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SOME GENERALIZED HERMITE-HADAMARD TYPE INEQUALITIES INVOLVING FRACTIONAL INTEGRAL OPERATOR FOR FUNCTIONS WHOSE SECOND DERIVATIVES IN ABSOLUTE VALUE ARE S-CONVEX

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Abstract. In this article, a general integral identity for twice differentiable mapping involving fractional integral operators is derived. Secondly by using this identity we obtain some new generalized Hermite-Hadamards type inequalities for functions whose absolute values of second derivatives are s-convex and concave. The main results generalize the existing Hermite-Hadamard type inequalities involving the Riemann-Liouville fractional integral. Also we point out, some results in this study in some special cases such as setting $s = 1$, $\lambda = \alpha$, $\sigma(0) = 1$ and $w = 0$, more reasonable than those obtained in [**8**].

1. Introduction and Preliminaries

Let $f\colon I \subset \mathbb{R} \to \mathbb{R}$ be a convex function on an interval I in the set of real numbers $\mathbb{R}$. Then, for a, $b \in I$ with $a < b$, the following so-called Hermite-Hadamard inequality (see, e.g., [**12**])

$$f\left(\frac{a+b}{2}\right) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{2} \tag{1.1}$$

holds true. Since its discovery in 1983, Hermite-Hadamard's inequality has been considered the most useful inequality in mathematical analysis. A number of the papers have been written on this inequality providing new proofs, noteworthy extensions, generalizations and numerous applications, see and references therein [**6, 7, 12, 15**].

Two definitions of s-convexity ($0 < s \leq 1$) of real-valued functions are well known in the literature.

Definition 1.1. Let $0 < s \leq 1$. A function $f\colon [0,\infty) \to \mathbb{R}$ is said to be s-Orlicz convex or s-convex in the first sense if for every $x, y \in [0,\infty)$ and $\alpha, \beta \geq 0$ with $\alpha^s + \beta^s = 1$, we have

$$f(\alpha x + \beta y) \leq \alpha^s f(x) + \beta^s f(y). \tag{1.2}$$

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We denote the set of all s-convex functions in the first sense by K_s^1. This definition of s-convexity was introduced by Orlicz in [**14**] and was used in the theory of Orlicz spaces. Then, s-convex function in the second sense was introduced in Breckner's paper [**4**] and a number of properties and connections with s-convexity in the first sense are discussed in paper [**10**].

Definition 1.2. [**4**] A function $f\colon \mathbb{R}_+ \to \mathbb{R}$ is said to be s-convex in the second sense if

$$f(\alpha x + \beta y) \leq \alpha^s f(x) + \beta^s f(y)$$

for all $x, y \in \mathbb{R}_+$ and all $\alpha, \beta \geq 0$ with $\alpha + \beta = 1$.

We denote this by K_s^2. It is obvious that the s-convexity means just the convexity when $s = 1$.

In [**6**] Dragomir and Fitzpatrick proved a variant of Hadamard's inequality which holds for s-convex functions in the second sense as follows

Theorem 1.1. *Suppose that $f\colon [0,\infty) \to [0,\infty)$ is an s-convex function in the second sense, where $s \in (0,1]$ and let $a, b \in [0,\infty)$, $a < b$. If $f \in L_1[a,b]$ then the following inequality holds*

$$2^{s-1} f\Big(\frac{a+b}{2}\Big) \leq \frac{1}{b-a}\int_a^b f(x)\mathrm{d}x \leq \frac{f(a)+f(b)}{s+1} \tag{1.3}$$

The constant $k = \frac{1}{s+1}$ is the best possible in the second inequality in (1.3). For more study, see ([**2, 3, 6, 11**]).

In the following, we give some necessary definitions and preliminary results which are used and referred to throughout this paper.

Definition 1.3. Let $f \in L_1[a,b]$. The Riemann-Liouville integrals $J_{a+}^{\alpha} f$ and $J_{b-}^{\alpha} f$ of order $\alpha > 0$ with $a \geq 0$ are defined by

$$J_{a^+}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)}\int_a^x (x-t)^{\alpha-1} f(t)\mathrm{d}t, \qquad x > a$$

and

$$J_{b^-}^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)}\int_x^b (t-x)^{\alpha-1} f(t)\mathrm{d}t, \qquad x < b,$$

respectively, where $\Gamma(\alpha) = \int_0^{\infty} \mathrm{e}^{-u} u^{\alpha-1} du$. Here $J_{a^+}^0 f(x) = J_{b^-}^0 f(x) = f(x)$.

