

Chapter 3

Stress scenarios for large number of risk factors

3.1 Introduction

Stress testing is one of the basic methods and should represent an integral part of the risk management framework. Within this context, we assess the impact of adverse scenarios of economic and financial situation which are severe but still plausible.

One of the important challenges within the process of stress testing is scenario design. The two most relevant issues are:

- **Type and magnitude of the scenario:** The scenario should capture the main risks and vulnerabilities which the portfolio is exposed to. In addition, the scenario must be sufficiently severe but its occurrence must still be possible at least with some low level of probability.
- **Capturing the dependences among risk factors:** In practice, portfolios consist of a large number of assets and are exposed to a large number of risk factors. When designing these scenarios, existing dependencies between various risk factors should be taken into account. In addition, any risk manager designing the stress scenarios must be aware that these dependencies tend to change during a market turmoil. In particular, the correlations tend to be higher during turbulent periods which restrict the diversification effects.

In this chapter, we will mainly address the second issue. In particular, we will present two possible approaches which can be applied to the problem of scenario design for portfolios with a large number of risk factors:

- (a) **Mixture models:** This approach focuses mainly on addressing the issue of elevated correlations in turbulent periods. In particular, it assumes that the distribution of

changes in risk factors is actually a mixture of two distributions: the first one represents quiet periods and the second one captures periods of market turmoil.

- (b) **Principal component analysis:** This approach addresses the issue of capturing the dependence structure by a significant reduction of dimensionality of the set of risk factors. In particular, the large number of risk factors is represented by a relatively small set of mutually uncorrelated principal components while preserving the largest possible amount of the variability of the original data. This condensed set of principal components can be then modeled by multidimensional models which would not be feasible for the original large set of risk factors.

3.2 Models based on the mixture of normal distributions

3.2.1 Motivation

Assume that FX rates represent important risk factors for a given portfolio. In particular, assume that significant open positions are mainly in the following six currencies: USD, JPY, GBP, CHF, AUD, CAD. For the purposes of the stress testing, we want to propose a relevant stress scenario.

One possible approach is to propose the scenario in accordance with a historical scenario which would cause a significant loss if repeated today. However, this might not be entirely sufficient approach, for example in case of several different portfolios with a different sensitivity to the same historical scenario or in case that we want to have a scenario which is consistent with a particular broader macroeconomic shock. Hence, we might want to impose a shock which did not occur in the past. For example, we want to assume e.g. a significant depreciation of euro with respect to US dollar by 30 %. Then the question arises what changes in the other FX rates are consistent with this change in EUR/USD. Note that the historical dependencies have to be taken into account in this case. Given the large magnitude of such a shock, we need to focus mainly on estimating dependencies which occur on the market during turbulent periods.

We know that in turmoil periods, both volatilities and correlations increase significantly. Hence we assume that data are generated from two normal distributions ("quiet" and "hectic") with different volatilities and correlations. Data from each distribution are drawn with some probability.

3.2.2 Two alternatives of model formulation

First, assume that we have only two FX rates. Denote their daily relative changes by x_t and y_t . Then the model is based on a mixture of two two-dimensional normal distribu-

tions, i.e.

$$(x, y) \sim \begin{cases} N \left(\begin{bmatrix} \mu_{x1} \\ \mu_{y1} \end{bmatrix}, \begin{bmatrix} \sigma_{x1}^2 & \rho_1 \sigma_{x1} \sigma_{y1} \\ \rho_1 \sigma_{x1} \sigma_{y1} & \sigma_{y1}^2 \end{bmatrix} \right) & \text{with probability } 1 - \omega \text{ (quiet days),} \\ N \left(\begin{bmatrix} \mu_{x2} \\ \mu_{y2} \end{bmatrix}, \begin{bmatrix} \sigma_{x2}^2 & \rho_2 \sigma_{x2} \sigma_{y2} \\ \rho_2 \sigma_{x2} \sigma_{y2} & \sigma_{y2}^2 \end{bmatrix} \right) & \text{with probability } \omega \text{ (hectic days).} \end{cases}$$

Note that this model has 11 parameters (5 for each distribution and 1 parameter for probability of hectic days). If we extend the model to all 6 FX rates, we would end up with 55 parameters! Hence this approach is not feasible for large portfolios.

