comparison of pricing European Call-option using different models.

we can also observe, the prices in both models have been changed because of incorporating nonconstant correlation. Thus, one could ask whether stochastic interest rates with nonconstant correlation can contribute to the performance improvement of the Black-Scholes model. To clarify this question, we run a calibration test in the next section.

3. Calibration to the market data

Both works [**12**] and [**13**] indicated that using a dynamic correlation can improve the model calibration. In the following, we examine both the Merton model and the Rabinovitch model whether incorporating stochastic interest rate contributes to the performance improvement of pring due to allowing a dynamic correlation.

We have seen that the bond price formula is available for both models, see (17) for the Merton model and (30) for the Rabinovitch model. Thus, one can directly estimate the parameters of the short rate model using the market yield curve Y_τ with the aid of the relation

$$Y_\tau = -\frac{1}{\tau}\ln P_\tau. \tag{31}$$

We consider the overnight rate on July 30, 2013, $r_0 = 0.26\%$, as the initial value of the short rate. One can then obtain the estimates by fitting (31) to the treasury yield curve[2] on this day for a maturity series, i.e. $1/12, 1/4, 1/2, 1, 2, 3, 5, \ldots 30$ years. Our results are: $\mu_r = 0.005$, $\sigma_r = 0.017$ (Merton short rate) and $\kappa_r = 0.111$, $\mu_r = 0.052$, $\sigma_r = 0.001$ (Vasicek short rate).

The parameters, which we do still need to estimate, are σ_S, ρ_c (for the case of using a constant correlation) or correlation function parameters (for using a dynamic correlation). For this purpose, we pick the market Call option prices on the $S\&P$ 500 on July 30, 2013 with the spot price $S = 169.1$, for the maturities $T = 30, 90, 180, 360$ days and the strikes $K/S = 0.9, 1, 1.1$. Then, we fit the model prices $V_{Mod}(T_i, K_j)$ to the market prices $V_{Mkt}(T_i, K_j)$ by minimizing the relative mean error sum of squares (RMSE)

$$\frac{1}{N}\sum_{i,j}\omega_{i,j}\frac{(V_{Mkt}(T_i, K_j) - V_{Mod}(T_i, K_j))^2}{V_{Mkt}(T_i, K_j)}, \tag{32}$$

where $\omega_{i,j}$ is an optional weight and N is number of prices. While minimizing we need to add some constraints on the parameters: the implied volatility σ_S must be positive, the constant correlation ρ_c must belong to the interval $(-1, 1)$. We know that the correlation function (20) stems from the expectation of the transformed Ornstein-Uhlenbeck process by $\tanh$. As mentioned before, the parameters of the correlation function must satisfy the following conditions

$$\kappa > 0, \quad \mu \in \mathbb{R}, \quad \sigma > 0, \quad \rho_0 \in (-1, 1). \tag{33}$$

We set the upper limit for κ to be 30 and the interval for μ to be $[-6, 6]$.

For this optimization problem we used the standard method of nonlinear optimization and report our results in Table 1 for using a constant correlation and in Table 2 for using a dynamic correlation. First, we look at Table 1 and find that the constant correlation ρ_c in both models tends to attain the boundary 0.99. We have checked that the option prices can not really be affected by varying the correlation value in the interval $(-0.7, 1)$, this result 0.99 could be thus justified in this case. However, the calibration to the chosen market data in both models

[2] available on http://www.treasury.gov

Model	σ_S	ρ_c	RMSE
The Merton model	0.12	0.99	0.115
The Rabinovitch model	0.12	0.99	0.141

Table 1. Parameter estimation for using constant correlation ($S\&P$ 500).

Model	σ_S	ρ_0	κ	μ	σ	RMSE
The Merton model	0.12	-0.99	18.14	6	5×10^{-4}	0.114
The Rabinovitch model	0.12	-0.99	22.90	6	6.6×10^{-3}	0.141

Table 2. Parameter estimation for using dynamic correlation ($S\&P$ 500).

is still not much satisfied. The reason could be that the option pricing within the Black-Scholes framework can not really be improved by incorporating stochastic interest rate.

