Acta Math. Univ. Comenianae
Vol. LXXXVII, 1 (2018), pp. 25–34

ON BOCHNER RICCI SEMI-SYMMETRIC HERMITIAN MANIFOLD

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Abstract. The aim of the present paper is to study a Bochner Ricci semi-symmetric quasi-Einstein Hermitian manifold $(QEH)_n$, a Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold $G(QEH)_n$ and a Bochner Ricci semi-symmetric pseudo generalised quasi-Einstein Hermitian manifold $P(GQEH)_n$.

1. Introduction

An even dimension differentiable manifold M^n is said to be a Hermitian manifold if a complex structure J of type $(1,1)$ and a pseudo-Riemannian metric g of the manifold M satisfy

$$J^2 = -I, \tag{1.1}$$

and

$$g(JX, JY) = g(X, Y), \tag{1.2}$$

for all $X, Y \in \chi(M)$, where $\chi(M)$ is Lie algebra of vector fields on M.

The notion of an Einstein manifold was introduced and studied by Albert Einstein and for this reason, this manifold is known as an Einstein manifold. In differential geometry and mathematical physics, an Einstein manifold is a Riemannian or pseudo-Riemannian manifold (M^n, g) $(n \geq 2)$ whose Ricci tensor satisfies the condition

$$S(X, Y) = \alpha g(X, Y), \tag{1.3}$$

where S denotes the Ricci tensor of the manifold $(M^n, g)(n \geq 2)$ and α is a non-zero scalar. An Einstein manifold plays an important role in Riemannian geometry as well as in general theory of relativity.

From the equation (1.3), we get

$$r = n\alpha. \tag{1.4}$$

Received February 8, 2017; revised April 27, 2017.
2010 *Mathematics Subject Classification.* Primary 53C25, 53D15.
Key words and phrases. Einstein manifold, quasi-Einstein manifold, generalised quasi-Einstein manifold, pseudo generalised quasi-Einstein manifold, Bochner curvature tensor.

In 2000, M. C. Chaki and R. K. Maity [**12**] introduced a new type of a non-flat Riemannian manifold whose non-zero Ricci tensor satisfies

$$S(X,Y) = \alpha g(X,Y) + \beta A(X)A(Y), \tag{1.5}$$

and they called it a quasi-Einstein manifold, where α, β are scalars such that $\beta \neq 0$ and A is a non-zero 1-form defined by $g(X,\rho) = A(X)$ for every vector field X. ρ denotes the unit vector called the generator of the manifold. An n-dimensional quasi-Einstein manifold is denoted by $(QE)_n$.

After contraction of the equation (1.5), we have

$$r = \alpha n + \beta. \tag{1.6}$$

From the equations, (1.2) and (1.5), we can easily obtain

$$\begin{aligned} &S(X,\rho) = (\alpha+\beta)A(X), \qquad && S(\rho,\,\rho) = (\alpha+\beta),\\ &g(J\rho,\,\rho) = 0, && S(J\rho,\,\rho) = 0. \end{aligned} \tag{1.7}$$

A quasi-Einstein manifold arises during the study of exact solutions of Einstein fields equations as well as considerations of a quasi-umblical hypesurfaces of semi-Euclidean space. The Walker-space time is an example of quasi-Einstein manifold. Also quasi-Einstein manifolds can be taken as a model of the perfect fluid space time in general theory of relativity [**22**]. So we can say that quasi-Einstein manifolds play an important role in the general theory of relativity. A quasi-Einstein manifold has been studied by several authors [**1, 6, 15, 16**] in different ways. In 2001, M. C. Chaki [**13**] introduced the notion of generalized quasi-Einstein manifolds. Also U. C. De and G. C. Ghosh [**20**] cited an example of a generalized quasi-Einstein manifold and studied its geometrical properties in 2004.