In order to reduce the number of parameters and make the implementation of the model feasible, we assume that one of the risk factors is core factor and others are peripheral factors. In our example, the core factor may be the FX rate for which the assumption of the magnitude in the stress scenario is made. The main simplification is mainly based on the assumption that the hectic and quiet periods are identified solely for this core risk factors and are assumed to be the same for all other peripheral risk factors.

Let us assume for illustration that the core risk factor is EUR/USD exchange rate. We assume that this core factor can be modeled by a one-dimensional mixture distribution with 5 parameters. In particular, when denoting the changes in the core FX rate by x_t , the mixture model for this risk factor is:

$$x \sim \begin{cases} N(\mu_{x1}, \sigma_{x1}^2) & \text{with probability } 1 - \omega \text{ (quiet days),} \\ N(\mu_{x2}, \sigma_{x2}^2) & \text{with probability } \omega \text{ (hectic days).} \end{cases}$$

Then we compute the conditional correlations between the core factor and each peripheral factor given that a hectic period occurred, as well as means and standard deviations for the hectic period. Based on these results, we can estimate the magnitude of the changes in other peripheral FX rates based on the change in the core FX rate.

One of the most important properties of this model is that the hectic period is not determined a priori. Instead, we calculate the probability that any given change in the core risk factor was drawn from the hectic distribution.

3.2.3 Estimating the parameters for the core risk factor

The model can be estimated using maximum likelihood method. The probability density function of the one-dimensional mixture distribution is

$$f(x_t | \mu_{x1}, \sigma_{x1}, \mu_{x2}, \sigma_{x2}, \omega) = (1 - \omega) \frac{1}{\sqrt{2\pi\sigma_{x1}^2}} e^{-\frac{(x_t - \mu_{x1})^2}{2\sigma_{x1}^2}} + \omega \frac{1}{\sqrt{2\pi\sigma_{x2}^2}} e^{-\frac{(x_t - \mu_{x2})^2}{2\sigma_{x2}^2}}.$$

Hence, the parameters are obtained by maximizing the log-likelihood function

$$\ln L(\mu_{x_1}, \sigma_{x_1}, \mu_{x_2}, \sigma_{x_2}, \omega | x_t) = \sum_{t=1}^n f(x_t | \mu_{x_1}, \sigma_{x_1}, \mu_{x_2}, \sigma_{x_2}, \omega).$$

Note that this is a problem of non-convex optimization. Hence, it is important to implement a method which is capable to find a true global maximum. One of the possible approaches is to use methods of stochastic optimization, e.g. the method of simulated annealing.

In Matlab, the estimation of the parameters can be easily implemented:

```
%Data for USD JPY GBP CHF AUD CAD, Jan 1999 - Dec 2012
Data=xlsread('fx_rates.xls');
Returns=log(Data(2:end,:)./Data(1:(end-1),:));

obj=gmdistribution.fit>Returns(:,1),2);
Proportions = obj.PComponents;
Means=obj.mu;
Volatilities=sqrt(obj.Sigma);

%Identifying hectic period
if Volatilities(:, :, 2)>Volatilities(:, :, 1)
    hectic=2;
else
    hectic=1;
end

%Calculating parameters for the primary risk factor
disp('Calculating parameters for the primary risk factor')
quiet=3-hectic;
omega=Proportions(hectic)
mu2=Means(hectic)
mu1=Means(quiet)
sigma2=Volatilities(hectic)
sigma1=Volatilities(quiet)
```

One of the questions that arises here is whether the model based on the mixture of two normal distributions is better than the model based on one normal distribution. Given that the former model is a generalization of the latter, we can apply the likelihood ratio test to assess whether the increase of the number of parameters significantly improves the value of the likelihood function. The likelihood ratio statistics has asymptotically χ^2 -distribution where the degree of freedom is equal to number of additional parameters (i.e. 3 in our case).