In the case of $\alpha = 1$, the fractional integral reduces to the classical integral. Some recent results and properties concerning this operator can be found [**5, 9, 27, 19, 16, 20, 21**].

The beta function is defined as follows:

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)} = \int_0^1 t^{a-1}(1-t)^{b-1}\mathrm{d}t, \qquad a, b > 0,$$

where $\Gamma(\alpha)$ is Gamma function. In [**27**], Sarıkaya et al. gave a remarkable integral inequalities of Hermite-Hadamard type involving Riemann-Liouville fractional integrals as follows.

Theorem 1.2. *Let $f\colon [a,b] \to \mathbb{R}$ be a positive function with $0 \le a < b$ and $f \in [a,b]$. If f is convex function on $[a,b]$, then the following inequality for fractional integrals holds.*

$$f\Big(\frac{a+b}{2}\Big) \le \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}[J^\alpha_{a^+}f(b) + J^\alpha_{b^-}f(a)] \le \frac{f(a)+f(b)}{2}. \tag{1.4}$$

It is obviously seen that if we take $\alpha = 1$ in Theorem 1.2, then the inequality (1.4) reduces to well known Hermite-Hadamard's inequality as (1.1).

Hermite Hadamard type inequality for s-convex functions on Riemann-Liouville fractional integral is given in [**19**] as follows.

Theorem 1.3. *Let $f\colon [a,b] \to \mathbb{R}$ be a positive function with $0 \le a < b$ and $f \in L_1[a,b]$. If f is s-convex mapping in the second sense on $[a,b]$, then the following inequality for fractional integral with $\alpha > 0$ and $s \in (0,1]$ hold*

$$\begin{aligned} 2^{s-1}f\Big(\frac{a+b}{2}\Big) &\le \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}[J^\alpha_{a^+}f(b) + J^\alpha_{b^-}f(a)] \\ &\le \alpha\Big[\frac{1}{\alpha+s} + B(\alpha, s+1)\Big]\frac{f(a)+f(b)}{2}, \end{aligned} \tag{1.5}$$

where $B(a,b)$ is beta function.

In [**8**] Dragomir et al proved the following identity and by using this identity they established new results involving Riemann-Liouville fractional integrals for twice differentiable convex mappings.

Lemma 1.1. [**8**] *Let $f\colon I \subset \mathbb{R} \to \mathbb{R}$ be a twice differentiable function on I°, the interior of I. Assume that $a, b \in I^\circ$ with $a < b$ and $f'' \in L[a,b]$, then the following identity for fractional integral with $\alpha > 0$ holds*

$$\begin{aligned} &\frac{f(a)+f(b)}{2} - \frac{\Gamma(\alpha+1)}{2(b-a)^\alpha}\left[J^\alpha_{b^-}f(a) + J^\alpha_{a^+}f(b)\right] \\ &= \frac{(b-a)^2}{2(\alpha+1)}\int_0^1 t(1-t^\alpha)\left[f''\left(ta+(1-t)b\right) + f''\left(tb+(1-t)a\right)\right]\mathrm{d}t. \end{aligned} \tag{1.6}$$

In [**25**], Raina introduced a class of functions defined formally by

$$\mathcal{F}^\sigma_{\rho,\lambda}(x) = \mathcal{F}^{\sigma(0),\sigma(1),\ldots}_{\rho,\lambda}(x) = \sum_{k=0}^{\infty}\frac{\sigma(k)}{\Gamma(\rho k+\lambda)}x^k \qquad (\rho,\lambda > 0;\ x \in \mathbb{R}), \tag{1.7}$$

where the coefficients $\sigma(k)$ ($k \in \mathbb{N} = \mathbb{N} \cup \{0\}$) are a bounded sequence of positive real numbers and $\mathbb{R}$ is the set of real numbers. With the help of (1.7), Raina [**25**] and Agarwal et al. [**1**] defined the following left-sided and right-sided fractional integral operators respectively, as follows

$$\left(\mathcal{J}^\sigma_{\rho,\lambda,a+;w}\varphi\right)(x) = \int_a^x (x-t)^{\lambda-1}\mathcal{F}^\sigma_{\rho,\lambda}[w(x-t)^\rho]\varphi(t)\mathrm{d}t \qquad (x > a > 0), \tag{1.8}$$

$$\left(\mathcal{J}^\sigma_{\rho,\lambda,b-;w}\varphi\right)(x) = \int_x^b (t-x)^{\lambda-1}\mathcal{F}^\sigma_{\rho,\lambda}[w(t-x)^\rho]\varphi(t)\mathrm{d}t \qquad (0 < x < b), \tag{1.9}$$

where $\lambda, \rho > 0$, $w \in \mathbb{R}$ and $\varphi(t)$ is such that the integral on the right side exits. Recently some new integral inequalities this operator have appeared in the literature (see, e.g., [**23, 22, 24, 1, 17, 18, 29**]).