A Riemannian manifold (M^n, g), $(n \geq 2)$ is said to be a generalized quasi-Einstein manifold if a non-zero Ricci tensor of type $(0,2)$ satisfies the condition

$$S(X,Y) = \alpha g(X,Y) + \beta A(X)A(Y) + \gamma C(X)C(Y), \tag{1.8}$$

where α, β and γ are scalars such that $\beta \neq 0$, $\gamma \neq 0$ and A, C are non-vanishing 1-forms such that

$$\begin{aligned} &g(X,\rho) = A(X), \qquad g(X,\mu) = C(X),\\ &g(\rho,\rho) = g(\mu,\mu) = 1, \end{aligned} \tag{1.9}$$

where ρ and μ are orthogonal unit vectors. An n-dimensional generalized quasi-Einstein manifold is denoted by $G(QE)_n$.

After contraction of equation (1.8), we get

$$r = \alpha n + \beta + \gamma. \tag{1.10}$$

From the equations (1.2), (1.8) and (1.9), we can easily obtain

$$\begin{aligned} &S(X,\,\rho) = (\alpha+\beta)A(X), \qquad && S(X,\,\mu) = (\alpha+\gamma)C(X),\\ &S(\mu,\,\mu) = \alpha+\gamma, && S(\rho,\,\rho) = \alpha+\beta,\\ &g(J\rho,\,\rho) = g(J\mu,\mu) = 0, &&\\ &S(J\mu,\,\mu) = S(J\rho,\,\rho) = 0. && \end{aligned} \tag{1.11}$$

In the continuation of above the studies several authors have generalized the notion of quasi-Einstein manifolds, for example, some results on generalized quasi-Einstein manifolds by Prakasha and Venkatesha [**9**], super quasi-Einstein manifolds by C. Özgür [**7**], pseudo quasi-Einstein manifolds by A. A. Shaikh [**2**] and also $N(k)$-quasi-Einstein manifolds studied by [**3, 5, 8, 11, 14**].

In 2008, De and Gazi [**21**] introduced the notion of nearly quasi-Einstein manifolds. A non-flat Riemannian manifold (M^n, g), $(n \geq 2)$ is called a nearly quasi-Einstein manifold if its Ricci tensor of type $(0, 2)$ is not identically zero and satisfies the condition

$$S(X, Y) = \alpha g(X, Y) + \beta E(X, Y), \tag{1.12}$$

where α, β are scalars such that $\beta \neq 0$ and E is a non-zero symmetric tensor of type $(0, 2)$. An n-dimensional nearly quasi-Einstein manifold is denoted by $N(QE)_n$.

In 2008, A. A. Shaikh and A. K. Jana introduced the concept of a pseudo generalized quasi-Einstein manifold and also verified it by a suitable example. A Riemannian manifold (M^n, g), $(n \geq 2)$ is called a pseudo generalized quasi-Einstein manifold if its Ricci tensor S of type $(0, 2)$ is not identically zero and satisfies the condition

$$S(X, Y) = \alpha g(X, Y) + \beta A(X)A(Y) + \gamma C(X)C(Y) + \delta D(X, Y), \tag{1.13}$$

where α, β, γ and δ are non-zero scalars and A, C are non-vanishing 1-forms such that

$$\begin{aligned} &g(X, \rho) = A(X), \qquad g(X, \mu) = C(X), \\ &g(\rho, \rho) = g(\mu, \mu) = 1 \end{aligned} \tag{1.14}$$

for every vector field X. ρ and μ are mutually orthogonal unit vector fields called the generators of the manifold. D is a non-zero symmetric tensor of type $(0, 2)$ with zero trace, which satisfies the condition

$$D(X, \rho) = 0 \tag{1.15}$$

for every vector field X. Also α, β, γ and δ are called the associated scalars. A and C are the associated 1-forms of the manifold and D is called the structure tensor of the manifold. Such type of the manifold is denoted by $P(GQE)_n$.

