

GENERATING FUNCTION FOR SOME GENERALIZED MOCK THETA FUNCTIONS

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ABSTRACT. In this paper, we provide the generating functions for partial generalized mock theta functions of the second order, as well as for some new partial generalized mock theta functions.

1. INTRODUCTION

In his last letter to G. H. Hardy [13], S. Ramanujan listed seventeen mock theta functions of order three, five, six and seven. According to Ramanujan, a mock theta function is a function $f(q)$, $|q| < 1$, satisfying the following two conditions:

- (0) For every root of unity ξ , there is a θ -function $\theta_\xi(q)$ such that the difference $f(q) - \theta_\xi(q)$ is bounded as $q \rightarrow \xi$ radially.
- (1) There is no single θ -function which works for all ξ , i.e., for every θ -function $\theta(q)$ there is some root of unity ξ for which $f(q) - \theta(q)$ is unbounded as $q \rightarrow \xi$ radially.

McIntosh [11] considered three second-order mock theta functions and gave transformation formulas for them. Andrews [3] generated some new mock theta functions and identified four of particular interest. Bringmann, Hikami and Lovejoy subsequently developed two additional mock theta functions. The works [18, 20, 21] provide a comprehensive study of these functions.

Following the study of these mock theta functions in [16, 17, 18], we derived the generating functions for the newly introduced partial generalized mock theta functions, as well as for the partial generalized second-order mock theta functions [14].

The four mock theta functions of Andrews [3] are stated as follows:

$$(1.1) \quad \bar{\psi}_0(q) = \sum_{n=0}^{\infty} \frac{q^{2n^2}}{(-q; q)_{2n}},$$

$$(1.2) \quad \bar{\psi}_1(q) = \sum_{n=0}^{\infty} \frac{q^{2n^2+2n}}{(-q; q)_{2n+1}},$$

$$(1.3) \quad \bar{\psi}_2(q) = \sum_{n=0}^{\infty} \frac{q^{2n^2+2n} (q; q^2)_n}{(q^2; q^2)_n (-q; q)_{2n}},$$

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$$(1.4) \quad \bar{\psi}_3(q) = \sum_{n=0}^{\infty} \frac{q^{n^2} (-q; q)_n^2}{(q; q)_{2n}}.$$

Andrews called $\bar{\psi}_3(q)$ as a companion to Ramanujan’s third-order mock theta function

$$\psi(q) = \sum_{n=0}^{\infty} \frac{q^{n^2}}{(q; q^2)_n}.$$

Bringmann, Hikami, and Lovejoy [16] defined two mock theta functions as follows:

$$(1.5) \quad \bar{\phi}_0(q) = \sum_{n=0}^{\infty} q^n (-q; q)_{2n+1},$$

$$(1.6) \quad \bar{\phi}_1(q) = \sum_{n=0}^{\infty} q^n (-q; q)_{2n}.$$

McIntosh [3] introduced the following three second-order mock theta functions:

$$(1.7) \quad A(q) = \sum_{n=0}^{\infty} \frac{q^{(n+1)^2} (-q; q^2)_n}{(q; q^2)_{n+1}^2} = \sum_{n=0}^{\infty} \frac{q^{(n+1)} (-q^2; q^2)_n}{(q; q^2)_{n+1}},$$

$$(1.8) \quad B(q) = \sum_{n=0}^{\infty} \frac{q^{n^2+n} (-q^2; q^2)_n}{(q; q^2)_{n+1}^2} = \sum_{n=0}^{\infty} \frac{q^n (-q; q^2)_n}{(q; q^2)_{n+1}},$$

$$(1.9) \quad \mu(q) = \sum_{n=0}^{\infty} \frac{(-1)^n q^{n^2} (q; q^2)_n}{(-q^2; q^2)_n^2}.$$