It is easy to verify that $\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi(x)$ and $\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi(x)$ are bounded integral operators on $L(a,b)$, if

$$\mathfrak{M} := \mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}] < \infty. \tag{1.10}$$

In fact, for $\varphi \in L(a,b)$, we have

$$\|\mathcal{J}^{\sigma}_{\rho,\lambda,a+;w}\varphi(x)\|_1 \leq \mathfrak{M}(b-a)^{\lambda}\|\varphi\|_1 \tag{1.11}$$

and

$$\|\mathcal{J}^{\sigma}_{\rho,\lambda,b-;w}\varphi(x)\|_1 \leq \mathfrak{M}(b-a)^{\lambda}\|\varphi\|_1, \tag{1.12}$$

where

$$\|\varphi\|_p := \left(\int_a^b |\varphi(t)|^p \mathrm{d}t\right)^{\frac{1}{p}}.$$

Here, many useful fractional integral operators can be obtained by specializing the coefficient $\sigma(k)$. For intance the classical Riemann-Liouville fractiona integrals J^{α}_{a+} and J^{α}_{b-} of order α follow easily by setting $\lambda = \alpha$, $\sigma(0) = 1$ and $w = 0$ in (1.8) and (1.9).

In [**26**] generalized Hermite-Hadamard's inequality for s-convex mapping fractional integral operators as follows.

Theorem 1.4. *Let $f\colon [a,b] \to \mathbb{R}$ be a function with $0 \leq a < b$ and $f \in L_1[a,b]$. If f is an s-convex function on $[a,b]$ then we have the following inequalities for generalized fractional integral operators*

$$\begin{aligned} 2^s f\left(\frac{a+b}{2}\right) &\leq \frac{1}{(b-a)^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(b-a)^{\rho}]}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a) + \left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right] \\ &\leq \frac{1}{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]}\left[A_1(\lambda,s) + \mathcal{F}^{\sigma_{0,s}}_{\rho,\lambda}[w(b-a)^{\rho}]\right][f(a)+f(b)], \end{aligned} \tag{1.13}$$

where

$$\sigma_{0,s}(k) = \frac{\sigma(k)}{\lambda+\rho k+s}, \qquad k = 0,1,2\ldots \quad \textit{and}$$

$$A_1(\lambda,s) = \int_0^1 t^{\lambda-1}(1-t)^s \mathcal{F}^{\sigma}_{\rho,\lambda}[w(b-a)^{\rho}t^{\rho}]\mathrm{d}t.$$

In this paper, first aim is to establish a new integral identity for a twice differentiable function via fractional integral operators. Using this new identity, we present some Hermite-Hadamard type inequalities for functions whose second-order derivatives absolute values are s-convex and concave in the second sense.

2. Main Results

Lemma 2.1. *Let $f\colon [a,b] \to \mathbb{R}$ be a twice differentiable mapping on (a,b) $(a<b)$. Also, $\rho,\lambda>0$ and $w\in\mathbb{R}$. If $f''\in L[a,b]$, then the following equality for generalized fractional integrals holds*

(2.1)
$$\begin{aligned}&\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\\&=\frac{(b-a)^2}{2}\times\int_0^1\Big\{t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\\&\qquad-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]\left[f''(ta+(1-t)b)+f''((1-t)a+tb)\right]\Big\}\mathrm{d}t.\end{aligned}$$

Proof. Integrating by parts and changing variables with $x=(ta+(1-t)b)$ we get

(2.2)
$$\begin{aligned}I_1&=\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\int_0^1 tf''(ta+(1-t)b)\,\mathrm{d}t\\&=\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\left\{\frac{1}{a-b}tf'(ta+(1-t)b)\Big|_0^1-\frac{1}{a-b}\int_0^1 f'(ta+(1-t)b)\,\mathrm{d}t\right\}\\&=\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\left\{-\frac{f'(a)}{b-a}-\frac{f(a)-f(b)}{(b-a)^2}\right\},\end{aligned}$$

by using same method

$$\begin{aligned}I_2&=\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\int_0^1 tf''((1-t)a+tb)\,\mathrm{d}t\\&=\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\left\{\frac{f'(b)}{b-a}-\frac{f(b)-f(a)}{(b-a)^2}\right\},\end{aligned}\tag{2.3}$$

