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SERIES SOLUTIONS FOR AN UNSTEADY FLOW AND HEAT TRANSFER OF A ROTATING DUSTY FLUID WITH RADIATION EFFECT

S. MANJUNATHA, B. J. GIREESHA AND C. S. BAGEWADI

Abstract. A theoretical analysis of free convective MHD flow of an unsteady rotating dusty fluid under the influence of hall current and radiation effect is carried out. The fluid flow is considered in the porous media under the influence of periodic pressure gradient and the fluid is assumed to be viscous, incompressible and electrically conducting with uniform distribution of dust particles. The governing partial differential equations are solved analytically using perturbation technique and the expressions for skin-friction is also derived. Further the effect of various pertinent parameter like magnetic parameter, rotation parameter and Hall current parameter on velocity of both fluid and dust phases are depicted graphically and the effect of radiation parameter, Grashof number and Prandtl number on temperature profile is also discussed in detail.

1. Introduction

The study of a two-phase flow of fluid-particle is important in power plant piping, petroleum transport, combustion, waste water treatment, corrosive particles in engine oil flow, smoke emission from vehicles and formation of raindrops. Its relevance is also seen in the field of mining, agriculture and food technologies. Particularly, the flow and heat transfer of a electrically conducting dusty fluid through a channel in the presence of a transverse magnetic field through porous medium occur in magnetohydrodynamic generators, pumps, accelerators, cooling systems, centrifugal separation of matter from fluid, petroleum industry, purification of crude oil, electrostatic precipitation, polymer technology and fluid droplets sprays.

Motivated by the applications of a two-phase flow, Saffman [**1**] initiated to study on the stability of a laminar flow of a dusty gas. The study of dusty viscous fluid under the influence of different physical conditions has been carried out by several authors; Nag et.al [**2**] studied an unsteady Couette flow of a dusty gas between two infinite parallel plates, when one plate of the channel is kept stationary and the other plate moves uniformly in its own plane. On a free convection flow of

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a dusty conducting fluid studied by Heamly [**3**]. Analytical solution for a free convective flow of a particle suspension past an infinite vertical surface carried out by Chamkha and Ramdan [**4**]. Ghosh et.al [**5**] discussed the hydromagnetic free convective flow with induced magnetic field. Later, MHD effects on a convective flow of a dusty viscous fluid with volume fraction investigated by Singh [**6**]. Attia [**7**] analyzed the unsteady hydromagnetic channel flow of dusty fluid with temperature dependent viscosity and thermal conductivity. An unsteady flow of a an electrically conducting dusty gas in channel due to an oscillating pressure gradient investigated by Chamkha [**8**]. Recently, Gireesha et.al ([**9**]-[**10**]) investigated the effects of dust particles in a flow and heat transfer of viscous fluid.

The theory of rotating fluids is highly important due to its occurrence in various natural phenomena. Rotating fluid flows have practical applications in many areas such as rotating machinery, lubrication, oceanography, computer storage devices, viscometry and crystal growth processes and also many engineering areas. The hydromagnetic flow due to rotating disk was first investigated by Von Karman [**11**]. However, when the medium is rarefied or if a strong magnetic field is present, the effect of Hall current cannot be neglected [**12**]. The study of MHD viscous flows with Hall current has important applications in problems of power generator and Hall accelerators as well as flight magnetohydrodynamics.

With the above understanding, a number of researchers have investigated the hydromagnetic flow of a viscous incompressible electrically conducting fluid in a rotating medium with Hall effect under the different conditions and configurations. Kanch and Jana [**13**] studied Hall effects on an unsteady hydromagnetic flow past a rotating disk when the fluid at infinity rotates about non-coincident axes, and they found that an increase in the hall current slow down the fluid flow. Hayat et.al [**14**] studied the effect of Hall current and heat transfer on a rotating flow of second grade fluid past a porous plate with the variable suction. The transient circular pipe MHD flow of a dusty fluid considering the hall effect investigated by Attia [**15**]. Later combined effects of Hall currents and rotation on MHD mixed convection in a rotating vertical channel studied by Guchhait et.al [**16**]. Gireesha et.al [**17**] studied the effect of Hall current on a flow of a viscous incompressible electrically conducting rotating dusty fluid with uniform distribution of dust particles bounded by a semi-infinite plate and they have obtained the solution using Laplace transform technique. They found that velocity profile decreases for an increase in hall effect. Very recently, Gireesha and Mahanthesh [**18**] reported the analytical solution for heat and mass transfer of a time-dependent MHD flow of an electrically conducting viscoelastic fluid in a nonuniform vertical channel with the convective boundary condition and Hall current. On the other hand, several authors from [**19**] to [**26**] have various aspects of a dusty fluid flow and heat transfer.

Motivated by the above mentioned investigations and applications, this work is aimed at providing the analytical solution to the combined effects of hall current and radiation on an unsteady free convective hydromagnetic rotating dusty fluid flow in a vertical channel. To the best of authors knowledge, this is yet to be addressed in a open literature. The flow in the channel under the influence of periodic pressure gradient is considered. A closed form solution is obtained by employing perturbation technique for fluid velocity, dust velocity, temperature, and as well as skinfriction co-efficient of both fluid and dust phase.

