BETA TYPE INTEGRAL FORMULA ASSOCIATED WITH
WRIGHT GENERALIZED BESSEL FUNCTION

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Abstract. The object of the present paper is to establish an integral formula involving Wright generalized Bessel function (or generalized Bessel-Maitland function) $J_{\mu,\gamma}^{\nu,q}(z)$ defined by Singh et al. [21], which is expressed in the terms of generalized (Wright) hypergeometric functions. Some interesting special cases involving Bessel functions, generalized Bessel functions, generalized Mittag-Leffler functions are deduced.

1. Introduction

In recent years, many integral formulas involving a variety of special functions have been developed by many authors (see [1, 2, 3, 4, 5], [7, 8], [10, 11, 12]). Several integral formulas involving product of Bessel functions have been developed and play an important role in several physical problems. In fact, Bessel functions are associated with a wide range of problems in diverse areas of mathematical physics. Here, we aim at presenting two generalized integral formulas involving the generalized Bessel-Maitland function, which are expressed in terms of generalized (Wright) hypergeometric functions. Some interesting special cases of our main results are also considered.

The Bessel-Maitland function (or the Wright-generalized Bessel function) is defined by (see [14]):

$$J_{\mu}^{\nu}(z) = \sum_{m=0}^{\infty} \frac{(-z)^m}{\Gamma(\nu + \mu m + 1)}, \quad (\mu > 0; z \in \mathbb{C}).$$

An interesting generalization of the Bessel-Maitland function $J_{\mu,\sigma}^{\nu}(z)$ is defined by (see [9]):

$$J_{\mu,\sigma}^{\nu}(z) = \sum_{m=0}^{\infty} \frac{(-1)^m(z)^{\nu+2\sigma+2m}}{\Gamma(\sigma + m + 1)\Gamma(\nu + \sigma + \mu m + 1)},$$

where $\mu > 0; z, \nu, \sigma \in \mathbb{C}$. Here and in the following, let $\mathbb{C}$ and $\mathbb{N}$ be the sets of all complex numbers and positive integers, respectively.
Further, another generalization of the generalized Bessel-Maitland function $J_{\nu,\sigma}^\alpha(z)$ is defined by (See [21]):

\begin{equation}
J_{\nu,\sigma}^\alpha(z) = \sum_{m=0}^{\infty} \frac{(\gamma)_q}{\Gamma(\nu + \mu m + 1)} \frac{(-z)^m}{m!},
\end{equation}

where $\mu, \nu, \gamma \in \mathbb{C}$; $\Re(\nu) > 0$, $\Re(\mu) > 0$, $\Re(\gamma) > 0$ and $q \in (0,1) \cup \mathbb{N}$ and $(\gamma)_0 = 1$, $(\gamma)_q = \frac{\Gamma(\gamma + qm)}{\Gamma(\gamma)}$, denote the generalized Pochhammer symbol.

We investigate some special cases of the generalized Bessel-Maitland function (1.3) by given particular values to the parameters $\mu, \nu, \gamma, q$.

(i) Setting $q = \gamma = 1$ and replacing $\nu$ by $\nu + \sigma$ and $z$ by $\frac{z^2}{4}$, to get

\begin{equation}
J_{\nu+\sigma}^{\mu,1} \left( \frac{z^2}{4} \right) = \Gamma(\sigma + m + 1) \left( \frac{z}{2} \right)^{-\nu-2\mu} J_{\nu,\sigma}^{\mu}(z),
\end{equation}

where $J_{\nu,\sigma}^{\mu}(z)$ denotes the Bessel-Maitland function defined by (1.2).

(ii) Letting $q = 0$, equation (1.3) reduces to

\begin{equation}
J_{\nu,0}^{\mu,\gamma}(z) = J_{\nu}^{\mu}(z),
\end{equation}

where $J_{\nu}^{\mu}(z)$ is the generalized Bessel function defined by (1.1).

(iii) Setting $q = 0$ and replacing $\nu$ by $\nu - 1$, equation (1.3) reduces to

\begin{equation}
J_{\nu-1,0}^{\mu,\gamma}(-z) = \Phi(\mu, \nu; z),
\end{equation}

where $\Phi(\mu, \nu; z)$ know as Wright function (see [23, 24, 25]).