analogously

(2.4)
$$\begin{aligned}I_3&=\int_0^1 t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]f''(ta+(1-t)b)\,\mathrm{d}t\\&=t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]\frac{f'(ta+(1-t)b)}{a-b}\Big|_0^1\\&\quad-\int_0^1 t^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}t^{\rho}]\frac{f'(ta+(1-t)b)}{a-b}\mathrm{d}t\\&=\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]\frac{f'(a)}{a-b}-\frac{1}{a-b}t^{\lambda}\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}t^{\rho}]\frac{f(ta+(1-t)b)}{a-b}\Big|_0^1\\&\quad+\frac{1}{a-b}\int_0^1 t^{\lambda-1}\mathcal{F}^{\sigma}_{\rho,\lambda}[w(b-a)^{\rho}t^{\rho}]\frac{f(ta+(1-t)b)}{a-b}\mathrm{d}t\end{aligned}$$

$$
\begin{aligned}
&= \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]f^{'}(a)}{a-b} - \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]f(a)}{(b-a)^2} \\
&\quad + \frac{1}{(b-a)^2}\int_a^b \left(\frac{b-x}{b-a}\right)^{\lambda-1} \mathcal{F}^{\sigma}_{\rho,\lambda}\left[w(b-a)^{\rho}\left(\frac{b-x}{b-a}\right)^{\rho}\right]\frac{f(x)}{b-a}\mathrm{d}x \\
&= \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]f^{'}(a)}{a-b} - \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]f(a)}{(b-a)^2} + \frac{\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)}{(b-a)^{\lambda+2}}
\end{aligned}
$$

and

$$
\begin{aligned}
I_4 &= \int_0^1 t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]f^{''}((1-t)a+tb)\,\mathrm{d}t \\
&= \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]f^{'}(b)}{b-a} - \frac{\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]f(b)}{(b-a)^2} + \frac{\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)}{(b-a)^{\lambda+2}}.
\end{aligned} \tag{2.5}
$$

Thus combining (2.2), (2.3), (2.4) and (2.5) as $I_1+I_2-I_3-I_4$ and multiplying both sides of the obtained equality with $\frac{(b-a)^2}{2}$, which proof is completed. □

Remark 2.1. Setting $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Lemma 2.1, we find the same identity as [**8**, Lemma 1].

Using this lemma, we can get the following results via generalized fractional integral operator for twice differentiable function whose absolute value is s-convex and s-concave.

Theorem 2.1. *Let $f\colon[a,b]\to\mathbb{R}$ be a twice differentiable function on (a,b) $(a<b)$. Also, $\rho,\lambda>0$ and $w\in\mathbb{R}$. If $|f''|$ is s-convex in the seconde sense on (a,b) then the following inequality for generalized fractional integral operators holds*

$$
\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2} - \frac{1}{2(b-a)^{\lambda}}\left[\left(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f\right)(a)+\left(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f\right)(b)\right]\right| \\
&\le \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{1,s}}_{\rho,\lambda+2}[|w|(b-a)^{\rho}]\left[\left|f''(a)\right|+\left|f''(b)\right|\right],
\end{aligned}
$$

where $s\in(0,1]$, $B(..,..)$ is Euler beta function and

$$
\sigma_{1,s}(k)=\sigma(k)\left[\frac{(\lambda+\rho k)}{(2+s)(\lambda+\rho k+s+2)}+B(2,s+1)-B(\lambda+\rho k+2,s+1)\right].
$$

Proof. From Lemma 2.1 with properties of modulus, we get

(2.6)
$$\begin{aligned}&\Big|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\Big|\\&\le\frac{(b-a)^2}{2}\int_0^1\Big\{\big|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]\big|\\&\quad\times\Big|\left[f''(ta+(1-t)b)+f''((1-t)a+tb)\right]\Big|\Big\}\mathrm{d}t\\&\le\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\big|t-t^{\lambda+\rho k+1}\big|\big|f''(ta+(1-t)b)\big|\mathrm{d}t\\&\quad+\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\big|t-t^{\lambda+\rho k+1}\big|\big|f''((1-t)a+tb)\big|\mathrm{d}t.\end{aligned}$$