Model	ω	μ	σ
Unconditional (one normal)		0.00003	0.0066
Conditional – quiet period	0.80	0.00002	0.0053
Conditional – hectic period	0.20	0.00007	0.0103

Table 3.1: Estimated values of parameters, USD/EUR (1999-2012)

This test can be easily implemented in Matlab as follows:

```
%TEST OF THE SPECIFICATION OF THE MODEL FOR 1 MAIN RISK FACTOR
temp_obj=gmdistribution.fit>Returns(:,1),2);
LogL2=temp_obj.NlogL;
temp_obj=gmdistribution.fit>Returns(:,1),1);
LogL1=temp_obj.NlogL;
Likelihood_ratio=-2*(LogL2-LogL1)
ch2_quantil=chi2inv(0.95,3)
```

In this case the likelihood ratio statistics equals to 156.4 which is higher than the quantile of χ^2 -distribution with 3 degrees of freedom and the confidence level 5 % (7.8). Hence we reject the null hypothesis that the extension of the model is not statistically significant.

Now, we calculate the probability that a particular relative change was drawn from the hectic distribution. In particular, it is the conditional probability of hectic period given that the change in the core risk factor was x_t . This probability can be calculated by the Bayes formula:

$$H(x_t) = P(\text{hectic} | x_t) = \frac{P(\text{hectic} \wedge x_t)}{P(x_t)} = \frac{\omega f(x_t | \mu_2, \sigma_2^2)}{(1 - \omega)f(x_t | \mu_1, \sigma_1^2) + \omega f(x_t | \mu_2, \sigma_2^2)}$$

The plot of this function can be obtained using Matlab (see Figure 3.1):

```
h=(omega.*pdf('Normal',Returns(:,1),mu2,sigma2))./((1-omega).*pdf('Normal',Returns(:,1),mu1,sigma1) ...
+omega.*pdf('Normal',Returns(:,1),mu2,sigma2)));
scatter>Returns(:,1),h)
```

3.2.4 Estimating the parameters and scenario design for peripheral risk factors

Assume that we have determined the scenario for the change in the "core" exchange rate. Now we describe how the parameters and scenarios for other (i.e. peripheral)

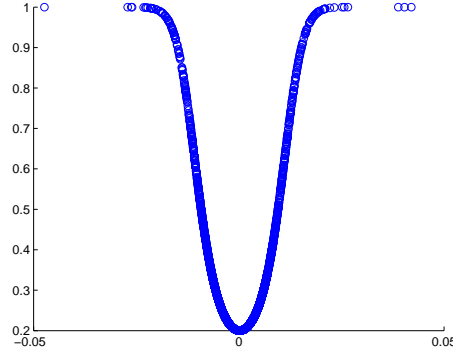


Figure 3.1: Conditional probability of hectic period. Horizontal axis represents x_t , vertical axis represents $H(x_t)$.

exchange rates can be calculated. We illustrate the procedure on one peripheral risk factor – denote the relative changes of this risk factor by y_t .

First we estimate the mean and standard deviation of changes of the peripheral risk factor as well as their correlation with the changes in the core risk factor. All these parameters will be estimated for the distribution corresponding to hectic days. This calculation is based on the standard formula for calculating this statistics, but all the sums in the formulas are adjusted to be weighted by the probability of hectic period ($H(x_t)$). In other words, replacing $H(x_t)$ by value 1 in the formulas below would yield classic formulas for calculation of mean, standard deviation and correlation.

$$\mu_{y2} = \frac{\sum_{t=1}^n H(x_t) y_t}{\sum_{t=1}^n H(x_t)},$$

$$\sigma_{y2} = \sqrt{\frac{\sum_{t=1}^n H(x_t) (y_t - \mu_{y2})^2}{\sum_{t=1}^n H(x_t)}},$$

$$\rho_2 = \frac{\sum_{t=1}^n H(x_t) (x_t - \mu_{x2}) (y_t - \mu_{y2})}{\sigma_{x2} \sigma_{y2} \sum_{t=1}^n H(x_t)}.$$

Note that the conditional values of parameters corresponding to the quiet period can be calculated analogously by replacing the conditional probability of hectic period ($H(x_t)$) by the conditional probability of quiet period ($1 - H(x_t)$).

Given that we know all the parameters (i.e. the mean and standard deviation for the core as well as for all peripheral FX rates together with their correlations) of the distribution corresponding to hectic period, the values of the stress scenarios can be calculated using the following formula:

$$\frac{y_t - \mu_{y2}}{\sigma_{y2}} = \rho_2 \frac{x_t - \mu_{x2}}{\sigma_{x2}} + \sqrt{1 - \rho_2^2} \varepsilon_t.$$

Given that the common shock expressed by the change in the core risk factor is the only shock on the market and there are no additional idiosyncratic shocks to the peripheral risk factors, ε_t vanishes and the previous equality can be rewritten to:

$$y_s = \mu_{y2} + \rho_2(x_s - \mu_{x2}) \frac{\sigma_{y2}}{\sigma_{x2}},$$

where x_s and y_s denote the scenario change in the core and peripheral risk factor.