(iv) Replacing $\nu$ by $\nu - 1$, (1.3) reduces to

\begin{equation}
J_{\nu-1,0}^{\mu,\gamma}(-z) = E_{\mu,\nu}^{\gamma}(z),
\end{equation}

where $\mu, \nu, \gamma \in \mathbb{C}$, $\Re(\mu) > 0$, $\Re(\nu) > 0$, $\Re(\gamma) > 0$, $q \in (0,1) \cup \mathbb{N}$ and $E_{\mu,\nu}^{\gamma}(z)$ denotes the generalized Mittag-Leffler function defined by Shukla and Prajapati [19].

(v) Setting $q = 1$ and replacing $\nu$ by $\nu - 1$, (1.3) reduces to

\begin{equation}
J_{\nu-1,1}^{\mu,\gamma}(-z) = E_{\mu,\nu}^{\gamma}(z),
\end{equation}

where $\alpha, \beta, \gamma \in \mathbb{C}$; $\Re(\alpha) > 0$, $\Re(\beta) > 0$, $\Re(\gamma) > 0$ and $E_{\mu,\nu}^{\gamma}(z)$ is the Mittag-Leffler function defined by Prabhakar [16].

(vi) For $\gamma = q = 1$ and replacing $\nu$ by $\nu - 1$, (1.3) reduces to

\begin{equation}
J_{\nu-1,1}^{\mu,1}(-z) = E_{\mu,\nu}(z),
\end{equation}

where $\nu \in \mathbb{C}$; $\Re(\mu) > 0$, $\Re(\nu) > 0$ and $E_{\mu,\nu}(z)$ is the Mittag-Leffler function defined by Wiman [22].

(vii) Setting $\nu = 0$, $q = \gamma = 1$, (1.3) reduces to

\begin{equation}
J_{0,1}^{\mu,1}(-z) = E_{\mu}(z),
\end{equation}

where $z \in \mathbb{C}$ and $\Gamma(s)$ is the Gamma function; $\alpha \geq 0$ and $E_{\mu}(z)$ is the Mittag-Leffler function defined by Goea Mittag-Leffler [15].
The generalization of the generalized hypergeometric series \( pF_q \) is due to Fox \([6]\) and Wright \([23, 24, 25]\) who studied the asymptotic expansion of the generalized (Wright) hypergeometric function defined by (see \([20, \text{ p.21}]\); see also \([18]\)):

\[
(1.11) \quad p\Psi_q \left[ \left( \frac{\alpha_1, A_1}{\beta_1, B_1}, \ldots, \frac{\alpha_p, A_p}{\beta_q, B_q} \right); z \right] = \sum_{k=0}^{\infty} \frac{\prod_{j=1}^{p} \Gamma(\alpha_j + A_j k)}{k!} \frac{\prod_{j=1}^{q} \Gamma(\beta_j + B_j k)}{z^k},
\]

where the coefficients \( A_1, \ldots, A_p \) and \( B_1, \ldots, B_q \) are positive real numbers such that

\[
\begin{align*}
(1.12) \quad & (i) \quad 1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j > 0 \quad \text{and} \quad 0 < |z| < \infty; \quad z \neq 0. \\
(1.13) \quad & (ii) \quad 1 + \sum_{j=1}^{q} B_j - \sum_{j=1}^{p} A_j = 0 \quad \text{and} \quad 0 < |z| < \frac{A_1 - A_1 - \ldots - A_p - A_p B_1 B_1 - \ldots - B_q B_q}.
\end{align*}
\]