Since $|f''|$ is s-convex, we have

(2.7)
$$\begin{aligned}&\int_0^1(t-t^{\lambda+\rho k+1})\big|f''(ta+(1-t)b)\big|\mathrm{d}t+\int_0^1(t-t^{\lambda+\rho k+1})\big|f''((1-t)a+tb)\big|\mathrm{d}t\\&\le\left[\int_0^1 t^{1+s}(1-t^{\lambda+\rho k})\mathrm{d}t+\int_0^1 t(1-t)^s(1-t^{\lambda+\rho k})\mathrm{d}t\right]\left[\big|f''(a)\big|+\big|f''(b)\big|\right]\\&=\Big[\frac{(\lambda+\rho k)}{(2+s)(\lambda+\rho k+s+2)}+B(2,s+1)-B(\lambda+\rho k+2,s+1)\Big]\left[\big|f''(a)\big|+\big|f''(b)\big|\right].\end{aligned}$$

Thus combining the inequalities (2.6) and (2.7), the requested result is obtained. □

Corollary 2.1. *If we take* $s=1$ *in* Theorem 2.1*, we get the following inequality*

$$\begin{aligned}&\Big|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\Big|\\&\le\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{1,1}}_{\rho,\lambda+2}[|w|(b-a)^{\rho}]\left[|f''(a)|+|f''(b)|\right],\end{aligned}$$

where

$$\sigma_{1,1}(k)=\sigma(k)\left[\frac{(\lambda+\rho k)}{2(\lambda+\rho k+2)}\right],\qquad\rho,\lambda>0,\quad w\in\mathbb{R}.$$

Corollary 2.2. *If we take* $\lambda=\alpha$*,* $\sigma(0)=1$ *and* $w=0$ *in* Corollary 2.1*, we get the following inequality*

$$\begin{aligned}&\Big|\frac{f(a)+f(b)}{2}-\frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J^{\alpha}_{b^-}f(a)+J^{\alpha}_{a^+}f(b)\right]\Big|\\&\le\frac{(b-a)^2\alpha}{4(\alpha+1)(\alpha+2)}\left[|f''(a)|+|f''(b)|\right],\end{aligned}$$

which is more reasonable than the result obtained [**8**, Theorem 2] *under the same assumptions.*

Remark 2.2. Setting $s = 1$, $\lambda = \alpha = 1$, $\sigma(0) = 1$ and $w = 0$ in Theorem 2.1 we come to the same result as [**28**, Proposition 2].

Theorem 2.2. *Let $f\colon [a,b] \to \mathbb{R}$ be a twice differentiable function on (a,b) $(a < b)$. Also, $\rho, \lambda > 0$ and $w \in \mathbb{R}$. If $|f''|^q$ is s-convex in the second sense and $q > 1$ with $\frac{1}{p} + \frac{1}{q} = 1$, then the following inequality for generalized fractional integral operators holds*

$$\begin{aligned}&\Big|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2} - \frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a) + (\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\Big|\\&\leq \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_2}_{\rho,\lambda+2}[|w|(b-a)^{\rho}]\left[\frac{|f''(a)|^q + |f''(b)|^q}{s+1}\right]^{\frac{1}{q}},\end{aligned}$$

where $s \in (0,1]$ and $B(..,..)$ is Euler beta function and

$$\sigma_2(k) = 2\sigma(k)\left[\frac{1}{\lambda+\rho k}B\Big(\frac{p+1}{\lambda+\rho k}, p+1\Big)\right]^{\frac{1}{p}}$$

Proof. From Lemma 2.1 have

(2.8)
$$\begin{aligned}&\Big|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2} - \frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a) + (\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\Big|\\&\leq \frac{(b-a)^2}{2}\int_0^1 \left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}] - t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]\right|\\&\left|\,[f''(ta+(1-t)b) + f''((1-t)a+tb)]\,\right|\mathrm{d}t\\&\leq \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1 \left|t - t^{\lambda+\rho k+1}\right|\left|f''(ta+(1-t)b)\right|\mathrm{d}t\\&+ \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1 \left|t - t^{\lambda+\rho k+1}\right|\left|f''((1-t)a+tb)\right|\mathrm{d}t.\end{aligned}$$

Using Hölder Inequality and the s-convexity of $|f''|^q$ we get the following inequality