2. Mathematical Formulation and Solution

Consider an unsteady MHD flow of a viscous incompressible electrically conducting dusty fluid with uniform distribution of dust particles in a vertical channel on taking hall currents into account. In vertical plate flow problems, the buoyancy forces significantly affect the flow and the thermal fields due to the temperature difference between the plate and the ambient fluid, i.e, the density variation due to buoyancy effects is taken into account in the momentum equation. A vertical channel saturated with porous medium placed in the plane $z = 0$ is of infinite extent, so all the physical quantities depend only on z and t. The x axis is taken in the direction along the channel which is set in motion, and the z axis is taken perpendicular to it. The inclusion of the Hall currents gives rise to the Lorentz force in y direction, which induces a cross flow in that direction. The y axis is assumed to be normal to the xz plane. Further, the fluid as well as the channel rotate with uniform angular velocity $\overrightarrow{\Omega}$ about the z-axis as shown in the Figure 1.

The unsteady hydromagnetic dusty fluid flow in a rotating co-ordinate system is governed by the following continuity and momentum equations for fluid and particle phases [**1**]:

$$\nabla \cdot \vec{U} = 0, \tag{2.1}$$

$$\begin{aligned} \frac{\partial \vec{U}}{\partial t} + (\vec{U} \cdot \nabla)\vec{U} + 2\vec{\Omega} \times \vec{U} = & -\frac{1}{\rho}\nabla p + \frac{1}{\rho}(\vec{J} \times \vec{B}) + \nu \nabla^2 \vec{U} \\ & + \frac{KN}{\rho}(\vec{U_p} - \vec{U}) - \frac{\nu}{k}\vec{U} + g\beta(T - T_2), \end{aligned} \tag{2.2}$$

$$\nabla \cdot \vec{U_p} = 0, \tag{2.3}$$

$$m_p\Big[\frac{\partial \vec{U_p}}{\partial t} + (\vec{U_p} \cdot \nabla)\vec{U_p} + 2\vec{\Omega} \times \vec{U_p}\Big] = K(\vec{U} - \vec{U_p}), \tag{2.4}$$

$$\frac{\partial T}{\partial t} + (\vec{U} \cdot \nabla)T = \frac{k_T}{\rho C_p}\nabla^2 T, \tag{2.5}$$

the following nomenclatures:
$\vec{U} = (u, v, w)$ and $\vec{U_p} = (u_p, v_p, w_p)$ are the velocity components of fluid and dust phase, respectively, $T-$fluid temperature, $p-$pressure field including the centrifugal term, $\vec{J}-$electric current density, $\vec{B}-$total magnetic field, $N-$number density of dust particles, m_p-mass of the dust particle, $K-$Stokes-co-efficient of resistance, $\rho-$density, $k-$permeability of the porous medium, $\nu-$kinematic viscosity of the fluid, $\beta-$thermal volumetric coefficient, $g-$acceleration due to gravity, k_T the thermal conductivity, c_p the specific heat, and $t-$time.

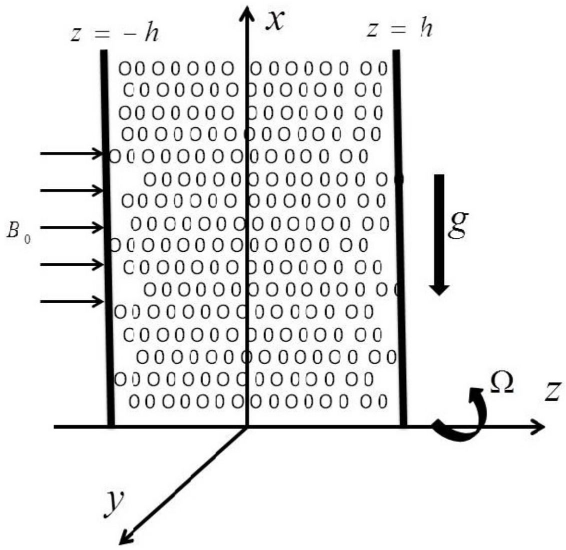

Figure 1. The physical configuration of the problem.

Neglecting ion-slip and thermoelectric effects, the generalized Ohm's law [**27**] is given as

$$\vec{J} + \frac{\omega_e \tau_e}{B_0}(\vec{J} \times \vec{B}) = \sigma \left[\vec{E} + \vec{u} \times \vec{B}\right], \tag{2.6}$$

where $\vec{B}$, $\vec{E}$, $\vec{u}$, $\vec{J}$, σ, ω_e, and τ_e are the magnetic field vector, electric field vector, fluid velocity vector, current density vector, conductivity of fluid, cyclotron frequency of electrons, electron collision time, respectively.

For the present problem, we assume that the magnetic Reynolds number for the flow is small so that the induced magnetic field can be neglected. The dust particles are assumed to be electrically non-conducting, spherical and uniformly distributed in the fluid. A periodic pressure gradient varying with time is applied in x-direction. A uniform magnetic field B_0 is applied in positive z-direction, i.e., $\vec{B} = (0, 0, B_0)$, and angular velocity is considered along z-direction, i.e., $\vec{\Omega} = (0, 0, \Omega)$ as shown in the above Figure 1. According to Boussinesq approximation, all physical quantities for this fully developed flow depend only on z and t. For the present problem, assume that $w(z,t) = 0 = w_p(z,t)$ and $N = N_0(constant)$.