The implementation in Matlab is as follows:

```
scenarios=[0.3 0 0 0 0 0];
for i=2:6
    N=size>Returns(:,i);
    mu_y2=sum(h.*Returns(:,i))/sum(h)
    mu_y1=sum((1-h).*Returns(:,i))/sum(1-h)
    sigma_y2=sqrt(sum(h.*(Returns(:,i)-mu_y2*ones(N)).^2)/sum(h))
    sigma_y1=sqrt(sum((1-h).*Returns(:,i)-mu_y1*ones(N)).^2)/sum(1-h))
    rho_y2=sum(h.*(Returns(:,i)-mu_y2*ones(N)).*(Returns(:,i)-mu_y1*ones(N)))/(sigma_y2*sigma_y1*sum(h))
    rho_y1=sum((1-h).*Returns(:,i)-mu_y1*ones(N)).*(Returns(:,i)-mu_y2*ones(N)))/(sigma_y1*sigma_y2*sum(1-h))
    scenarios(i)=rho_y2*(scenarios(1)-mu_y2)*sigma_y2/sigma_y1+mu_y2
end
```

The value of parameters as well as scenarios for all the risk factors are summarized in Table 3.2. Note that the correlations between the changes in core and peripheral risk factors are indeed higher in case of the distribution corresponding to hectic period compared to the quiet period.

	USD	JPY	GBP	CHF	AUD	CAD
μ_1	0.0020%	0.0014%	0.0031%	-0.0030%	-0.0157%	-0.0135%
μ_2	0.0070%	-0.0287%	0.0068%	-0.0293%	0.0066%	0.0104%
σ_1	0.53%	0.71%	0.46%	0.37%	0.67%	0.61%
σ_2	1.03%	1.10%	0.67%	0.46%	0.84%	0.90%
ρ_1		54%	44%	13%	20%	52%
ρ_2		75%	61%	25%	33%	70%
Scenario	30%	24%	12%	3%	8%	18%

Table 3.2: Parameters and scenarios for all FX rates

3.2.5 References

The following references might be useful in case of a deeper interest:

[1] Description of the model and its implementation:

Kim, J., Finger, Ch. C. (2000): A Stress Test to Incorporate Correlation Breakdown, RiskMetrics Journal, Volume 1

[2] Application to the stress testing of FX risk in the NBS:

Jurča, P., Rychtárik, Š. (2006): Stress testing the Slovak banking sector, Banking journal BIATEC, Vol. XIV, No. 4, National Bank of Slovakia

[3] Implementation of the method of simulated annealing for the maximization of the likelihood function:

Lucia Potisková (2007): Stresové testovanie s využitím modelu založenom na kombinácii normálnych rozdelení (Bachelor thesis, in Slovak)

3.3 Principal component analysis

3.3.1 Motivation

Assume that we want to propose stress scenarios of the changes in the yield curve for the purpose of stress testing of the bond portfolio. The main issue is the high dimensionality of this problem: The yield curve consists of a number of points representing individual maturities (tenors). The changes of the interest rate corresponding to different maturities are partially mutually interrelated, but they are not fully correlated.

One of the possible solutions to this issue is to explore which types of changes were the most typical in the historical data. The time series of EUR interest rates for 1 year, 3 years, 5 years and 7 years are depicted in Figure 3.2.¹ We can see that one of the most typical changes was a parallel shift in the whole yield curve. This was mainly prevailing in the market from 2006 until the first half of 2008 as well as in 2012 and 2013. However, there are also other types of changes which should be captured by the stress testing. For example, there was a significant increase in slope of the yield curve during the second half of 2008 with a large drop in short-term interest rates and only a mild decrease in long-term rates. An additional type of changes can be observed at the end of 2011 characterised by a significant jump in both 1-year and 7-year interest rates while the increase in 5-year rates was more limited. Hence, we will apply the principal component analysis which will allow us to rigorously identify the most common types of changes which the yield curve has been exposed to.