(2.9)
$$\begin{aligned}&\int_0^1 t(1-t^{\lambda+\rho k})\left[\left|f''(ta+(1-t)b)\right| + \left|f''((1-t)a+tb)\right|\right]\mathrm{d}t\\&\leq \left[\int_0^1\big(t(1-t^{\lambda+\rho k})\big)^p\mathrm{d}t\right]^{\frac{1}{p}}\left\{\left[\int_0^1\left|f''(ta+(1-t)b)\right|^q\mathrm{d}t\right]^{\frac{1}{q}} + \left[\int_0^1\left|f''((1-t)a+tb)\right|^q\mathrm{d}t\right]^{\frac{1}{q}}\right\}\\&\leq \left[\int_0^1 t^p(1-t^{(\lambda+\rho k)})^p\mathrm{d}t\right]^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left[|f''(a)|^q + |f''(b)|^q\right]^{\frac{1}{q}}\\&= 2\left[\frac{1}{\lambda+\rho k}B\Big(\frac{p+1}{\lambda+\rho k}, p+1\Big)\right]^{\frac{1}{p}}\left(\frac{1}{s+1}\right)^{\frac{1}{q}}\left[|f''(a)|^q + |f''(b)|^q\right]^{\frac{1}{q}}.\end{aligned}$$

By changing $x = t^{\lambda+\rho k}$ and simple calculating we get

$$\int_0^1 t^p(1-t^{(\lambda+\rho k)})^p \mathrm{d}t = \frac{1}{\lambda+\rho k} B\Big(\frac{p+1}{\lambda+\rho k}, p+1\Big).$$

Thus combining (2.8) and (2.9), the desired result is obtained. □

Remark 2.3. Setting $\lambda = \alpha = 1$, $\sigma(0) = 1$ and $w = 0$ in Theorem 2.2 the same result as in [**11**, Theorem 10].

Corollary 2.3. *Taking $s = 1$ in* Theorem 2.2*, the following inequality holds*

$$\Big|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2} - \frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a) + (\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\Big|$$
$$\leq \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_2}_{\rho,\lambda+2}[|w|(b-a)^{\rho}]\left[\frac{|f''(a)|^q + |f''(b)|^q}{2}\right]^{\frac{1}{q}},$$

where $\rho, \lambda > 0$, $w \in \mathbb{R}$, $B(..,..)$ is Euler beta function and

$$\sigma_2(k) = 2\sigma(k)\left[\frac{1}{(\lambda+\rho k)}B\Big(\frac{p+1}{\lambda+\rho k}, p+1\Big)\right]^{\frac{1}{p}}.$$

Corollary 2.4. *Taking $\lambda = \alpha$, $\sigma(0) = 1$ and $w = 0$ in* Corollary 2.3*, the following inequality holds*

$$\Big|\frac{f(a)+f(b)}{2} - \frac{\Gamma(\alpha+1)}{2(b-a)^{\alpha}}\left[J^{\alpha}_{b^-}f(a) + J^{\alpha}_{a^+}f(b)\right]\Big|$$
$$\leq \frac{(b-a)^2}{(\alpha+1)}\left[\frac{1}{\alpha}B\Big(\frac{p+1}{\alpha}, p+1\Big)\right]^{\frac{1}{p}}\left[\frac{|f''(a)|^q + |f''(b)|^q}{2}\right]^{\frac{1}{q}},$$

which is more reasonable than obtained [**8**, Theorem 3] *under the same assumptions.*

Theorem 2.3. *Let $f\colon [a,b] \to \mathbb{R}$ be a twice differentiable function on (a,b) $(a < b)$. Also, $\rho, \lambda > 0$ and $w \in \mathbb{R}$. If $|f''|^q$ is s-convex in the second sense and $q \geq 1$, then the following inequality for generalized fractional integral operators holds*

$$\Big|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2} - \frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a) + (\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\Big|$$
$$\leq \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{3,s}}_{\rho,\lambda+2}[|w|(b-a)^{\rho}],$$

where $s \in (0,1]$, $B(..,..)$ *is Euler beta function and*

$$\sigma_{3,s}(k) = \sigma(k)\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}}$$
$$\times\left\{\left[\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(a)|^q+(B(2,s+1)-B(\lambda+\rho k+2,s+1))\,|f''(b)|^q\right]^{\frac{1}{q}}\right.$$
$$\left.+\left[(B(2,s+1)-B(\lambda+\rho k+2,s+1))\,|f''(a)|^q+\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(b)|^q\right]^{\frac{1}{q}}\right\}.$$

Proof. From Lemma 2.1 with properties of modulus we get

(2.10)
$$\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\Big(\frac{f(a)+f(b)}{2}\Big)-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\right|$$
$$\le\frac{(b-a)^2}{2}\int_0^1\left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]\right|$$
$$\times\left|\,[f''(ta+(1-t)b)+f''((1-t)a+tb)]\,\right|\mathrm{d}t$$
$$\le\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f''(ta+(1-t)b)\right|\mathrm{d}t$$
$$+\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left|f''((1-t)a+tb)\right|\mathrm{d}t.$$