A special case of (1.11) is

\[
(1.14) \quad p\Psi_q \left[ \left( \frac{\alpha_1, 1}{\beta_1, 1}, \ldots, \frac{\alpha_p, 1}{\beta_q, 1} \right); z \right] = \frac{\prod_{j=1}^{p} \Gamma(\alpha_j)}{\prod_{j=1}^{q} \Gamma(\beta_j)} pF_q \left[ \left( \frac{\alpha_1, \ldots, \alpha_p}{\beta_1, \ldots, \beta_q} \right); z \right],
\]

where \( pF_q \) is the generalized hypergeometric series defined by \([17]\)

\[
(1.15) \quad pF_q \left[ \frac{\alpha_1, \ldots, \alpha_p}{\beta_1, \ldots, \beta_q}; z \right] = \sum_{n=0}^{\infty} \frac{(\alpha_1)_n \ldots (\alpha_p)_n z^n}{(\beta_1)_n \ldots (\beta_q)_n n!} = pF_q(\alpha_1, \ldots, \alpha_p; \beta_1, \ldots, \beta_q; z),
\]

where \((\lambda)_n\) is the Pochhammer’s symbol \([17]\).

For our present investigation, the following interesting and useful result due to MacRobert \([13]\) will be required:

\[
(1.16) \quad \int_{0}^{1} x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{-\alpha - \beta} dx = \frac{\Gamma(\alpha) \Gamma(\beta)}{a^\alpha b^\beta \Gamma(\alpha + \beta)},
\]

provided \( \Re(\alpha) > 0 \) and \( \Re(\beta) > 0 \).
2. Integral formula involving generalized Bessel-Maitland function

**Theorem 2.1.** If \( \alpha, \beta, \gamma, \mu, \nu, \lambda \in \mathbb{C}, \ 1 + \mu - \lambda > 0, \ \Re(\alpha) > 0, \ \Re(\beta) > 0, \ \Re(\gamma) > 0, \ \Re(\nu) > 0, \) then

\[
\int_{0}^{1} x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} J_{\mu,\gamma}^{\nu,\lambda} \left[ \frac{2abx(1-x)}{ax+b(1-x)} \right]^2 \, dx
\]

(2.1)

\[
= \frac{1}{a^{\alpha} b^{\beta} \Gamma(\gamma)^{3} \Psi_2} \left[ \begin{array}{c} (\gamma, \lambda), (\beta, 1), (\alpha, 1); \\ (\nu + 1, \mu), (\alpha + \beta, 2); \\ -2 \end{array} \right].
\]

**Proof.** To establish our main result (2.1), we denote the left-hand side of (2.1) by \( I \) and then using (1.7), we have

\[
I = \int_{0}^{1} x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} J_{\mu,\gamma}^{\nu,\lambda} \left[ \frac{2abx(1-x)}{ax+b(1-x)} \right]^2 \, dx.
\]

(2.2)

Now changing the order of integration and summation, which is justified by the uniform convergence of the series in the interval \((0,1)\), we arrive

\[
I = \frac{1}{a^{\alpha} b^{\beta} \Gamma(\gamma)^{3}} \sum_{m=0}^{\infty} \frac{\Gamma(\gamma + \lambda m) \Gamma(\alpha + m) \Gamma(\beta + m) - 2^m}{\Gamma(\nu + 1 + \mu m) \Gamma(\alpha + \beta + 2m) \, m!}.
\]

(2.3)

Finally, summing up the above series with the help of (1.11), we easily arrive at the right-hand side of (2.1). This completes the proof. \( \square \)

Next, we consider other variation of (2.1). In fact, we establish an integral formula for the Bessel-Maitland function \( J_{\mu,\gamma}^{\nu,\lambda}(z) \), which is expressed in terms of the generalized hypergeometric function \( pF_q \).

**Variation of (2.1).** Let the conditions of our main result be satisfied. Then the following integral formula holds true:

\[
\int_{0}^{1} x^{\alpha-1}(1-x)^{\beta-1}[ax+b(1-x)]^{-\alpha-\beta} J_{\mu,\gamma}^{\nu,\lambda} \left[ \frac{2abx(1-x)}{ax+b(1-x)} \right]^2 \, dx
\]

(2.4)

\[
= \frac{\Gamma(\alpha) \Gamma(\beta)}{a^{\alpha} b^{\beta} \Gamma(\nu + 1) \Gamma(\alpha + \beta)^{\lambda + 2} F_{\mu+2}} \left[ \begin{array}{c} \Delta(\lambda; \gamma), \alpha; \\ \Delta(\mu; \nu + 1), \beta; \frac{\lambda^{2} q^{\nu}}{2 \mu^{\alpha}} \end{array} \right],
\]

where \( \Delta(m; l) \) abbreviates the array of \( m \) parameters \( \frac{l}{m}, \frac{l+1}{m}, \ldots, \frac{l+m-1}{m}, m \in \mathbb{N} \).