In addition, CRD IV² prescribes that supervisors should apply supervisory measures in case that bank's economic value declines by more than 20 % of its own funds as a result of a sudden and unexpected change in interest rates of 200 basis points. The principal component analysis allows us to assess how representative is such a change and estimate its probability level.

¹Source: ECB, EUR spot rates, all credit ratings.

²Directive 2013/36/EU of the European Parliament and of the Council of 26 June 2013 on access to the activity of credit institutions and the prudential supervision of credit institutions and investment firms, Article 98 paragraph 5.

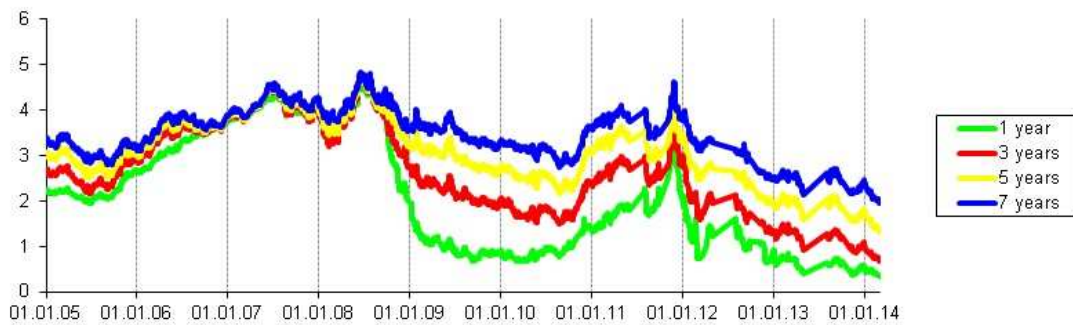


Figure 3.2: Interest rates for 1 year, 3 years, 5 years and 7 years.

3.3.2 Main idea

Let us start with exploring only two time series, in particular 1-year and 3 year interest rates. The scatter plot of the daily absolute changes of these two interest rates is in Figure 3.3. It can be seen that the majority of the data are represented by an ellipse. If we want to reduce the dimensions of the data from 2 to 1 while keeping the largest possible part of the variability of the original data, we can represent the dataset by the main axis of this ellipse.

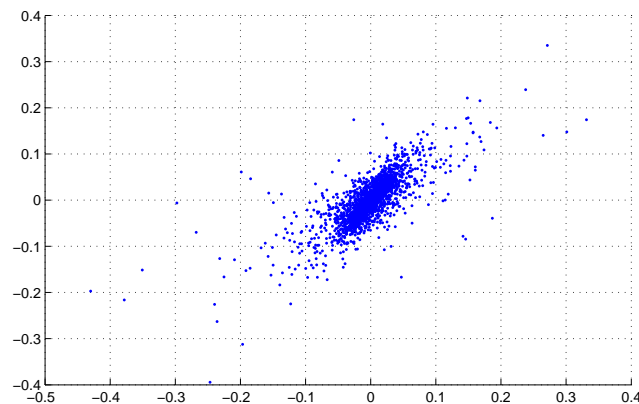


Figure 3.3: Interest rates for 1 year, 3 years, 5 years and 7 years.

In case of 3 interest rates, the data can be represented by an ellipsoid. In this case, the data can be compressed (projected) into an ellipse, while neglecting the third dimension. It is however important to choose the right direction of the compression. In order to preserve the largest possible part of the data variability, we have to find a plane where the intersection of the ellipsoid with this plane forms the largest possible ellipse, and to compress the data in the direction which is orthogonal to this plane. In other words, we have to identify the vectors and lengths corresponding to the three main axes of the ellipsoid and then to compress the data in the direction of the shortest main axis.

We now that the vectors of the main axes of the ellipsoid can be obtained as the eigenvectors of the variance-covariance matrix of the original data. Their lengths are represented by the eigenvalues of this matrix. Based on this insight, we will now rigorously describe the theoretical background of the principal component analysis.

3.3.3 Theoretical background

Let us denote the $N \times T$ matrix of the changes in the risk factors by X (i.e. time series are in rows of this matrix).