Under these assumptions the equations of motion (2.1) to (2.5) for the fluid and dust phase velocity in its component form is given by

$$\begin{aligned} \frac{\partial u}{\partial t} - 2\Omega v = & -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial z^2} + \frac{\sigma B_0}{\rho(1+m^2)}(mv - u) \\ & + \frac{l}{\tau_p}(u_p - u) - \frac{\nu}{k}u + g\beta(T - T_2), \end{aligned} \tag{2.7}$$

$$\frac{\partial v}{\partial t} + 2\Omega u = \nu \frac{\partial^2 v}{\partial z^2} - \frac{\sigma B_0}{\rho(1+m^2)}(v+mu) + \frac{l}{\tau_p}(v_p - v) - \frac{\nu}{k} v, \tag{2.8}$$

$$\frac{\partial u_p}{\partial t} - 2\Omega v_p = \frac{1}{\tau_p}(u - u_p), \tag{2.9}$$

$$\frac{\partial v_p}{\partial t} + 2\Omega u_p = \frac{1}{\tau_p}(v - v_p). \tag{2.10}$$

Energy equation with thermal radiation effect is given by

$$\rho C_p \frac{\partial T}{\partial t} = K_T \frac{\partial^2 T}{\partial z^2} - 16 k^* \alpha^* {T_2}^3 (T - T_2), \tag{2.11}$$

where $m = \omega_e \tau_e$ (hall parameter), $l = \frac{m_p N_0}{\rho}$ (mass concentration), $\tau_p = \frac{m_p}{K}$ (relaxation time), α^* is the Stefan-Boltzman constant and k^* is the spectral mean absorption coefficient of the medium.

The boundary conditions for velocity of the fluid and dust particles and temperature distributions are

$$\begin{aligned} &u = v = u_p = v_p = 0, \qquad T = T_2 + (T_1 - T_2)\cos\omega t \qquad \text{at } z = -h,\\ &u = v = u_p = v_p = 0, \qquad T = T_2 \qquad \text{at } z = +h. \end{aligned} \tag{2.12}$$

To make the above system dimensionless, introduce the following non-dimensional variables and parameters as

$$\begin{aligned} &(\eta, \xi) = \frac{(z, x)}{h}, \qquad (u^*, v^*) = \frac{h}{\nu}(u, v), \qquad ({u_p}^*, {v_p}^*) = \frac{h}{\nu}(u_p, v_p),\\ &\tau = \frac{\nu t}{h^2}, \qquad \theta = \frac{T - T_2}{T_1 - T_2}. \end{aligned} \tag{2.13}$$

By using (2.13) and neglecting $*$, the equations (2.7) to (2.11) become

$$\begin{aligned} \frac{\partial u}{\partial \tau} - 2Ev = &-\frac{\partial P}{\partial \xi} + \frac{\partial^2 u}{\partial \eta^2} + lL(u_p - u) + \frac{M^2}{(1+m^2)}(mv - u)\\ &- \frac{1}{k^+} u + G_r \theta, \end{aligned} \tag{2.14}$$

$$\frac{\partial v}{\partial \tau} + 2Eu = \frac{\partial^2 v}{\partial \eta^2} + lL(v_p - v) - \frac{M^2}{(1+m^2)}(v + mu) - \frac{1}{k^+} v, \tag{2.15}$$

$$\frac{\partial u_p}{\partial \tau} - 2Ev_p = L(u - u_p), \tag{2.16}$$

$$\frac{\partial v_p}{\partial \tau} + 2Eu_p = L(v - v_p), \tag{2.17}$$

$$P_r \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} - R\theta, \tag{2.18}$$

where $E = \frac{\Omega h^2}{\nu}$ is the rotation parameter, $M = B_0 h \sqrt{\frac{\sigma}{\nu \rho}}$ is the magnetic parameter, $G_r = \frac{g\beta(T_1 - T_2)h^3}{\nu^2}$ is the Grashof number, $P_r = \frac{\rho C_p \nu}{k}$ is the Prandtl number,

$R = \frac{16k^*\sigma^* T_2^3 h^2}{k}$ is the radiation parameter, $L = \frac{h^2}{\tau_p \nu}$ is the reciprocal of relaxation parameter, $k^+ = \frac{k}{h^2}$ is permiability parameter and $P = \frac{h^2 p}{\rho \nu^2}$ is the non-dimensional fluid pressure.

The boundary conditions (2.12) become

$$\begin{aligned} &u = v = u_p = v_p = 0, \quad &\theta = \cos n\tau \quad &\text{at } \eta = -1, \\ &u = v = u_p = v_p = 0, \quad &\theta = 0 \quad &\text{at } \eta = +1, \end{aligned} \tag{2.19}$$

where $n = \frac{\omega h^2}{\nu}$ is frequency parameter. Now by introducing $F = u + \mathrm{i}\, v$ and $G = u_p + \mathrm{i}\, v_p$, equations (2.14) to (2.18) are reduced to

$$\begin{aligned} \frac{\partial F}{\partial \tau} = &- \frac{\partial P}{\partial x\,\mathrm{i}} + \frac{\partial^2 F}{\partial \eta^2} - \left[\left(lL + \frac{1}{k^+} + \frac{M^2}{(1+m^2)} \right) + \mathrm{i} \left(2E + \frac{mM^2}{1+m^2} \right) \right] F \\ &+ G_r \theta + lLG, \end{aligned} \tag{2.20}$$

$$\frac{\partial G}{\partial \tau} = -\,\mathrm{i}\, 2EG + L(F - G), \tag{2.21}$$

$$P_r \frac{\partial \theta}{\partial \tau} = \frac{\partial^2 \theta}{\partial \eta^2} - R\theta. \tag{2.22}$$