**Proof.** In order to prove the result (2.4), using the results

\[
\Gamma(\alpha + n) = \Gamma(\alpha)(\alpha)_n
\]

and

\[
(l)_n = k^{\alpha n} \left( \frac{l}{k} \right)_n \left( \frac{l+1}{k} \right)_n \ldots \left( \frac{l+k-1}{k} \right)_n,
\]

(Gauss multiplication theorem) in (2.3) and summing up the given series with the help of (1.15), we easily arrive at our required result (2.4). \( \square \)
3. Special Cases

(i). On setting \( \lambda = \gamma = 1 \) and replacing \( \nu \) by \( \nu + \sigma \) and \( z \) by \( (\frac{z^2}{x^2}) \) in (2.1) and then by using (1.4), we get

\[
\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} [ax + b(1-x)]^{-\alpha-\beta} J_{\nu,\sigma}^{\mu} \left[ \frac{2abx(1-x)}{[ax + b(1-x)]^2} \right] \, dx
\]

\[
= \frac{1}{a^\alpha b^\beta} \Psi_2 \begin{bmatrix} \alpha, 1; & \beta, 2; & 1 \end{bmatrix},
\]

where \( \alpha, \beta, \nu, \mu, \sigma \in \mathbb{C}; \ Re(\alpha) > 0, Re(\beta) > 0, Re(\nu) > 0, Re(\sigma) > 0. \)

(ii). On setting \( \lambda = \gamma = 1 \) and replacing \( \nu \) by \( \nu + \sigma \) and \( z \) by \( (\frac{z^2}{x^2}) \) in (2.4) and then by using (1.4), we find:

\[
\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} [ax + b(1-x)]^{-\alpha-\beta} J_{\nu,\sigma}^{\mu} \left[ \frac{2abx(1-x)}{[ax + b(1-x)]^2} \right] \, dx
\]

\[
= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^\alpha b^\beta \Gamma(\nu + \sigma)\Gamma(\alpha + \beta)} \left[ 1, \Delta(2;\alpha), \Delta(2;\beta); -1 \right] \begin{bmatrix} \Delta(2;\alpha), \Delta(2;\beta); -1 \end{bmatrix},
\]

where \( \alpha, \beta, \nu, \mu, \sigma \in \mathbb{C}; \ Re(\alpha) > 0, Re(\beta) > 0, Re(\nu) > 0, Re(\sigma) > 0. \)

(iii). On setting \( \lambda = 0 \) in (2.1) and then by using (1.5), we attain:

\[
\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} [ax + b(1-x)]^{-\alpha-\beta} J_{\nu,\sigma}^{\mu} \left[ \frac{2abx(1-x)}{[ax + b(1-x)]^2} \right] \, dx
\]

\[
= \frac{1}{a^\alpha b^\beta} \Psi_2 \begin{bmatrix} \alpha, 1; & \beta, 1; & 1 \end{bmatrix},
\]

where \( \alpha, \beta, \nu, \mu, \sigma \in \mathbb{C}; \ Re(\alpha) > 0, Re(\beta) > 0, Re(\nu) > 0. \)

(iv) On setting \( \lambda = 0 \) in (2.4), and then by using (1.5), we acquire:

\[
\int_0^1 x^{\alpha-1} (1-x)^{\beta-1} [ax + b(1-x)]^{-\alpha-\beta} J_{\nu,\sigma}^{\mu} \left[ \frac{2abx(1-x)}{[ax + b(1-x)]^2} \right] \, dx
\]

\[
= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^\alpha b^\beta \Gamma(\nu + 1)\Gamma(\alpha + \beta)} \left[ \alpha, \beta; -1 \right] \begin{bmatrix} \alpha, \beta; -1 \end{bmatrix},
\]