Using Power-mean Inequality and s-convexity of $|f''|^q$, we obtain the following inequality

(2.11)
$$\int_0^1 t(1-t^{\lambda+\rho k})\left[\left|f''(ta+(1-t)b)\right|+\left|f''((1-t)a+tb)\right|\right]\mathrm{d}t$$
$$\le\left[\int_0^1(t-t^{\lambda+\rho k+1})\mathrm{d}t\right]^{1-\frac{1}{q}}\left[\int_0^1(t-t^{\lambda+\rho k+1})\left|f''(ta+(1-t)b)\right|^q\mathrm{d}t\right]^{\frac{1}{q}}$$
$$+\left[\int_0^1(t-t^{\lambda+\rho k+1})\mathrm{d}t\right]^{1-\frac{1}{q}}\left[\int_0^1(t-t^{\lambda+\rho k+1})\left|f''((1-t)a+tb)\right|^q\mathrm{d}t\right]^{\frac{1}{q}}$$
$$\le\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}}$$
$$\times\left\{\left[\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(a)|^q+(B(2,s+1)-B(\lambda+\rho k+2,s+1))\,|f''(b)|^q\right]^{\frac{1}{q}}\right.$$
$$\left.+\left[(B(2,s+1)-B(\lambda+\rho k+2,s+1))|f''(a)|^q+\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(b)|^q\right]^{\frac{1}{q}}\right\}.$$

Combining the inequalities (2.10) and (2.11) we have

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\right|\\
&\le \frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}}\\
&\quad\times\left\{\left[\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(a)|^q+(B(2,s+1)-B(\lambda+\rho k+2,s+1))\,|f''(b)|^q\right]^{\frac{1}{q}}\right.\\
&\quad+\left.\left[(B(2,s+1)-B(\lambda+\rho k+2,s+1))|f''(a)|^q+\frac{\lambda+\rho k}{(s+2)(\lambda+\rho k+s+2)}|f''(b)|^q\right]^{\frac{1}{q}}\right\}\\
&=\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{3,s}}_{\rho,\lambda+2}[w(b-a)^{\rho}].
\end{aligned}$$

Thus the proof is completed. □

Corollary 2.5. *Taking $s=1$ with $\rho,\lambda>0$ and $w\in\mathbb{R}$ in* Theorem 2.3*, the following inequality holds*

$$\begin{aligned}
&\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\right|\\
&\le \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{3,1}}_{\rho,\lambda+2}[|w|(b-a)^{\rho}],
\end{aligned}$$

where

$$\begin{aligned}
\sigma_{3,1}(k)=\sigma(k)&\left[\frac{\lambda+\rho k}{2(\lambda+\rho k+2)}\right]^{1-\frac{1}{q}}\\
&\times\left\{\left[\frac{\lambda+\rho k}{3(\lambda+\rho k+3)}|f''(a)|^q+\frac{(\lambda+\rho k)(\lambda+\rho k+5)}{6(\lambda+\rho k+2)(\lambda+\rho k+3)}|f''(b)|^q\right]^{\frac{1}{q}}\right.\\
&\quad+\left.\left[\frac{(\lambda+\rho k)(\lambda+\rho k+5)}{6(\lambda+\rho k+2)(\lambda+\rho k+3)}|f''(a)|^q+\frac{\lambda+\rho k}{3(\lambda+\rho k+3)}|f''(b)|^q\right]^{\frac{1}{q}}\right\}
\end{aligned}$$

Remark 2.4. Setting $\lambda=\alpha=1$, $\sigma(0)=1$ and $w=0$ in Theorem 2.3 the same result as [**11**, Theorem 8]

Remark 2.5. Setting $s=1$, $\lambda=\alpha$, $\sigma(0)=1$ and $w=0$ in Theorem 2.3 the same result as [**8**, Theorem 4]

Theorem 2.4. *Let $f\colon[a,b]\to\mathbb{R}$ be a twice differentiable function on (a,b) $(a<b)$. Also, $\rho,\lambda>0$ and $w\in\mathbb{R}$. If $|f''|^q$ is s-concave in the second sense and $q>1$ with $\frac{1}{p}+\frac{1}{q}=1$, then the following inequality for generalized fractional* *integral operators holds*

$$\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\right|$$
$$\leq\frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{4,s}}_{\rho,\lambda+2}[|w|(b-a)^{\rho}]\left|f''\left(\frac{a+b}{2}\right)\right|,$$

where $s\in(0,1]$ *and* $\sigma_{4,s}(k)=\sigma(k)2^{\frac{s}{q}}\left[\frac{2}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right)\right]^{\frac{1}{p}}$.