The boundary conditions (2.19) are reduced to

$$\begin{aligned} &F = G = 0, \quad &\theta = \cos n\tau \quad &\text{at } \eta = -1, \\ &F = G = 0, \quad &\theta = 0 \quad &\text{at } \eta = +1. \end{aligned} \tag{2.23}$$

Non-dimensional equations (2.20)–(2.22) represent the set of partial differential equations which cannot be solved in the closed form due to the non-linearity. However, these equations can be solved analytically after being reduced to set of ordinary differential equations. To this end, the fluid velocity (F), dust phase velocity (G) and the temperature (θ) can be expressed as follows[**16**]:

$$\begin{aligned} -\frac{\partial P}{\partial \xi} &= \frac{1}{2}\left(\mathrm{e}^{\mathrm{i}\, n\tau} + \mathrm{e}^{-\,\mathrm{i}\, n\tau} \right), \\ F(\eta, \tau) &= f_1(\eta)\, \mathrm{e}^{\mathrm{i}\, n\tau} + f_2(\eta)\, \mathrm{e}^{-\,\mathrm{i}\, n\tau}, \\ G(\eta, \tau) &= g_1(\eta)\, \mathrm{e}^{\mathrm{i}\, n\tau} + g_2(\eta)\, \mathrm{e}^{-\,\mathrm{i}\, n\tau}, \\ \theta(\eta, \tau) &= \theta_1(\eta)\, \mathrm{e}^{\mathrm{i}\, n\tau} + \theta_2(\eta)\, \mathrm{e}^{-\,\mathrm{i}\, n\tau} \end{aligned} \tag{2.24}$$

where $f_1(\eta)$, $f_2(\eta)$, $g_1(\eta)$, $g_2(\eta)$, $\theta_1(\eta)$ and $\theta_2(\eta)$ are unknown functions.

In view of the above relations (2.24), the equations (2.20) to (2.22) take the following forms

$$f_1''(\eta) - r_1^2 f_1(\eta) = -\frac{1}{2} - G_r \theta_1(\eta) - lLg_1(\eta), \tag{2.25}$$

$$f_2''(\eta) - r_2^2 f_2(\eta) = -\frac{1}{2} - G_r \theta_2(\eta) - lLg_2(\eta), \tag{2.26}$$

$$g_1(\eta) = f_1(\eta) \left(\frac{L}{L + \mathrm{i}(2E + n)} \right), \tag{2.27}$$

$$g_2(\eta) = f_2(\eta)\left(\frac{L}{L + \mathrm{i}(2E - n)}\right), \tag{2.28}$$

$$\theta_1''(\eta) - r_3^2\theta_1(\eta) = 0, \tag{2.29}$$

$$\theta_2''(\eta) - r_4^2\theta_2(\eta) = 0 \tag{2.30}$$

with the boundary conditions

$$\begin{aligned} f_1 = f_2 = g_1 = g_2 = 0, \quad & \theta_1 = \theta_2 = \frac{1}{2} \quad && \text{at } \eta = -1, \\ f_1 = f_2 = g_1 = g_2 = 0, \quad & \theta_1 = \theta_2 = 0 \quad && \text{at } \eta = +1. \end{aligned} \tag{2.31}$$

The solutions of the equations (2.25) to (2.30) with subjected to the boundary condition (2.31) are

$$\begin{aligned} F(\eta,\tau) = {} & \frac{1}{2}\left[\frac{1}{r_1^2}\left(1 - \frac{\cosh r_1\eta}{\cosh r_1}\right)\right. \\ & \left. + \frac{G_r}{(r_3^2 - r_1^2)}\left(\frac{\sinh r_1(1-\eta)}{\sinh 2r_1} - \frac{\sinh r_3(1-\eta)}{\sinh 2r_3}\right)\right]\mathrm{e}^{\mathrm{i}\,n\tau} \\ & + \frac{1}{2}\left[\frac{1}{r_2^2}\left(1 - \frac{\cosh r_2\eta}{\cosh r_2}\right)\right. \\ & \left. + \frac{G_r}{(r_4^2 - r_2^2)}\left(\frac{\sinh r_2(1-\eta)}{\sinh 2r_2} - \frac{\sinh r_4(1-\eta)}{\sinh 2r_4}\right)\right]\mathrm{e}^{-\mathrm{i}\,n\tau}, \end{aligned} \tag{2.32}$$

$$\begin{aligned} G(\eta,\tau) = {} & \frac{L}{2(L + \mathrm{i}(2E+n))}\left[\frac{1}{r_1^2}\left(1 - \frac{\cosh r_1\eta}{\cosh r_1}\right)\right. \\ & \left. + \frac{G_r}{(r_3^2 - r_1^2)}\left(\frac{\sinh r_1(1-\eta)}{\sinh 2r_1} - \frac{\sinh r_3(1-\eta)}{\sinh 2r_3}\right)\right]\mathrm{e}^{\mathrm{i}\,n\tau} \\ & + \frac{L}{2(L + \mathrm{i}(2E-n))}\left[\frac{1}{r_2^2}\left(1 - \frac{\cosh r_2\eta}{\cosh r_2}\right)\right. \\ & \left. + \frac{G_r}{(r_4^2 - r_2^2)}\left(\frac{\sinh r_2(1-\eta)}{\sinh 2r_2} - \frac{\sinh r_4(1-\eta)}{\sinh 2r_4}\right)\right]\mathrm{e}^{-\mathrm{i}\,n\tau}, \end{aligned} \tag{2.33}$$