where \( \alpha, \beta, \nu, \mu, \sigma \in \mathbb{C}; \ Re(\alpha) > 0, Re(\beta) > 0, Re(\nu) > 0. \)
(v) On setting $\lambda = 0$ and replacing $\nu$ by $\nu - 1$ and then by using (1.6), we find:

\[
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{\gamma - \beta} \Phi \left( \frac{2abx(1 - x)}{ax + b(1 - xy)} \right) dx
\]

\[= \frac{1}{a^\alpha b^\beta} \Psi_2 \left[ \begin{array}{c} (\alpha, 1), & (\beta, 1); \\ \mu, & (\alpha + \beta, 2) \end{array} \right], \]

where $\alpha, \beta, \nu, \mu \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\nu) > 0$.

(vi) On setting $\lambda = 0$ and replacing $\nu$ by $\nu - 1$ and then by using (1.6), we get:

\[
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{\gamma - \beta} \Phi \left( \frac{2abx(1 - x)}{ax + b(1 - xy)} \right) dx
\]

\[= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^\alpha b^\beta \Gamma(\alpha + \beta)} \Psi_2 \left[ \begin{array}{c} \alpha, & \beta; \\ \mu, & \frac{1}{2}\mu^\nu \end{array} \right], \]

where $\alpha, \beta, \nu, \mu \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\nu) > 0$.

(vii) On replacing $\nu$ by $\nu - 1$ and then by using (1.7), we obtain:

\[
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{\gamma - \beta} E_{\mu, \nu} \left[ \frac{-2abx(1 - x)}{ax + b(1 - xy)} \right] dx
\]

\[= \frac{1}{a^\alpha b^\beta \Gamma(\gamma)} \Psi_2 \left[ \begin{array}{c} \gamma, \lambda, & (\alpha, 1), & (\beta, 1); \\ \nu, \mu, & (\alpha + \beta + 2), & 2 \end{array} \right], \]

where $\alpha, \beta, \gamma, \mu, \lambda \in \mathbb{C}; \Re(\lambda) > 0, \Re(\beta) > 0, \Re(\lambda) > 0, \Re(\gamma) > 0, \Re(\nu) > 0$.

(viii) Replacing $\nu$ by $\nu - 1$ and then by using (1.7), we get:

\[
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{\gamma - \beta} E_{\mu, \nu}^{\gamma, \lambda} \left[ \frac{-2abx(1 - x)}{ax + b(1 - xy)} \right] dx
\]

\[= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^\alpha b^\beta \Gamma(\mu)\Gamma(\alpha + \beta)} \lambda^+2 \Psi_2 \left[ \begin{array}{c} \Delta(\lambda; \gamma), & \Delta(\mu; \nu), & \Delta(2; \alpha + \beta); \\ \alpha, & \beta; \frac{\lambda}{2}\mu^\nu \end{array} \right], \]

where $\alpha, \beta, \gamma, \mu \in \mathbb{C}; \Re(\mu) > 0, \Re(\lambda) > 0, \Re(\gamma) > 0, \Re(\nu) > 0$. 
(ix) Setting $\lambda = 1$ and replacing $\nu$ by $\nu - 1$ in (2.1) and then by using (1.8), we get

$$
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{-\alpha - \beta} E_{\mu, \nu}^{\gamma} \left[ \frac{2abx(1 - x)}{[ax + b(1 - xy)]^2} \right] dx
$$

(3.9)

$$
= \frac{1}{a^{\alpha}b^{\beta}\Gamma(\gamma)} \psi_2 \left[ \begin{array}{c} \gamma, 1, \alpha, 1, (\beta, 1); \\ (\nu, \mu), (\alpha + \beta + 2); \end{array} 2 \right]
$$

where $\alpha, \beta, \gamma, \mu \in \mathbb{C}; \Re(\mu) > 0, \Re(\lambda) > 0, \Re(\gamma) > 0, \Re(\nu) > 0.$

(x) Setting $\lambda = 1$ and replacing $\nu$ by $\nu - 1$ in (2.4) and then by using (1.8), we get

$$
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{-\alpha - \beta} E_{\mu, \nu}^{\gamma} \left[ \frac{2abx(1 - x)}{[ax + b(1 - xy)]^2} \right] dx
$$