Partial generalized mock theta functions

- Partial generalized new mock theta functions:

$$(1.10) \quad \bar{\psi}_{0,m}(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m \frac{(t)_n q^{2n^2 - 3n + n\alpha} z^{2n}}{\left(\frac{-z^2}{q}; q\right)_{2n}},$$

$$(1.11) \quad \bar{\psi}_{1,m}(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m \frac{(t)_n q^{2n^2 - n + n\alpha} z^{2n}}{\left(\frac{-z^2}{q}; q\right)_{2n+1}},$$

$$(1.12) \quad \bar{\psi}_{2,m}(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m \frac{(t)_n q^{2n^2 + 2n - 2n\alpha} z^{2n} (z; q^2)_n}{(z^2; q^2)_n \left(\frac{-z^2}{q}; q\right)_{2n}},$$

$$(1.13) \quad \bar{\psi}_{3,m}(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m \frac{(t)_n q^{n^2 - n + n\alpha} z^{2n} (-z; q)_n^2}{\left(\frac{-z^2}{q}; q\right)_{2n}},$$

$$(1.14) \quad \bar{\phi}_{0,m}(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m (t)_n q^{n - 2n\alpha} z^{2n} \left(\frac{-z^2}{q}; q\right)_{2n+1},$$

$$(1.15) \quad \bar{\phi}_{1,m}(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m (t)_n q^{n - 2n\alpha} z^{2n} \left(\frac{-z^2}{q}; q\right)_{2n}.$$

- Partial generalized second-order mock theta functions:

$$(1.16) \quad A_m(q) = \frac{1}{(t)_\infty} \sum_{n=0}^m \frac{(t)_n q^{n^2-n+n\alpha} z^{2n+1} \left(\frac{-z^2}{q}; q^2\right)_n}{\left(\frac{z^2}{q}; q^2\right)_{n+1}^2},$$

$$(1.17) \quad B_m(q) = \frac{1}{(t)_\infty} \sum_{n=0}^m \frac{(t)_n q^{n^2-2n+n\alpha} z^{2n} (-z^2; q^2)_n}{\left(\frac{z^2}{q}; q^2\right)_{n+1}^2},$$

$$(1.18) \quad \mu_m(q) = \frac{1}{(t)_\infty} \sum_{n=0}^m \frac{(-1)^n (t)_n q^{n^2-3n+n\alpha} z^{2n} \left(\frac{z^2}{q}; q^2\right)_n}{(-z^2; q^2)_n^2}.$$

2. DEFINITIONS AND NOTATIONS

We will use the following standard basic hypergeometric notations:

$$(a; q^k)_n = \prod_{m=1}^n (1 - aq^{k(m-1)}), \quad |q^k| < 1, \quad n \text{ a non-negative integer,}$$

$$(a; q^k)_0 = 1,$$

$$(a; q^k)_\infty = \prod_{m=1}^{\infty} (1 - aq^{k(m-1)}),$$

$$(a_1, a_2, a_3, \dots, a_m; q^k)_n = (a_1; q^k)_n (a_2; q^k)_n \cdots (a_m; q^k)_n$$

$${}_A\phi_{A-1} \left[\begin{matrix} a_1, a_2, \dots, a_A; \\ b_1, b_2, \dots, b_{A-1}; \end{matrix} q_1, z \right] = \sum_{n=0}^{\infty} \frac{(a_1; q_1)_n \cdots (a_A; q_1)_n z^n}{(b_1; q_1)_n \cdots (b_{A-1}; q_1)_n (q_1; q_1)_n}, \quad |z| < 1$$

A generalized basic hypergeometric function with base q is defined as

$${}_r\phi_s \left[\begin{matrix} a_1, a_2, \dots, a_r; \\ b_1, b_2, \dots, b_s; \end{matrix} q, z \right] = \sum_{n=0}^{\infty} \frac{(a_1; q)_n \cdots (a_r; q)_n}{(b_1; q)_n \cdots (b_s; q)_n (q; q)_n} \left[(-1)^n q^{\frac{n^2-n}{2}} \right]^{1+s-r} z^n.$$

For $r = s + 1$, the above series is convergent for $|z| < 1$.

For $r \leq s$, the above series is convergent for all z .

For $r > s + 1$, the above series diverges for all z except $z = 0$.

3. MAIN RESULT

We shall use the following identity to obtain the generating functions for the generalized partial mock theta functions.

$$(3.1) \quad \sum_{m=0}^{\infty} t^m \sum_{r=0}^m \alpha_r = \left[\sum_{r=0}^{\infty} \alpha_r t^r \right] \sum_{n=0}^{\infty} t^n, \quad |t| < 1$$

The following identity can be easily deduced from the identity of Srivastava [10]:

$$\sum_{r=0}^p \alpha_r \beta_r = \beta_{p+1} \sum_{r=0}^p \alpha_r + \sum_{m=0}^p (\beta_m - \beta_{m+1}) \sum_{r=0}^m \alpha_r$$