Now contracting the equation (1.13), we have

$$r = \alpha n + \beta + \gamma + \delta D. \tag{1.16}$$

From equations (1.13)–(1.15) and (1.2), we have

$$\begin{aligned} &S(X, \rho) = (\alpha + \beta)A(X), && S(X, \mu) = (\alpha + \gamma)C(X), \\ &S(\rho, \rho) = (\alpha + \beta) + \delta D(\rho, \rho), && S(\mu, \mu) = (\alpha + \gamma) + \delta D(\mu, \mu), \\ &g(J\rho, \rho) = g(J\mu, \mu) = 0, && \\ &S(J\mu, \mu) = \delta D(J\mu, \mu), && S(J\rho, \rho) = \delta D(J\rho, \rho). \end{aligned} \tag{1.17}$$

The notion of a Bochner curvature tensor was introduced by S. Bochner [**18**]. The Bochner curvature tensor B is defined by

(1.18)

$$\begin{aligned}B(Y,Z,U,V) = R(Y,Z,U,V) &- \frac{1}{2(n+2)}\Big\{S(Y,V)g(Z,U) - S(Y,U)g(Z,V)\\ &+ g(Y,V)S(Z,U) - g(Y,U)S(Z,V) + S(JY,V)g(JZ,U)\\ &- S(JY,U)g(JZ,V) + S(JZ,U)g(JY,V) - g(JY,U)S(JZ,V)\\ &- 2S(JY,Z)g(JU,V) - 2g(JY,Z)S(JU,V)\Big\}\\ &+ \frac{r}{(2n+2)(2n+4)}\Big\{g(Z,U)g(Y,V) - g(Y,U)g(Z,V)\\ &+ g(JZ,U)g(JY,V) - g(JY,U)g(JZ,V) - 2g(JY,Z)g(JU,V)\Big\},\end{aligned}$$

where r is a scalar curvature of the manifold.

In a Hermitian manifold, a Bochner curvature tensor satisfies the condition

$$B(X,Y,U,V) = -B(X,Y,V,U). \tag{1.19}$$

In 2012, S. K. Hui and R. S. Lemence [**17**] studied a generalised quasi-Einstein manifold addmitting a W_2-curvature tensor and proved that if a W_2-curvature tensor satisfies $W_2 \cdot S = 0$, then either the associated scalars β and γ are equal or the curvature tensor R satisfies a definite condition. D. G. Prakasha and H. Venkatesha [**9**] studied some results on generalised quasi-Einstein manifolds and proved that in generalized quasi-Einstein manifold, if a conharmonic curvature tensor satisfies $L \cdot S = 0$, then either M is a nearly quasi-Einstein manifold $N(QE)_n$ or the curvature tensor R satisfies a definite condition. After studying of these developments in quasi-Einstein manifold $(QE)_n$, generalized quasi-Einstein manifold $G(QE)_n$ and a pseudo generalized quasi-Einstein manifold $P(GQE)_n$, we plan to study another type of Bochner Ricci semi-symmetric Hermitian manifold.

2. Bochner Ricci semi-symmetric Hermitian manifold

Let (M^n, g) be a Riemannian manifold and ∇ be the Levi-Civita connection on (M^n, g) then, a Riemannian manifold is said to be locally symmetric if $\nabla R = 0$, where R is the Riemannian curvature tensor of (M^n, g). The locally symmetric manifold has been studied by different geometers through different aproaches. The notion has been developed, e.g., a semi-symmetric manifold by Szabo [**23**], recurrent manifold by Walker [**4**], conformally recurrent manifold by Adati and Miyazawa [**19**].

According to Z. I. Szabo [**23**], if the manifold M satisfies the condition

$$(R(X,Y).R)(U,V)W = 0, \qquad X,Y,U,V,W \in \chi(M), \tag{2.1}$$

for all vector fields X and Y, then the manifold is called a semi-symmetric manifold.

For a $(0,k)$-tensor field T on M, $k \geq 1$, and a symmetric $(0,2)$-tensor field A on M, the $(0,k+2)$-tensor fields R, T and Q(A, T) are defined by

$$(R.T)(X_1,\dots,X_k;X,Y) = -T(R(X,Y)X_1,X_2,\dots,X_k) - \dots - T(X_1,\dots,X_{k-1},R(X,Y)X_k) \tag{2.2}$$

and

$$Q(A,T)(X_1,\dots,X_k;X,Y) = -T((X\wedge_A Y)X_1,X_2,\dots,X_k) - \dots - T(X_1,\dots,X_{k-1},(X\wedge_A Y)X_k), \tag{2.3}$$

where $X \wedge_A Y$ is the endomorphism given by

$$(X\wedge_A Y)Z = A(Y,Z)X - A(X,Z)Y. \tag{2.4}$$

Definition 2.1 ([**10**])**.** A semi-Riemannian manifold is said to be Ricci semi-symmetric if the following condition is satisfied

$$(R(X,\,Y).S)(Z,W) = -S(R(X,Y)Z,W) - S(Z,R(X,Y)W) = 0 \tag{2.5}$$

for all $X,Y,Z \in \chi(M^n)$.