(3.10)

$$
= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^{\alpha}b^{\beta}\Gamma(\nu)\Gamma(\alpha + \beta)} \psi_2 \left[ \begin{array}{c} \gamma, \alpha, \beta; \\ \Delta(\mu; \nu), (\alpha + \beta + 2); \frac{1}{2\mu^2} \end{array} 2 \right]
$$

where $\alpha, \beta, \gamma, \mu \in \mathbb{C}; \Re(\mu) > 0, \Re(\lambda) > 0, \Re(\gamma) > 0, \Re(\nu) > 0.$

(xi) Setting $\lambda = \gamma = 1$ and replacing $\nu$ by $\nu - 1$ in (2.1) and then by using (1.9), we get

$$
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{-\alpha - \beta} E_{\mu, \nu}^{\gamma} \left[ \frac{2abx(1 - x)}{[ax + b(1 - xy)]^2} \right] dx
$$

(3.11)

$$
= \frac{1}{a^{\alpha}b^{\beta}} \psi_2 \left[ \begin{array}{c} (1, 1), \alpha, 1, (\beta, 1); \\ (\nu, \mu), (\alpha + \beta + 2); \end{array} 2 \right]
$$

where $\alpha, \beta, \nu, \mu \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\nu) > 0, \Re(\mu) > 0.$

(xii) Setting $\lambda = \gamma = 1$ and replacing $\nu$ by $\nu - 1$ in (2.4) and then by using (1.9), we get

$$
\int_0^1 x^{\alpha - 1}(1 - x)^{\beta - 1}[ax + b(1 - x)]^{-\alpha - \beta} E_{\mu, \nu}^{\gamma} \left[ \frac{2abx(1 - x)}{[ax + b(1 - xy)]^2} \right] dx
$$

(3.12)

$$
= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^{\alpha}b^{\beta}\Gamma(\nu)\Gamma(\alpha + \beta)} \psi_2 \left[ \begin{array}{c} 1, \alpha, \beta; \\ \Delta(\mu; \nu), \Delta(2; \alpha + \beta); \frac{1}{2\mu^2} \end{array} 2 \right]
$$

where $\alpha, \beta, \nu, \mu \in \mathbb{C}; \Re(\alpha) > 0, \Re(\beta) > 0, \Re(\nu) > 0, \Re(\mu) > 0.$
On setting $\nu = 0, \lambda = \gamma = 1$ in (2.1) and then by using (1.10), we get

\begin{equation}
\int_0^1 x^{\alpha-1}(1-x)^{\beta-1} \left[ ax + b(1-x) \right]^{-\alpha-\beta} E_\mu \left[ \frac{2abx(1-x)}{(ax+b(1-xy))^2} \right] \, dx
\end{equation}

(3.13)

\begin{equation}
= \frac{1}{a^\alpha b^\beta} \Psi_2 \left[ \begin{array}{ccc}
(1, 1), & (\alpha, 1), & (\beta, 1); \\
(1, \mu), & (\alpha + \beta, 2) \\
\end{array} \right],
\end{equation}

where $\alpha, \beta, \mu \in \mathbb{C}; \Re(\mu) > 0, \Re(\alpha) > 0, \Re(\beta) > 0$.

On setting $\nu = 0, \lambda = \gamma = 1$ in (2.4) and then by using (1.10), we get

\begin{equation}
\int_0^1 x^{\alpha-1}(1-x)^{\beta-1} \left[ ax + b(1-x) \right]^{-\alpha-\beta} E_\mu \left[ \frac{2abx(1-x)}{(ax+b(1-xy))^2} \right] \, dx
\end{equation}

(3.14)

\begin{equation}
= \frac{\Gamma(\alpha)\Gamma(\beta)}{a^\alpha b^\beta \Gamma(\alpha + \beta)} \left[ \begin{array}{ccc}
1, & \alpha, & \beta; \\
\Delta(\mu; 1), & \Delta(2; \alpha + \beta) \\
\end{array} \right],
\end{equation}

where $\alpha, \beta, \mu \in \mathbb{C}; \Re(\mu) > 0, \Re(\alpha) > 0, \Re(\beta) > 0$.

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