Proof. From Lemma 2.1 and Hölder inequality with properties of modulus, we have

(2.12)
$$\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2}-\frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^-;w}f)(a)+(\mathcal{J}^{\sigma}_{\rho,\lambda,a^+;w}f)(b)\right]\right|$$
$$\leq\frac{(b-a)^2}{2}\int_0^1\left|t\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}]-t^{\lambda+1}\mathcal{F}^{\sigma}_{\rho,\lambda+2}[w(b-a)^{\rho}t^{\rho}]\right|$$
$$\times\left|[f''(ta+(1-t)b)+f''((1-t)a+tb)]\right|\mathrm{d}t$$
$$\leq\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}$$
$$\times\int_0^1\left|t-t^{\lambda+\rho k+1}\right|\left[\left|f''(ta+(1-t)b)\right|+\left|f''((1-t)a+tb)\right|\right]\mathrm{d}t$$
$$\leq\frac{(b-a)^2}{2}\sum_{k=0}^{\infty}\frac{\sigma(k)|w|^k(b-a)^{\rho k}}{\Gamma(\rho k+\lambda+2)}\times\left[\int_0^1(t-t^{\lambda+\rho k+1})^p\mathrm{d}t\right]^{\frac{1}{p}}$$
$$\times\left\{\left[\int_0^1\left|f''(ta+(1-t)b)\right|^q\mathrm{d}t\right]^{\frac{1}{q}}+\left[\int_0^1\left|f''((1-t)a+tb)\right|^q\mathrm{d}t\right]^{\frac{1}{q}}\right\}.$$

Since $|f''|^q$ is s-concave, we can write

(2.13)
$$\int_0^1\left|f''((1-t)a+tb)\right|^q\mathrm{d}t\leq 2^{s-1}\left|f''\left(\frac{a+b}{2}\right)\right|^q,$$
$$\int_0^1\left|f''(ta+(1-t)b)\right|^q\mathrm{d}t\leq 2^{s-1}\left|f''\left(\frac{a+b}{2}\right)\right|^q.$$

On the other hand, by simple calculating we establish

(2.14)
$$\int_0^1(t-t^{\lambda+\rho k+1})^p\mathrm{d}t=\frac{1}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k},p+1\right).$$

Thus combining (2.13), (2.14) and (2.12) the requested result is obtained. □

Corollary 2.6. *Taking* $s = 1$ *in* Theorem 2.4*, the following inequality holds*

$$\left|\mathcal{F}^{\sigma}_{\rho,\lambda+1}[w(b-a)^{\rho}]\frac{f(a)+f(b)}{2} - \frac{1}{2(b-a)^{\lambda}}\left[(\mathcal{J}^{\sigma}_{\rho,\lambda,b^{-};w}f)(a) + (\mathcal{J}^{\sigma}_{\rho,\lambda,a^{+};w}f)(b)\right]\right|$$
$$\leq \frac{(b-a)^2}{2}\mathcal{F}^{\sigma_{4,1}}_{\rho,\lambda+2}[|w|(b-a)^{\rho}]\left|f''\left(\frac{a+b}{2}\right)\right|,$$

where $\rho, \lambda > 0$*,* $w \in \mathbb{R}$*,*

$$\sigma_{4,1}(k) = \sigma(k)2^{\frac{1}{q}}\left[\frac{2}{\lambda+\rho k}B\left(\frac{p+1}{\lambda+\rho k}, p+1\right)\right]^{\frac{1}{p}}.$$

Corollary 2.7. *If we take* $\lambda = \alpha$*,* $\sigma(0) = 1$ *and* $w = 0$ *in* Corollary 2.6*, the following inequality holds*

$$\left|\frac{f(a)+f(b)}{2} - \frac{\Gamma(\alpha+1)}{2(b-a)^{\lambda}}\left[J^{\alpha}_{b^-}f(a) + J^{\alpha}_{a^+}f(b)\right]\right|$$
$$\leq \frac{(b-a)^2}{(\alpha+1)}\left[\frac{1}{\alpha}B\left(\frac{p+1}{\alpha}, p+1\right)\right]^{\frac{1}{p}}\left|f''\left(\frac{a+b}{2}\right)\right|,$$

which is more reasonable than [**8**, Theorem 5] *under the same assumptions.*

Remark 2.6. Setting $s = 1$, $\lambda = \alpha = 1$, $\sigma(0) = 1$ and $w = 0$ in Theorem 2.3 the same result as [**11**, Theorem 9].

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