Now we set the following definition.

Definition 2.2. An even dimensional Hermitian manifold (M^n,g) is said to be Bochner Ricci semi-symmetric Hermitian manifold if the Bochner curvature tensor satisfies the condition $B\cdot S = 0$, i.e.,

$$(B(X,\,Y).S)(Z,W) = -S(B(X,Y)Z,W) - S(Z,(B(X,Y)W) = 0 \tag{2.6}$$

for all $X,Y,Z \in \chi(M^n)$.

3. Bochner Ricci semi-symmetric quasi-Einstein Hermitian manifold $(QEH)_n$

In this section, we introduce the following definitions.

Definition 3.1. A Hermitian manifold is said to be a quasi-Einstein Hermitian manifold if it satisfies the equation (1.5). Throughout this paper, we denote the quasi-Einstein Hermitian manifold by $(QEH)_n$.

Definition 3.2. A quasi-Einstein Hermitian manifold is said to be a Bochner Ricci semi-symmetric quasi-Einstein Hermitian manifold $(QEH)_n$ if it satisfies the equation (2.6).

If we take a Bochner Ricci semi-symmetric quasi-Einstein Hermitian manifold, then from the equations (1.5) and (2.6), we have

$$\alpha[B(X,Y,Z,W)+B(X,Y,W,Z)] + \beta[A(B(X,Y)Z)A(W)+A(Z)A(B(X,Y)W)] = 0, \tag{3.1}$$

where $g(B(X,Y)W,Z) = B(X,Y,W,Z)$.

Now from the equations (1.19) and (3.1), we have

$$\beta[A(B(X,Y)Z)A(W)+A(Z)A(B(X,Y)W)] = 0, \tag{3.2}$$

this implies either

$$\beta = 0, \tag{3.3}$$

or

$$A(B(X,Y)Z)A(W) + A(Z)A(B(X,Y)W) = 0. \tag{3.4}$$

From the equation (3.2), if $\beta = 0$, then from the equation (1.5), we have

$$S(X,Y) = \alpha g(X,Y). \tag{3.5}$$

Thus we come to the conclusion.

Theorem 3.3. *Every Bochner Ricci semi-symmetric quasi-Einstein Hermitian manifold $(QEH)_n$ is either Bochner Ricci semi-symmetric Einstein manifold or 1-form A satisfying*

$$A(B(X,Y)Z)A(W) + A(Z)A(B(X,Y)W) = 0.$$

4. Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold $G(QEH)_n$

In this section, we introduce the following definitions.

Definition 4.1. A Hermitian manifold is said to be generalised quasi-Einstein Hermitian manifold if it satisfies the equation (1.8). Throughout this paper, we denote the generalised quasi-Einstein Hermitian manifold by $G(QEH)_n$.

Definition 4.2. A generalised quasi-Einstein Hermitian manifold is said to be a Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold $G(QEH)_n$ if it satisfies the equation (2.6).

If we take a Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold then from the equations (1.8) and (2.6), we have

$$\begin{aligned} &\alpha[B(X,Y,Z,W) + B(X,Y,W,Z)] \\ &+ \beta[A(B(X,Y)Z)A(W) + A(Z)A(B(X,Y)W)] \\ &+ \gamma[C(B(X,Y)Z)C(W) + C(Z)C(B(X,Y)W)] = 0, \end{aligned} \tag{4.1}$$

where $g(B(X,Y)W,Z) = B(X,Y,W,Z)$.