Taking $\beta_r = t^r$ with $|t| < 1$, we obtain

$$\sum_{r=0}^p \alpha_r t^r = t^{p+1} \sum_{r=0}^p \alpha_r + (1-t) \sum_{m=0}^p t^m \sum_{r=0}^m \alpha_r.$$

Letting $p \rightarrow \infty$, we get

$$\sum_{m=0}^{\infty} t^m \sum_{r=0}^m \alpha_r = \frac{1}{1-t} \sum_{r=0}^{\infty} \alpha_r t^r = \sum_{r=0}^{\infty} \alpha_r t^r \sum_{n=0}^{\infty} t^n.$$

4. GENERATING FUNCTIONS FOR GENERALIZED PARTIAL NEW MOCK THETA FUNCTIONS

(I)

$$\bar{\psi}_0(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m \frac{(t)_n q^{2n^2-3n+n\alpha} z^{2n}}{\left(\frac{-z^2}{q}; q\right)_{2n}}.$$

At $t = 0$,

$$\implies \bar{\psi}_0(0, \alpha, z; q) = \sum_{n=0}^m \frac{q^{2n^2-3n+n\alpha} z^{2n}}{\left(\frac{-z^2}{q}; q\right)_{2n}}.$$

Taking $\alpha_r = \frac{q^{2r^2-3r+\alpha r} z^{2r}}{\left(\frac{-z^2}{q}; q\right)_{2r}}$ in (3.1), we get

$$(4.1) \quad \sum_{m=0}^{\infty} t^m \bar{\psi}_{0,m}(0, \alpha, z; q) = \sum_{r=0}^m \frac{q^{2r^2-3r+\alpha r} z^{2r} t^r}{\left(\frac{-z^2}{q}; q\right)_{2r}} \sum_{n=0}^{\infty} t^n,$$

or

$$(4.2) \quad \sum_{m=0}^{\infty} t^m \bar{\psi}_{0,m}(0, \alpha, z; q) = {}_1\phi_2 \left[\begin{matrix} q^2 \\ \frac{-z^2}{q}, -z^2 \end{matrix}; q^2, tz^2 q^{\alpha-1} \right] \sum_{n=0}^{\infty} t^n.$$

Special case:

For $z = q, \alpha = 1$ in (4.1) and (4.2), we get the generating function for partial mock theta function of Andrews:

$$(4.3) \quad \sum_{m=0}^{\infty} t^m \bar{\psi}_{0,m}(q) = \sum_{r=0}^{\infty} \frac{q^{2r^2} t^r}{\left(\frac{-z^2}{q}; q\right)_{2r}} \sum_{n=0}^{\infty} t^n = {}_1\phi_2 \left[\begin{matrix} q^2 \\ q, -q^2 \end{matrix}; q^2, tq^2 \right] \sum_{n=0}^{\infty} t^n.$$

Here, we are listing the generating functions for other generalized partial mock theta functions, omitting calculations and giving only the value of α_r in parentheses.

$$\sum_{m=0}^{\infty} t^m \bar{\psi}_{0,m}(q) = \sum_{r=0}^{\infty} \frac{q^{2r^2} t^r}{\left(\frac{-z^2}{q}; q\right)_{2r}} \sum_{n=0}^{\infty} t^n = {}_1\phi_2 \left[\begin{matrix} q^2 \\ q, -q^2 \end{matrix}; q^2, tq^2 \right] \sum_{n=0}^{\infty} t^n.$$

(II)

$$(4.4) \quad \sum_{m=0}^{\infty} t^m \bar{\psi}_{1,m}(0, \alpha, z; q) = \frac{1}{\left(1 + \frac{z^2}{q}\right)} {}_1\phi_2 \left[\begin{matrix} q^2 \\ -z^2, -z^2q \end{matrix}; q^2, tz^2q^{\alpha-1} \right] \sum_{n=0}^{\infty} t^n$$

$$\left(\alpha_r = \frac{q^{2r^2-r+\alpha r} z^{2r}}{\left(\frac{-z^2}{q}; q\right)_{2r+1}} \text{ in (3.1)} \right).$$

Here $\bar{\psi}_{1,m}(q)$ is the partial mock theta function of Andrews.