$$\theta(\eta,\tau) = \frac{1}{2}\left[\mathrm{e}^{\mathrm{i}\,n\tau}\left(\frac{\sinh r_3(1-\eta)}{\sinh 2r_3}\right) + \mathrm{e}^{-\mathrm{i}\,n\tau}\left(\frac{\sinh r_4(1-\eta)}{\sinh 2r_4}\right)\right] \tag{2.34}$$

where,

$$\begin{aligned} r_1^2 = {} & \left[\left(lL + \frac{1}{k^+} + \frac{M^2}{(1+m^2)} - \frac{lL^3}{L^2 + (2E+n)^2}\right)\right. \\ & \left. + \mathrm{i}\left(2E + n + \frac{mM^2}{1+m^2} + \frac{lL^2(2E+n)}{L^2 + (2E+n)^2}\right)\right], \\ r_2^2 = {} & \left[\left(lL + \frac{1}{k^+} + \frac{M^2}{(1+m^2)} - \frac{lL^3}{L^2 + (2E-n)^2}\right)\right. \\ & \left. - \mathrm{i}\left(2E - n + \frac{mM^2}{1+m^2} + \frac{lL^2(2E-n)}{L^2 + (2E-n)^2}\right)\right], \end{aligned}$$

$$r_3^2 = (R + \mathrm{i}\, nP_r),$$
$$r_4^2 = (R - \mathrm{i}\, nP_r).$$

By separating into a real and imaginary parts, one can easily obtain the primary and secondary velocity components of both fluid and particle phases from (2.32) and (2.33), respectively.

Skin friction:

Let τ_f and τ_p be the skin friction for the fluid phase and dust phase, respectively. Then we have

$$\tau_f = \left(\frac{\partial F}{\partial \eta}\right)_{\eta=1} = \frac{1}{2}\left[\frac{G_r}{r_3^2 - r_1^2}\left(\frac{r_3}{\sinh 2r_3} - \frac{r_1}{\sinh 2r_1}\right) - \frac{\tanh r_1}{r_1}\right]\mathrm{e}^{\mathrm{i}\, n\tau}$$

(2.35)

$$+\frac{1}{2}\left[\frac{G_r}{r_4^2 - r_2^2}\left(\frac{r_4}{\sinh 2r_4} - \frac{r_2}{\sinh 2r_2}\right) - \frac{\tanh r_2}{r_2}\right]\mathrm{e}^{-\mathrm{i}\, n\tau}.$$

$$\tau_p = \left(\frac{\partial G}{\partial \eta}\right)_{\eta=1} = \frac{L}{2(L+\mathrm{i}(2E+n))}\left[\frac{G_r}{r_3^2 - r_1^2}\left(\frac{r_3}{\sinh 2r_3} - \frac{r_1}{\sinh 2r_1}\right) - \frac{\tanh r_1}{r_1}\right]\mathrm{e}^{\mathrm{i}\, n\tau}$$

(2.36)

$$+\frac{L}{2(L+\mathrm{i}(2E-n))}\left[\frac{G_r}{r_4^2 - r_2^2}\left(\frac{r_4}{\sinh 2r_4} - \frac{r_2}{\sinh 2r_2}\right) - \frac{\tanh r_2}{r_2}\right]\mathrm{e}^{-\mathrm{i}\, n\tau}.$$

By separating into a real and imaginary parts, one can easily obtain the skin friction for primary and secondary velocity components of both fluid and particle phases from (2.35) and (2.36), respectively. The correctness of the present solution is verified by taking $l = 0$ (mass concentration of dust particles) and $k^+ = 0$ (permeability). Then our solution is reduced to well reported solution by Guchhait et.al [**16**].

3. Results and Discussion

The governing partial differential equations of velocity and temperature profiles (2.7) to (2.11) subject to boundary conditions (2.12) describe the MHD free convective dusty fluid flow in a rotating vertical channel with hall current and radiation effect. The flow is considered in the porous media under the influence of periodic pressure gradient. The solutions of the governing equations are obtained by employing the perturbation technique. In order to have a physical insight into the problem, we have written a MATLAB program to compute and generate the graphs for the primary velocity of the fluid phase, dust phase and temperature profiles verses η for different values of physical parameters (like $M^2, m, E, R, k^+, Pr, Gr, n$ and $n\tau$). Further, the skin-friction of primary velocities of both fluid and dust phases against different values of physical parameter (like M^2, Gr, E and n) are depicted in the Figures14–17.

Figure 2 has been plotted to depict the variation of velocity profiles against η for different values of hall parameter (m) by fixing other physical parameters. From this graph we observe that the primary velocity profile for both fluid and dust phases notably decreases with an increase in hall parameter (m). Figure 3 depicts that the primary velocity profile for both fluid and dust phases decreases with an increase in magnetic parameter (M^2) since the application of the transverse magnetic field plays the important role of a resistive type force (Lorentze force) similar to drag force (that acts in the opposite direction of the fluid motion) which tends to resist the flow thereby reducing its velocity.

The primary velocity profile of both fluid and dust phases decreases with an increase in the rotation parameter (E) which is observed from the Figure 4 with $n\tau = \pi/3$. Figure 5 has been plotted to depict the variation of velocity profiles against η for different values of porous parameter (k^+) by fixing other physical parameters ($m = 0.4$ & $L = 3$). This figure gives clear picture of the primary velocity profile of both fluid and dust phases and it is observed that both fluid and dust phase velocity decreases with increase in the porous parameter (k+). Nevertheless, this effect is quite opposite for increase in Grashof number (Gr) that is noticed in the Figure 6. This is because an increase in Grashof number (Gr) means more heating and less density.