Now from the equations (1.19) and (4.1), we have

$$\begin{aligned} &\beta[A(B(X,Y)Z)A(W) + A(Z)A(B(X,Y)W)] \\ &+ \gamma[C(B(X,Y)Z)C(W) + C(Z)C(B(X,Y)W)] = 0. \end{aligned} \tag{4.2}$$

Putting $Z = \rho$ and $W = \mu$ in a equation (4.2), we have

$$\beta A(B(X,Y)\mu) + \gamma C(B(X,Y)\rho) = 0. \tag{4.3}$$

Since we know that

$$C(B(X,Y)\rho) = g(B(X,Y)\rho,\mu) = B(X,Y,\rho,\mu)$$

and

$$A(B(X,Y)\mu) = g(B(X,Y)\mu,\rho) = B(X,Y,\mu,\rho),$$

then from the equation (4.3), we get

$$\beta B(X,Y,\mu,\rho)+\gamma B(X,Y,\rho,\mu)=0. \tag{4.4}$$

From the equations (1.19) and (4.4), we have

$$(\beta-\gamma)B(X,Y,\mu,\rho)=0, \tag{4.5}$$

this implies either $\beta=\gamma$ or $B(X,Y,\mu,,\rho)=0$.

From the equation (4.5), if $\beta=\gamma$, then from the equation (1.8), we have

$$S(X,Y)=\alpha g(X,Y)+\beta E(X,Y), \tag{4.6}$$

where $E(X,Y)=A(X)A(Y)+C(X)C(Y)$. This is a nearly quasi-Einstein manifold.

Thus we state the conclusion.

Theorem 4.3. *Every Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold $G(QEH)_n$ is either a Bochner Ricci semi-symmetric nearly quasi-Einstein manifold $N(QE)_n$ or*

$$B(X,Y,\mu,\rho)=0.$$

Putting $Z=W=\rho$ in the equation (4.2), we have

$$\beta A(B(X,\,Y)\rho)=0. \tag{4.7}$$

This implies either $\beta=0$ or $A(B(X,Y)\rho)=B(X,Y,\rho,\rho)=0$. If $\beta=0$, then from the equation (1.8), we have

$$S(X,Y)=\alpha g(X,Y)+\gamma C(X)C(Y), \tag{4.8}$$

which implies that the manifold is a quasi-Einstein manifold.

Thus, we have the lollowing conclusion.

Theorem 4.4. *A Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold $G(QEH)_n$ is a quasi-Einstein manifold.*

Now again putting $Z=\mu$ and $W=\mu$ in the the equation (4.2), we get

$$\gamma C(B(X,\,Y)\mu)=0, \tag{4.9}$$

this implies either $\gamma=0$ or $C(B(X,Y)\mu)=g(B(X,Y)\mu,\mu)=B(X,Y,\mu,\mu)=0$. If $\gamma=0$, then from the equation (1.8), we have

$$S(X,Y)=\alpha g(X,Y)+\beta A(X)A(Y), \tag{4.10}$$

this is a quasi-Einstein manifold.

Thus we conclude.

Theorem 4.5. *A Bochner Ricci semi-symmetric generalised quasi-Einstein Hermitian manifold $G(QEH)_n$ is a quasi-Einstein manifold.*

5. Bochner Ricci semi-symmetric pseudo generalised quasi-Einstein Hermitian manifold $P(GQEH)_n$

In this section, we introduce the following definitions.

Definition 5.1. A Hermitian manifold is said to be a pseudo generalised quasi-Einstein Hermitian manifold if it satisfies the equation (1.13). Throughout this paper, we denote the generalised quasi-Einstein Hermitian manifold by $P(GQEH)_n$.

Definition 5.2. A pseudo generalised quasi-Einstein Hermitian manifold is said to be a Bochner Ricci semi-symmetric pseudo generalised quasi-Einstein Hermitian manifold $P(GQEH)_n$ if it satisfies the equation (2.6).

If we take a Bochner Ricci semi-symmetric pseudo generalised quasi-Einstein Hermitian manifold, then from the equations (1.13) and (2.6), we have

$$\begin{aligned}\alpha[B(X,Y,Z,W)+B(X,Y,W,Z)]\\ +\beta[A(B(X,Y)Z)A(W)+A(Z)A(B(X,Y)W)]\\ +\gamma[C(B(X,Y)Z)C(W)+C(Z)C(B(X,Y)W)]\\ +\delta[D(B(X,Y)Z,W)+D(Z,B(X,Y)W)]=0,\end{aligned} \tag{5.1}$$

where $g(B(X,Y)W,Z)=B(X,Y,W,Z)$. Now from the equations (1.19) and (5.1), we get

$$\begin{aligned}\beta[A(B(X,Y)Z)A(W)+A(Z)A(B(X,Y)W)]\\ +\gamma[C(B(X,Y)Z)C(W)+C(Z)C(B(X,Y)W)]\\ +\delta[D(B(X,Y)Z,W)+D(Z,B(X,Y)W)]=0.\end{aligned} \tag{5.2}$$