Special case:

$$\sum_{m=0}^{\infty} t^m \bar{\psi}_{1,m}(q) = \frac{1}{\left(1 + \frac{z^2}{q}\right)} {}_1\phi_2 \left[\begin{matrix} q^2 \\ -q^2, -q^3 \end{matrix}; q^2, tq^2 \right] \sum_{n=0}^{\infty} t^n \quad (z=q, \alpha=1 \text{ in (4.4)}).$$

(III)

$$(4.5) \quad \sum_{m=0}^{\infty} t^m \bar{\psi}_{2,m}(0, \alpha, z; q) = {}_2\phi_3 \left[\begin{matrix} q^2, z \\ z^2, \frac{-z^2}{q}, -z^2 \end{matrix}; q^2, tz^2q^{4-2\alpha} \right] \sum_{n=0}^{\infty} t^n$$

$$\left(\alpha_r = \frac{q^{2r^2+2r-2r\alpha} z^{2r} (z; q^2)_r}{(z^2; q^2)_r \left(\frac{-z^2}{q}; q\right)_{2r}} \text{ in (3.1)} \right).$$

Here $\bar{\psi}_{2,m}(q)$ is the partial mock theta function of Andrews.

Special case :

$$\sum_{m=0}^{\infty} t^m \bar{\psi}_{2,m}(q) = {}_2\phi_3 \left[\begin{matrix} q^2, q \\ q^2, -q, -q^2 \end{matrix}; q^2, tq^4 \right] \sum_{n=0}^{\infty} t^n \quad (z=q, \alpha=1 \text{ in (4.5)})$$

(IV)

$$(4.6) \quad \sum_{m=0}^{\infty} t^m \bar{\phi}_{0,m}(0, \alpha, z; q) = \left(1 + \frac{z^2}{q}\right) {}_3\phi_2 \left[\begin{matrix} q^2, -z^2, -z^2q \\ 0, 0 \end{matrix}; q^2, tz^2q^{1-2\alpha} \right] \sum_{n=0}^{\infty} t^n$$

$$\left(\alpha_r = q^{r-2r\alpha} z^{2r} \left(\frac{-z^2}{q}; q\right)_{2r+1} \text{ in (3.1)} \right).$$

Here $\bar{\phi}_{0,m}(q)$ is the partial mock theta function of Bringmann, Hikami and Lovejoy.

Special case:

$$\sum_{m=0}^{\infty} t^m \bar{\phi}_{0,m}(q) = \left(1 + \frac{z^2}{q}\right) {}_3\phi_2 \left[\begin{matrix} q^2, -q^2, -q^3 \\ 0, 0 \end{matrix}; q^2, tq \right] \sum_{n=0}^{\infty} t^n \quad (z=q, \alpha=1 \text{ in (4.6)}).$$

(V)

$$(4.7) \quad \sum_{m=0}^{\infty} t^m \bar{\phi}_{1,m}(0, \alpha, z; q) {}_3\phi_2 \left[\begin{matrix} q^2, \frac{-z^2}{q}, -z^2 \\ 0, 0 \end{matrix}; q^2, tz^2q^{1-2\alpha} \right] \sum_{n=0}^{\infty} t^n$$

(Taking $\alpha_r = q^{r-2r\alpha} z^{2r} \left(\frac{-z^2}{q}; q\right)_{2r}$ in (3.1)).

Here $\bar{\phi}_{1,m}(q)$ is the partial mock theta function of Bringmann, Hikami and Lovejoy.

Special case:

$$\sum_{m=0}^{\infty} t^m \bar{\phi}_{1,m}(q) = {}_3\phi_2 \left[\begin{matrix} q^2, -q, -q^2 \\ 0, 0 \end{matrix}; q^2, tq \right] \sum_{n=0}^{\infty} t^n \quad (z = q, \alpha = 1 \text{ in (4.7)}).$$

5. GENERATING FUNCTIONS FOR GENERALIZED PARTIAL SECOND-ORDER MOCK THETA FUNCTIONS

(I)

$$A_m(t, \alpha, z; q) = \frac{1}{(t)_{\infty}} \sum_{n=0}^m \frac{(t)_n q^{n^2-n+n\alpha} z^{2n+1} \left(\frac{-z^2}{q}; q^2\right)_n}{\left(\frac{z^2}{q}; q^2\right)_{n+1}^2}.$$

At $t = 0$,

$$A_m(0, \alpha, z; q) = \frac{q^{n^2-n+n\alpha} z^{2n+1} \left(\frac{-z^2}{q}; q^2\right)_n}{\left(\frac{z^2}{q}; q^2\right)_{n+1}^2}.$$