Figure 7 depicts that the primary velocity of fluid phase and dust phase decreases with an increase in the radiation parameter (R). Figure 8 indicates that an increase in phase angle decreases the fluid phase and dust phase primary velocity with $L = 0.6$. The effect of frequency parameter on velocity of both fluid and dust phases is shown in Figure 9. It is observed that the primary velocity of both fluid phase and dust phase increases with an increase in frequency parameter n with $L = 1$.

The variation of temperature profiles are illustrated in the Figures 10 to 13. Figure 10 reveals an interesting phenomenon. It shows that temperature θ decreases with an increase of radiation parameter (R). This result qualitatively agrees with expectations, since the effect of radiation is to decrease the rate of energy transport to the fluid, thereby decreasing the temperature of the fluid. Figure 11 has been plotted to depict the variation of temperature profiles against η for different values of Prandtl number (Pr) by fixing other physical parameters ($n\tau = \pi/3$). Figure 10 displays that the fluid temperature θ decreases with an increase of prandtl number (Pr). This is because fluids with large (Pr) have low thermal diffusivity which causes low heat penetration.

Figure 12 depicts that temperature profile notably decreases with an increase in the phase angle ($n\tau$). The fluid temperature θ increases near the plate at $\eta = -1$ and it decreases away from the plate at $\eta = -1$ with an increase in frequency parameter (n) as observed from Figure 13 with $n\tau = \pi/3$.

The Figures 14, 15, 16, and 17 depict that an increase in magnetic parameter (M^2), rotation parameter (E) and frequency parameter (n) result in an increase in the skin-friction of primary velocity of both phases. While an increase in the Grashof number (Gr) decreases the skin-friction of primary velocities of both phases which is illustrated in Figure 16.

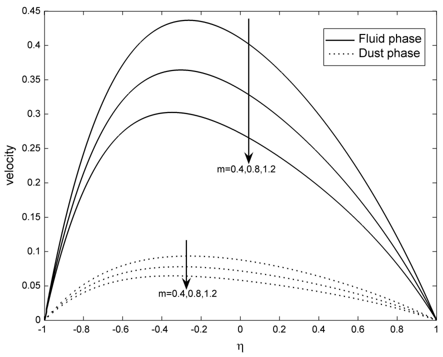

Figure 2. Variation of velocity profile for the different values of m.

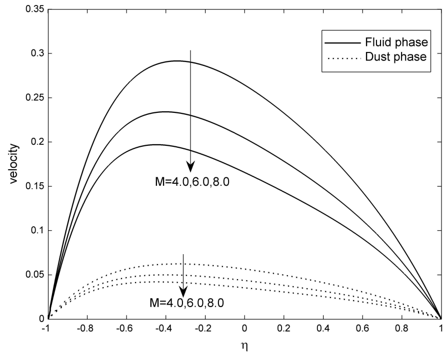

Figure 3. Variation of velocity profile for the different values of M^2.

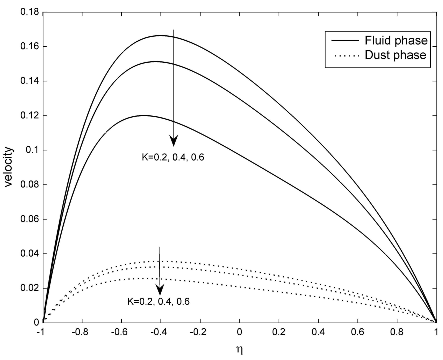

Figure 4. Variation of velocity profile for the different values of k^+.

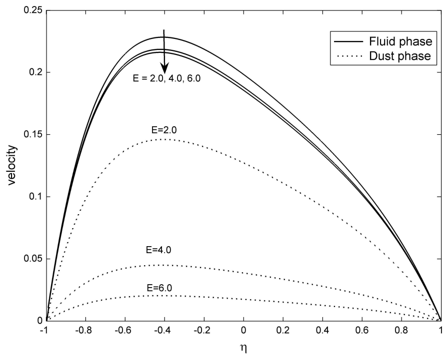

Figure 5. Variation of velocity profile for the different values of E.

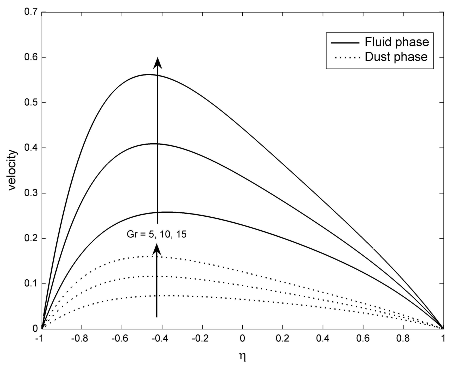

Figure 6. Variation of velocity profile for the different values of Gr.

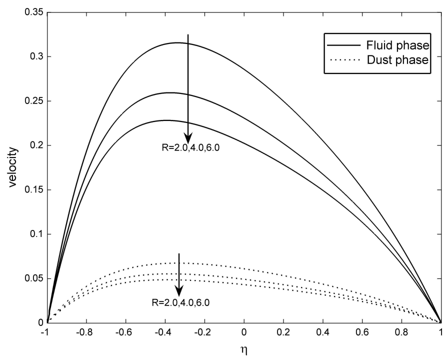

Figure 7. Variation of velocity profile for the different values of R.