We know that any regular symmetric matrix A with mutually distinct eigenvalues can be decomposed as follows:

$$A = P\Lambda P^\top,$$

where Λ is a diagonal matrix of eigenvalues of the matrix A and P is an orthonormal matrix of the standardised eigenvectors of the matrix A (represented by columns of matrix P).

Based on the main idea of the principal component analysis that has been introduced above, we use the following procedure:

1. First, we identify the main axes of the N -dimensional ellipsoid of the data. In particular, we can apply the spectral decomposition to the variance-covariance matrix Σ of the original variables:

$$\Sigma = P\Lambda P^\top.$$

We know that the directions of the main axes are represented by the columns of matrix P and their length by eigenvalues, which are on the diagonal of matrix Λ .

2. We now transform the original data into the new coordinate system with origin at the centre of the ellipsoid and with coordinates given by the eigenvectors. In particular, the transformed data can be calculated as follows:

$$Y = P^\top(X - \mu),$$

where μ is the vector of means of the original time series X . The rows of matrix Y represent the original data transformed to the new coordinate system and they are called **principal components**. Note that one has

$$E(Y) = 0$$

and

$$\text{cov}(Y) = P^\top \Sigma P = P^\top P \Lambda P^\top P = \Lambda.$$

Hence, individual principal components are mutually uncorrelated and their variances are equal to the eigenvalues.

Let us note that the columns of matrix P represents coefficients which are used to calculate individual principal components (i.e. coefficients for the linear combination). These coefficients are also called **component loadings**.

3. Finally, we identify the most important principal components together with the share of the variability of the original dataset that is captured by these factors. Recall that the variance of any principal component is given by its corresponding eigenvalue. Hence, we sort the principal components in descending order according to their corresponding eigenvalues. Since the total variability of the original data is

$$\sum_{i=1}^N \text{var}(X_i) = \text{trace}(\Sigma) = \sum_{i=1}^N \lambda_i,$$

the share of total variability corresponding to the j -th principal component is

$$\frac{\lambda_j}{\sum_{i=1}^N \lambda_i}.$$

Based on this procedure, the first principal component corresponds to such linear combination of X , which has a maximum variance among all other linear combinations.

A question arises here whether principal components calculated on sample data (so-called sample principal components) are identical to the principal components of the true underlying distribution. This is indeed the case provided that some assumptions are satisfied. In particular, if X has a normal distribution and all the eigenvalues of $\text{cov}(X)$ are different, then the sample principal components are estimates of the actual principal components of the random vector X provided that the sample principal component are based on a variance-covariance matrix estimated using the maximum likelihood (i.e. not adjusted for degrees of freedom).

One of the important issues of the principal component analysis is that principal components are not invariant to the scaling of the original variables. In other words, if we calculate principal components for a set of returns of equity prices, the result (including matrix P) will be different when the price of one of the equities is expressed in euro cents compared to the case when this price is expressed in euros. Due to this reason, it is common to apply the principal component analysis to standardized variables, i.e. using the correlation matrix instead of variance-covariance matrix of the original data. On the other hand, if the different volatilities are present a homogeneous scale and we want to capture this property in the model, it could be appropriate to use the variance-covariance matrix.

Another important question is how to determine the number of principal components. Clearly, the decision is based on the corresponding eigenvalues. In particular, the following approaches can be used:

1. Select eigenvalues higher than their average (Kaiser)

2. Impose a floor on the percentage of the explained variability
3. Identify a break in the plot of the sorted eigenvalues (scree plot)

3.3.4 Interpretation of results

We will now apply the principal component analysis to the example of designing stress scenarios for yield curve changes. The implementation of the method will be done in Matlab. In particular, the Matlab code presented below implements the following steps:

- (a) Load the data (dates and values) and calculate the centralised daily absolute changes.
- (b) Apply the principal component analysis. In order to illustrate the differences, the principal component analysis is applied based on both correlation as well as variance-covariance matrix.
- (c) Calculate the share of the variability which is represented by the first three principal components.
- (d) Plot the component loadings for the first three principal components.

```
%Read data
[YC,txt]=xlsread('YC.xls');
Changes=YC(2:end,1:177)-YC(1:end-1,1:177);
Changes_central=Changes-repmat(mean(Changes),size(Changes,1),1);
Dates=datenum(txt(3:end,1),'dd.mm.yy');
axis=3/12:1/12:179/12; %convert months to years

%PCA based on correlation matrix
[loadings1,components1,eigenvalues1] = ...
    princomp(zscore(Changes_central));
%PCA based on variance-covariance matrix
[loadings2,components2,eigenvalues2] =princomp(Changes_central);
%Part of volatility explained by first three principal components
cumsum(eigenvalues1(1:3))./sum(eigenvalues1)

%Loadings
figure(1)
plot(axis,loadings1(:,1:3),'LineWidth',3)
legend('1st PC','2nd PC','3rd PC')
hold on
plot(axis,loadings2(:,1:3))
```