Taking $\left(\alpha_r = \frac{q^{r^2-r+r\alpha} z^{2r+1} \left(\frac{-z^2}{q}; q^2\right)_r}{\left(\frac{z^2}{q}; q^2\right)_{r+1}^2} \text{ in (3.1)}\right)$, we get

$$(5.1) \quad \sum_{m=0}^{\infty} t^m A_m(0, \alpha, z; q) = \sum_{r=0}^{\infty} \frac{q^{r^2-r+r\alpha} z^{2r+1} \left(\frac{-z^2}{q}; q^2\right)_r}{\left(\frac{z^2}{q}; q^2\right)_{r+1}^2} \sum_{n=0}^{\infty} t^n,$$

or

$$(5.2) \quad \sum_{m=0}^{\infty} t^m A_m(0, \alpha, z; q) = \frac{z}{\left(1 - \frac{z^2}{q}\right)^2} {}_2\phi_2 \left[\begin{matrix} q^2, \frac{-z^2}{q} \\ z^2 q, z^2 q \end{matrix}; q^2, -tz^2 q^{\alpha} \right] \sum_{n=0}^{\infty} t^n.$$

Special case:

For $z = q, \alpha = 1$ in (5.1) and (5.2), we get the generating function for partial mock theta function of McIntosh

$$(5.3) \quad \begin{aligned} \sum_{m=0}^{\infty} t^m A_m(q) &= \sum_{r=0}^{\infty} \frac{q^{r^2+2r+1} \left(\frac{-z^2}{q}; q^2\right)_r}{\left(\frac{z^2}{q}; q^2\right)_{r+1}^2} t^r \sum_{n=0}^{\infty} t^n \\ &= \frac{q}{(1-q)^2} {}_2\phi_2 \left[\begin{matrix} q^2, -q \\ q^3, q^3 \end{matrix}; q^2, -tq^3 \right] \sum_{n=0}^{\infty} t^n. \end{aligned}$$

We list the generating functions for the other generalized and partial mock theta functions omitting calculations only given the value of α_r in parenthesis.

(II)

$$(5.4) \quad \sum_{m=0}^{\infty} t^m B_m(0, \alpha, z; q) = \frac{1}{\left(1 - \frac{z^2}{q}\right)^2} {}_2\phi_2 \left[\begin{matrix} q^2, -z^2 \\ z^2 q, z^2 q \end{matrix}; q^2, -tz^2 q^{\alpha-1} \right] \sum_{n=0}^{\infty} t^n$$

$$\left(\alpha_r = \frac{q^{r^2-2r+r\alpha} z^{2r} (-z^2; q^2)_r}{\left(\frac{z^2}{q}; q^2\right)_{r+1}} \text{ in (3.1)} \right)$$

Here $B_m(q)$ is the partial mock theta function of McIntosh.

Special case :

$$\sum_{m=0}^{\infty} t^m B_m(q) = \frac{1}{(1-q)^2} {}_2\phi_2 \left[\begin{matrix} q^2, -q^2 \\ q^3, q^3 \end{matrix}; q^2, -tq^2 \right] \sum_{n=0}^{\infty} t^n \quad (z = q, \alpha = 1 \text{ in (5.4)}).$$

(III)

$$(5.5) \quad \sum_{m=0}^{\infty} t^m \mu_m(0, \alpha, z; q) = {}_2\phi_2 \left[\begin{matrix} q^2, \frac{z^2}{q} \\ -z^2, -z^2 \end{matrix}; q^2, tz^2 q^{\alpha-2} \right] \sum_{n=0}^{\infty} t^n$$

$$\left(\alpha_r = \frac{q^{r^2-3r+r\alpha} z^{2r} \left(\frac{z^2}{q}; q^2\right)_r}{(-z^2; q^2)_r} \text{ in (3.1)} \right).$$

Here $\mu_m(q)$ is the partial mock theta function of McIntosh.

Special case :

$$\sum_{m=0}^{\infty} t^m \mu_m(q) = {}_2\phi_2 \left[\begin{matrix} q^2, q \\ -q^2, -q^2 \end{matrix}; q^2, tq \right] \sum_{n=0}^{\infty} t^n \quad (z = q, \alpha = 1 \text{ in (5.5)})$$

CONCLUSION

Mock theta functions are mysterious functions. These investigations will be helpful in understanding more about these functions. In Section 4 and 5, we have given generating function of generalized new mock theta functions and generalized second-order mock theta functions.

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