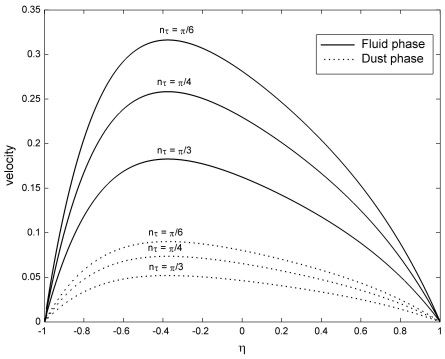

Figure 8. Variation of velocity profile for the different values of $n\tau$.

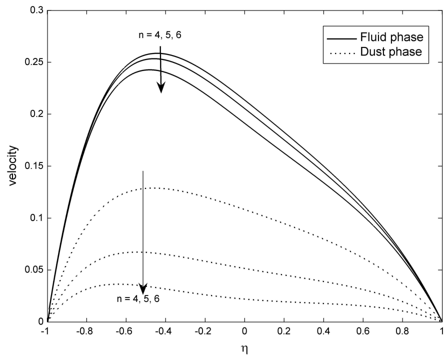

Figure 9. Variation of velocity profile for the different values of n.

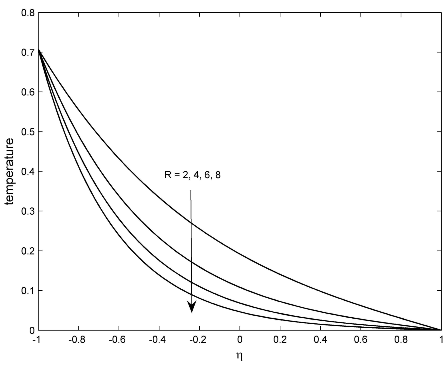

Figure 10. Variation of temperature profile for the different values of R.

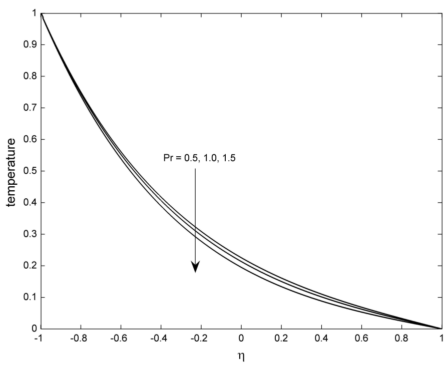

Figure 11. Variation of temperature profile for the different values of Pr.

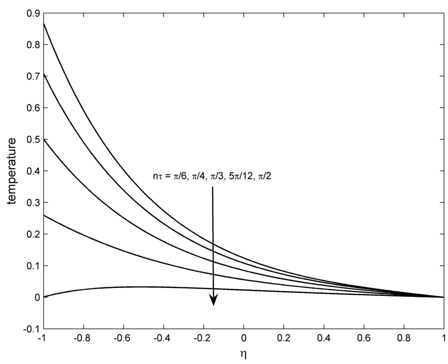

Figure 12. Variation of temperature profile for the different values of $n\tau$.

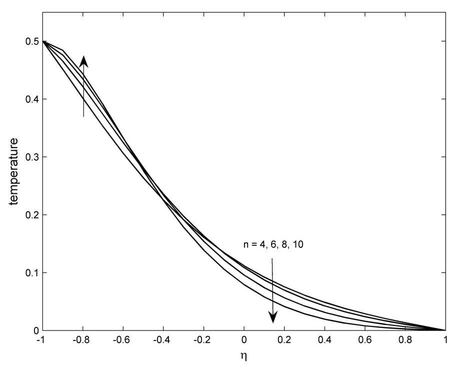

Figure 13. Variation of temperature profile for the different values of n.

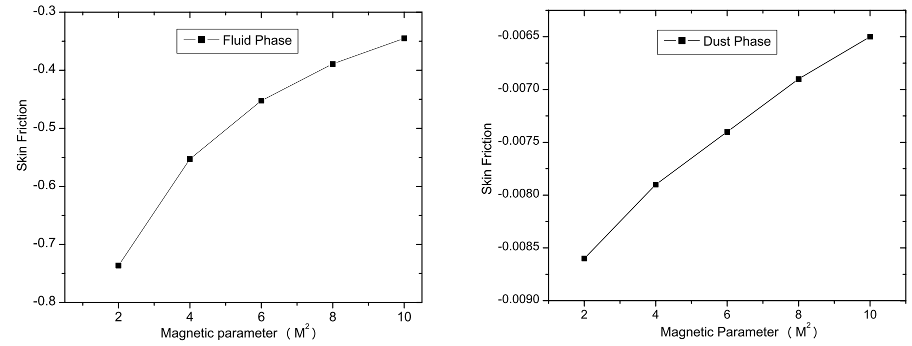

Figure 14. Skin friction of fluid phase and dust phase for different values of M^2 ($L = 1$, $k = 10$, $l = 0.5$, $Gr = 5$, $R = 5$, $n\tau = \pi/4$, $E = 10$, $Pr = 0.72$, $n = 1.0$).