As a results of this procedure, we can see that using the correlation matrix, the share of the overall variability of the original dataset captured by the first three principal components is very high – it is 96.9 %. In particular, the first component itself explains 78.4 %, the second one 13.3 % and the third one 5.2 %. In addition, based on the component loadings depicted in Figure 3.4, these principal components have a nice interpretation: The component loadings of the first principal component are between -0.6 and -0.8 for the majority of maturities (except the shortest maturities). Hence, this principal component can be interpreted as approximately a parallel shift of the whole yield curve. Similarly, the second principal component corresponds to a change in the slope of the yield curve, since its loadings are negative for short-term maturities and positive for long-term maturities. Finally, the third principal component represents some kind of butterfly effect.

Figure 3.4 also illustrates the differences in the component loadings for the method based on correlation and variance-covariance matrix. We can see that the differences are not significant in this case. Note however that this is a result of a relative homogeneity of the time series (all represents changes in interest rates) and generally it should not be considered as a valid result.

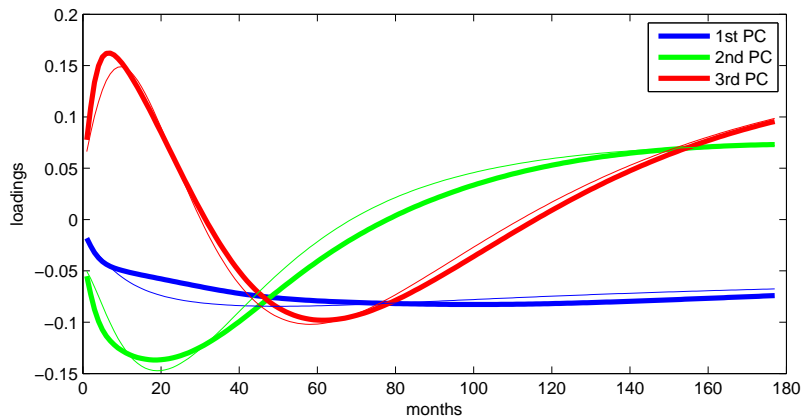


Figure 3.4: Component loadings for the first three principal components. Thick lines represent loadings calculated from correlation matrix, thin lines represent loadings based on variance-covariance matrix.

To distinguish between the types of effects that were prevailing in the different time periods, Figure 3.5 plots the principal components themselves. Note that for easier interpretation, the cumulative values of the principal components are plotted. For example, we can see that the period between 2004 and first half of 2008 was mainly characterised by parallel shift in the yield curve. During the second half of 2008, all three effects took place. In particular, the yield curve was shifted downward while its slope increased significantly. This figure has been obtained by the following Matlab code:

```

plot(Dates,cumsum(components1(:,1:3)),'LineWidth',3)
datetick('x','mm/yy','keepticks')
legend('1st PC','2nd PC','3rd PC')

```

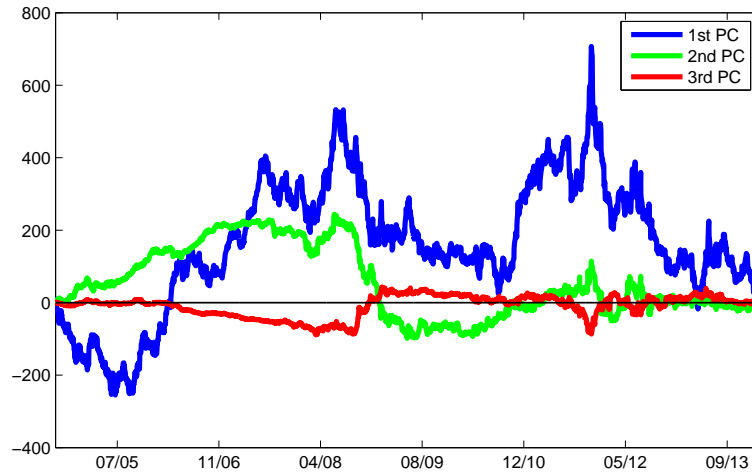


Figure 3.5: Times series of the first three principal components (cumulated).