Naturally, one may think that the unsatisfactory calibration might be caused by the drawback of the chosen interest rate model, e.g. the linear drift in the Merton model as mentioned before. Ignoring this, one could think whether the calibration will be thus improved by allowing a nonconstant correlation between stock and the stochastic interest rate, namely whether option pricing could be improved by imposing a nonconstant correlation? Unfortunately, from Table 2 we see although the dynamic correlation has changed from the initial value -0.99 to the boundary 0.99 over time, there is almost no improvement of the RMSE compared to the RMSE in Table 1, and both RMSEs are quite large.

To further confirm the outcome of the experiment above, we repeat the calibration using other market data. We fit both stochastic interest rate models to $r_0 = 4.2\%$ and the yield curve on June 27, 2007: $\mu_r = 0.003$, $\sigma_r = 0.016$ (Merton short rate) and $\kappa_r = 7.225$, $\mu_r = 0.058$, $\sigma_r = 0.8278$ (Vasicek short rate). We use the market option prices on the DAX on the same day with the spot price $S = 7858.5$, for the maturities $T = 0.02,\ 0.52,\ 1.02,\ 1.52$ years and the strikes $K/S = 0.9,\ 1,\ 1.1$. We present our results in Table 3 for using constant correlation and in Table 4 for using dynamic correlation.

Model	σ_S	ρ_c	RMSE
The Merton model	0.18	0.99	1.960
The Rabinovitch model	0.18	-0.304	1.312

Table 3. Parameter estimation for using constant correlation (DAX).

Model	σ_S	ρ_0	κ	μ	σ	RMSE
The Merton model	0.19	-0.99	3.73	6	0.001	1.916
The Rabinovitch model	0.19	-0.501	6.054	-0.339	0.156	1.234

Table 4. Parameter estimation for using dynamic correlation (DAX).

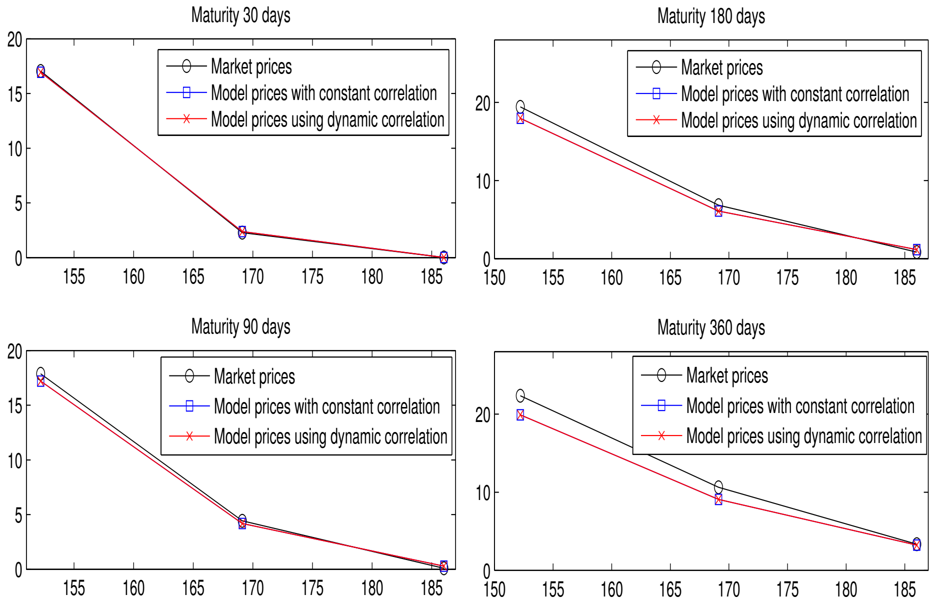


Figure 2. Comparison of the market and model prices in the Merton model ($S\&P$ 500).

Although the calibration of the Rabinovitch model is a bit better than using the Merton model in this experiment, the Merton model exhibits a drawback mentioned before. Unfortunately, we still observe no significant improvements by incorporating a time-varying correlation.