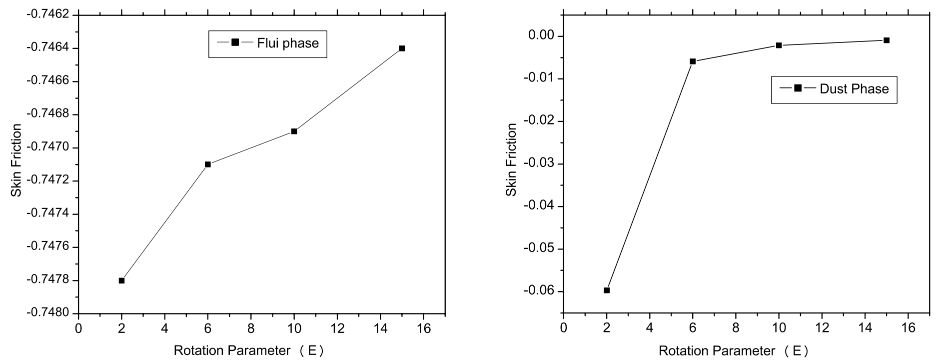

Figure 15. Skin friction of fluid phase and dust phase for different values of -E ($L = 1$, $k = 10$, $l = 0.5$, $Gr = 5$, $R = 5$, $n\tau = \pi/4$, $M^2 = 5$, $Pr = 0.72$, $n = 1.0$).

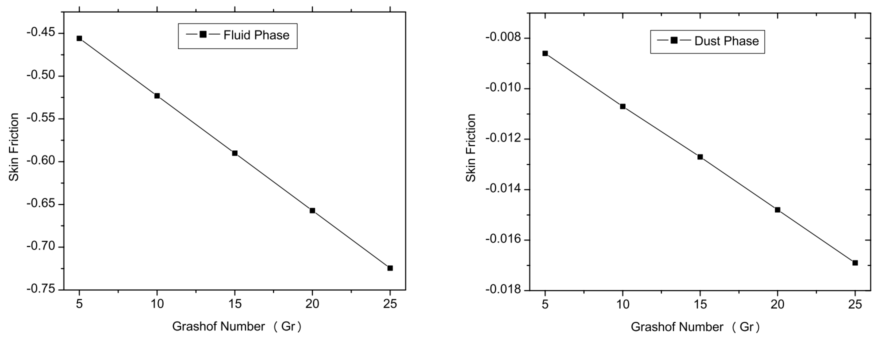

Figure 16. Skin friction of fluid phase and dust phase for different values of -Gr ($L = 1$, $k = 10$, $l = 0.5$, $M^2 = 5$, $R = 5$, $n\tau = \pi/4$, $E = 10$, $Pr = 0.72$, $n = 1.0$).

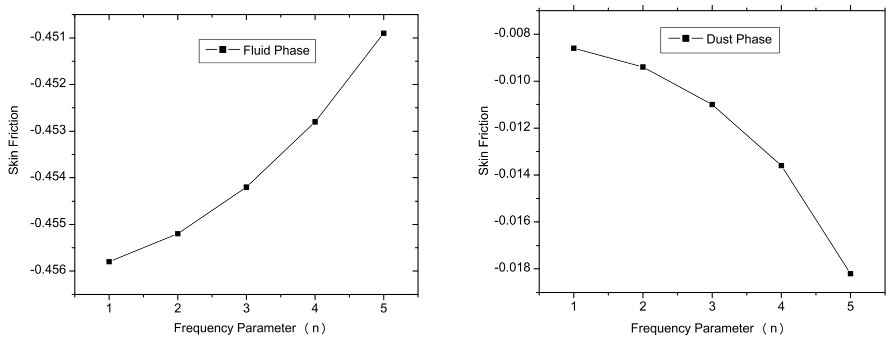

Figure 17. Skin friction of fluid phase and dust phase for different values of n ($L = 1$, $k = 10$, $l = 0.5$, $Gr = 5$, $R = 5$, $n\tau = \pi/4$, $E = 10$, $Pr = 0.72$, $M^2 = 5$).

4. Conclusions

In this paper, a mathematical analysis has been carried out on the MHD free convective dusty fluid flow in a rotating porous vertical channel with hall current and the effect of radiation under a periodic pressure gradient. The governing partial differential equations are solved analytically using the perturbation method. The effect of various physical parameters (like rotation parameter (E), magnetic parameter (M^2) and Hall current parameter (m) etc.) are examined. Some of the important findings of our analysis obtained by the graphical representation are listed below:

- The rotation and hall effect on the MHD convection flow have more significant effect on the dusty fluid flow in a vertical channel through porous medium.
- The primary velocity of the dust phase is lower than the primary velocity of the fluid phase.
- The effect of increasing values of rotation parameter is to decrease of the primary velocity of the fluid and dust phases.
- The combined effect of increasing values of magnetic parameter and Hall current parameter is to decrease the primary velocity of the fluid and dust phases.
- The boundary layer thickness increases with an increase in the Hall current parameter.
- The thermal boundary layer thickness decreases with increase in the Prandle number.
- The effect increasing values of magnetic parameter, rotation parameter, frequency parameter is to increase the skin-friction of primary velocity of both phases.

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S. Manjunatha, Department of Engineering Mathematics, Christ University Faculty of Engineering Kumbalagodu, Mysore Road, Bangalore-560074, Karnataka-State, India,
e-mail: manjubhushana@gmail.com

B. J. Gireesha, Department of Studies and Research in Mathematics, Kuvempu University, Shankaraghatta-577 451, Shimoga, Karnataka, India,
e-mail: bjgireesu@rediffmail.com

C. S. Bagewadi, Department of Studies and Research in Mathematics, Kuvempu University, Shankaraghatta-577 451, Shimoga, Karnataka, India,
e-mail: prof_bagewadi@yahoo.co.in