Putting $Z=\rho$ and $W=\mu$ in the equation (5.2), we have

$$\beta A(B(X,Y)\mu)+\gamma C(B(X,Y)\rho))+\delta[D(B(X,Y)\rho,\mu)+D(\rho,B(X,Y)\mu)]=0. \tag{5.3}$$

Now if we take $D(B(X,Y)\rho,\mu)=D(\rho,B(X,Y)\mu)=0$, then from the equation (5.3), we obtain

$$\beta A(B(X,Y)\mu)+\gamma C(B(X,Y)\rho)=0. \tag{5.4}$$

Since we know that

$$C(B(X,Y)\rho)=g(B(X,Y)\rho,\mu)=B(X,Y,\rho,\mu)$$

and

$$A(B(X,Y)\mu)=g(B(X,Y)\mu,\rho)=B(X,Y,\mu,\rho),$$

then from the equation (5.4), we have

$$\beta B(X,Y,\mu,\rho)+\gamma B(X,Y,\rho,\mu)=0. \tag{5.5}$$

From the equations (1.19) and (5.5), we have

$$(\beta-\gamma)B(X,Y,\mu,\rho)=0, \tag{5.6}$$

this implies either $\beta=\gamma$ or $B(X,Y,\mu,\rho)=0$.

Thus we state the conclusion.

Theorem 5.3. *In a Bochner Ricci semi-symmetric pseudo generalised quasi-Einstein Hermitian manifold, if* $D(B(X,Y)\rho,\mu) = D(\rho, B(X,Y)\mu) = 0$*, then either the scalars* β *and* γ *are equal or*

$$B(X,Y,\mu,\rho) = 0.$$

Now we propose the following corollary.

Corollary 5.4. *In a Bochner Ricci semi-symmetric pseudo generalised quasi Einstein Hermitian manifold, if* $D(B(X,Y)\rho,\rho) = 0$*, then*

$$S(X,Y) = \alpha g(X,Y) + \gamma C(X)C(Y) + \delta D(X,Y).$$

Proof. Putting $Z = W = \rho$ and $D(B(X,Y)\rho,\rho) = 0$ in the equation (5.2), we have

$$\beta A(B(X,Y)\rho) = 0, \tag{5.7}$$

this implies either $\beta = 0$ or $A(B(X,Y)\rho) = B(X, Y, \rho, \rho) = 0$. If $\beta = 0$, then from the equation (1.13), we have

$$S(X,Y) = \alpha g(X,Y) + \gamma C(X)C(Y) + \delta D(X,Y). \tag{5.8}$$

□

Similarly, putting $Z = W = \mu$ and $D(B(X,Y)\mu,\mu) = 0$ in the equation (5.2), we get

$$\gamma C(B(X, Y)\mu) = 0, \tag{5.9}$$

this implies either $\gamma = 0$ or $C(B(X, Y)\mu) = B(X, Y, \mu, \mu) = 0$.

If $\gamma = 0$, then from the equation (1.13), we have

$$S(X,Y) = \alpha g(X,Y) + \beta A(X)A(Y) + \delta D(X,Y). \tag{5.10}$$

Thus we conclude the following corollary.

Corollary 5.5. *In a Bochner Ricci semi-symmetric pseudo generalised quasi-Einstein Hermitian manifold, if* $D(B(X,Y)\mu,\mu) = 0$*, then*

$$S(X,Y) = \alpha g(X,Y) + \beta A(X)A(Y) + \delta D(X,Y).$$

Acknowledgment. We are grateful to the referee for his valuable suggestions for improvement of the paper. Also, the second author expresses his thanks to the University Grants Commission (UGC), India, for providing Senior Research Fellowship (SRF).

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