Furthermore, we can observe that the value of κ is quite large and the value of σ is small in both examples above, this means that the dynamic correlation will rapidly tend to its equilibrium which is close to the value of the applied constant correlation. This means that no big differences between using constant and dynamic correlation are expected. To confirm this statement, for the first example ($S\&P$ 500) we compare the model prices using constant and dynamic correlation to the market prices in Figure 2 for the Merton model and in Figure 3 for the Rabinovitch model. As expected, in both models there is almost no difference between prices using constant and dynamic correlation, especially, for a longer maturity.

Thus, we conclude that allowing a dynamic correlation in this example does not improve the calibration as in [**12**] and [**13**]. Incorporating a stochastic volatility could probably solve this calibration problem. This means also that our experimental results do not only coincide with the statement that only incorporating stochastic interest rate does not improve pricing performance. Furthermore, our results show that the calibration is not getting better for allowing a dynamic correlation between stochastic interest rate and stock process.

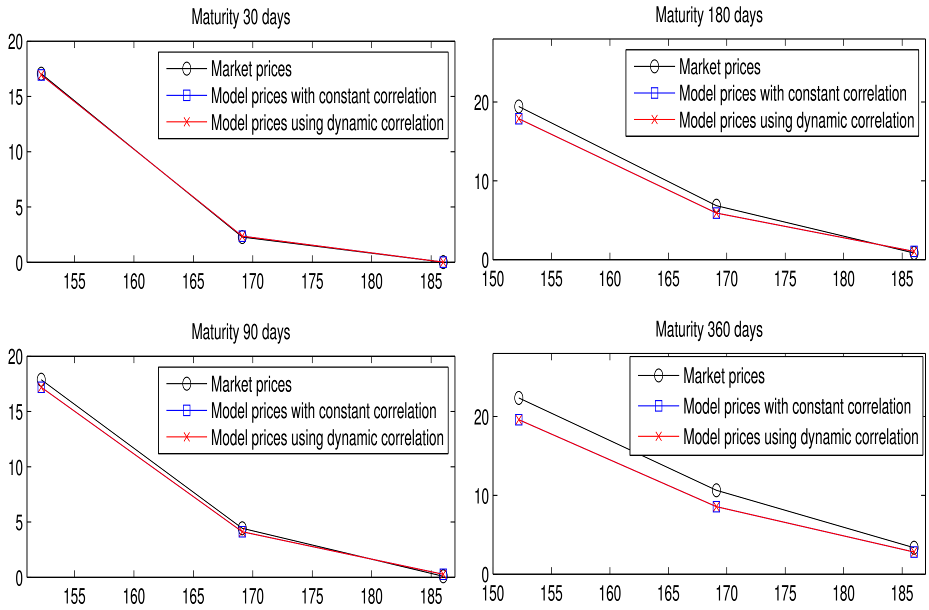


Figure 3. Comparison of the market and model prices in the Rabinovitch model ($S\&P$ 500).

4. Conclusion

In this work, we reviewed two European option pricing models with stochastic interest rates: the Merton model (interest rate given by Gauss-Wiener process) and the Rabinovitch model (interest rate given by Vasicek process). We extend both models by incorporating a local time dependent correlation between the underlying process and the stochastic interest rate. We presented numerical results to show the difference between using a constant and a dynamic correlation. Furthermore, we conducted experiments on fitting the model to the market price. Our result has justified firstly the earlier empirical finding that the option pricing within the Black-Scholes framework could not be improved by incorporating stochastic interest rate and secondly showed no significant improvement even when using a nonlinear time-varying correlation between the stock process and the short rate process.

Acknowledgment. We thank the anonymous referees for their very helpful comments.

The work was partially supported by the European Union in the FP7-PEOPLE-2012-ITN Program under Grant Agreement Number 304617 (FP7 Marie Curie Action, Project Multi-ITN STRIKE - Novel Methods in Computational Finance).

The authors acknowledge partial support from the German-Slovakian Project *NL-BS – Numerical Solution of Nonlinear Black-Scholes Equations*, financed by DAAD.

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