3.3.5 Using PCA for stress scenario design

Having identified the most common types of changes in the yield curve, we now know how to design the most representative stress scenarios that can be used in the management of the interest rate risk.

First, let us focus on the first principal component. We assume that the values of this principal component are normally distributed from $N(\mu_{HK}, \sigma_{HK}^2)$.

Hence, the quantile corresponding the probability level of $\alpha = 99\%$ is

$$y = (\mu_{HK} - u_{\alpha}\sigma_{HK}, 0, \dots, 0)^{\top}$$

(assuming zero value of all others principal components).

Using backward transformation, this scenario for the first principal component can be expressed as the change in the yield curve as follows:

$$x = Py + \mu.$$

In this backward transformation, we have used that $P^{\top} = P^{-1}$, given that the matrix P of eigenvectors is orthonormal.

After simplification, this yields

$$x = P_1(\mu_{HK} - u_\alpha \sigma_{HK}) + \mu,$$

where P_1 is the eigenvector corresponding to the largest eigenvalue and $\mu = E(X)$.

Using Matlab, this procedure can be easily implemented:

```
scenario=loadings1(:,1)*(mean(components1(:,1)) ...
    -std(components1(:,1))*norminv(0.99,0,1))+mean(Changes)';
plot(axis,scenario)
```

When applying this procedure to the analysed data, we obtain the scenario depicted in Figure 3.6.

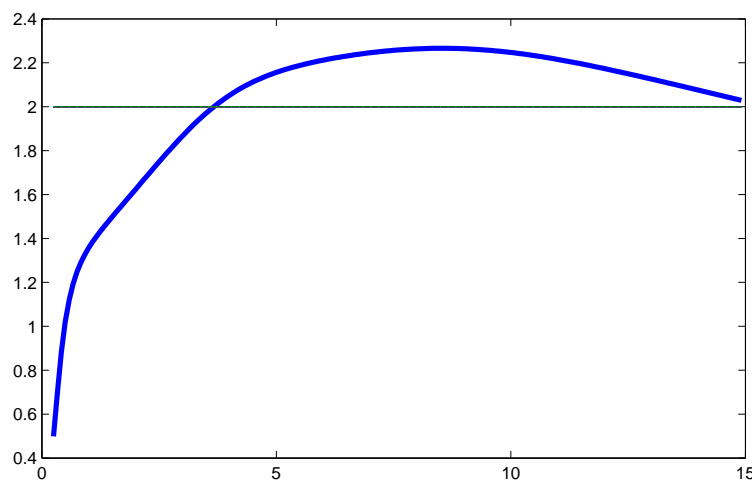


Figure 3.6: The result for the "most typical" scenario on the probability level of 99%. Horizontal axis represents maturities (tenors) in years, vertical axis represents interest rates in percentages.

3.3.6 Main weaknesses of the principal components analysis

The principal component analysis suffers from several weaknesses that should be taken into account:

- The interpretation of the principal components is often difficult. A straightforward interpretation as in the example presented above is a rare exception.

- Given that the principal component analysis is based on correlation / variance-covariance matrix, it only addresses the linear dependencies (i.e. correlations). It captures neither more complex in the dependency structure nor the pattern of an increasing dependence in market turmoils.
- The principal component analysis does not allow changing weights (i.e. component loadings) when constructing the individual principal components, even if this would be appropriate for the real data. For example, the typical type of change in the yield curve might change in time over a longer time period.
- Although the principal components are mutually uncorrelated, the more complex interdependencies between principal components exist. This issue might be illustrated by the dependencies between volatilities of the principal components.

3.3.7 Possible extensions

There are several possible extensions of the basic principal component analysis which might be able to address some of the above-mentioned shortcomings:

- (a) The calculation of the principal component can be based on conditional instead of unconditional correlation / covariance matrix.
- (b) Orthogonal GARCH models.
- (d) Factor analysis – rotation of principal components can improve their interpretation.