Sensitivity Analysis for a Dynamic Stochastic Accumulation Model for Optimal Pension Savings Management[1]

Tibor JAKUBÍK – Igor MELICHERČÍK – Daniel ŠEVČOVIČ***

Abstract

Since January 2005, pensions in Slovakia are operated by a three-pillar system. This paper concentrates on the mandatory, fully funded second pillar. In our analysis we follow the dynamic stochastic accumulation model proposed by the authors in (Kilianová et al., 2006). Recently pension asset managers tend to be very cautious and they hold low stock to bond proportions in the pension funds. We discuss the sensitivity of the level of savings with respect to the proportion of stocks in the portfolios. Furthermore, we perform the sensitivity analysis with respect to correlation between stock and bond returns and risk aversion. Finally, we prove linearity of the level of savings with respect to the contribution rate.

Keywords: *dynamic stochastic programming, funded pillar, utility function, Bellman equation, Slovak pension system, correlation, risk aversion, pension portfolio simulations*

JEL Classification: C15, E27, G11, G23

Introduction

Since January 2005, pensions in Slovakia are operated by a three-pillar system proposed by the World Bank:

1. Mandatory non-funded first pillar (pay-as-you-go pillar),
2. Mandatory fully funded second pillar,
3. Voluntary fully funded third pillar.

* Tibor JAKUBÍK, Allianz-Slovenská d. s. s., a. s., Allianz Asset Management, správ. spol., a. s., Račianska 62, 831 02 Bratislava 3, Slovakia; e-mail: jakubikt@asdss.sk

** Igor MELICHERČÍK – Daniel ŠEVČOVIČ, Department of Applied Mathematics and Statistics, Faculty of Mathematics, Physics and Informatics, Comenius University, Mlynská dolina, 842 48 Bratislava 4, Slovakia; e-mail: igor.melichercik@fmph.uniba.sk; sevcovic@fmph.uniba.sk

[1] The research was supported by VEGA 1/0381/09 and CESIUK grants.

Contribution rates were set for the first pillar at 19.75% (old age 9%, disability and survival 6% and reserve fund 4.75%) and for the second pillar 9%. In the second pillar, pension asset administrators are responsible for management of pensioners' savings. Each pension administrator manages three funds: Growth Fund, Balanced Fund and Conservative Fund, each of them with different limits for investment (see Tab. 1). At the same time instant savers may hold assets in one fund only. According to Slovak legislature, in the last 15 years preceding retirement a saver may not hold assets in the Growth Fund and in the last 7 years all assets must be deposited in the Conservative Fund. Even with these restrictions contributors have some freedom for individual decisions which fund is optimal in a specific situation, e.g. the age of the contributor, the saved amount, the past performance of the pension funds, etc. A thorough description of the Slovak pension reform with calculations of the balance of the pension system and expected level of pensions in the new system can be found in Goliaš, (2003), Melicherčík and Ungvarský (2004), and Thomay et al. (2002).

Table 1

Limits for Investment for the Pension Funds in Slovakia

Fund type	Stocks (in %)	Bonds and Money Market Instruments (in %)
Growth fund	up to 80	at least 20
Balanced fund	up to 50	at least 50
Conservative fund	no stocks	100

Source: Melicherčík and Ungvarský (2004).

The idea of dynamic stochastic pension planning comes from seminal works by Merton (1969) and Samuelson (1969). The idea was further developed in Bodie et al. (1992; 2003). Based on these ideas, a dynamic stochastic accumulation model for optimal decision between Growth, Balanced and Conservative Funds was proposed and analyzed in Kilianová et al. (2006). The model was tested using historical data of asset returns and forecasts of wage growth. The authors assumed the highest possible stocks proportions in the funds (i.e. 80%, 50%, 0% in the Growth, Balanced and Conservative Funds, respectively). However, the current situation in Slovak pension funds is rather different and pension asset managers are very cautious holding significantly lower proportions of stocks in the funds than it is allowed by governmental regulations shown in Table 1. A natural question arises, whether such cautious strategies could lead to sufficient level of future pensions. The main goal of this paper is to perform a sensitive analysis of the model taking into account cautious investment strategies of pension asset managers.

The paper is organized as follows: In section 2 we recall the dynamic stochastic accumulation model proposed in Kilianová et al. (2006). Section 3 contains sensitivity analysis of the model. At the end of this section we prove linearity of the level of savings with respect to the contribution rate. The last section contains final remarks and conclusions.

2. Dynamic Stochastic Programming Accumulation Model

2.1. The Model Description

In this section, we recall the dynamic stochastic programming model proposed and derived in Kilianová et al. (2006). For reader's convenience, we present key ideas of derivation of the accumulation model. Suppose that the future pensioner deposits once a year a τ-part of his/her yearly salary w_t to a pension fund $j \in \{1, 2, ..., m\}$. Denote by s_t, $t = 1, 2, \ldots, T$ the accumulated sum at time t where T is the retirement time. Then the budget-constraint equations read as follows:

$$s_{t+1} = s_t\left(1 + r_t^j\right) + w_{t+1}\tau, \quad t = 1,\ 2, ..., T-1$$
$$s_1 = w_1\tau$$

Here r_t^j is the return of the fund j in the time period $[t,\ t + 1)$. When retiring the pensioner will strive to maintain his living standard compared to the level of his last salary. From this point of view, the saved sum s_T at the time of retirement T is not precisely what the future pensioner cares about. For a given life expectancy, the ratio of the cumulative sum s_T to the yearly salary w_T, i.e. $d_T = s_T / w_T$ is more important. Using the quantity $d_t = s_t/w_t$ one can reformulate the budget-constraint equation:

$$d_{t+1} = F_t(d_t, j), t = 1,\ 2, ..., T-1$$
$$d_1 = \tau$$

where

$$F_t(d, j) = d\frac{1 + r_t^j}{1 + \rho_t} + \tau, t = 1,\ 2, ..., T-1$$

and ρ_t denotes the wage growth defined by the equation

$$w_{t+1} = w_t(1 + \rho_t)$$

Suppose that each year the saver has an opportunity to choose a fund $j(t,\ I_t) \in \{1,\ 2, ..., m\}$, where I_t denotes the information set consisting of the history of returns $r_{t'}^j$, $t' = 1, 2, \ldots, t - 1$, $j \in \{1, 2, ..., m\}$ and the wage growth $\rho_{t'}$, $t' = 1, 2, \ldots, t - 1$. Now suppose that the history of the wage growth

ρ_t, $t = 1, 2,...,T-1$ is deterministic and the returns r_t^j are assumed to be random and they are independent for different times $t = 1, 2,...,T-1$. Then the only relevant information is the quantity d_t. Hence $j(t, I_t) \equiv j(t, d_t)$. One can formulate a problem of dynamic stochastic programming as follows:

$$\max_j E(U(d_T))$$

with the recurrent budget constraint:

$$d_{t+1} = F_t(d_t, j(t,d_t)), \quad t = 1, 2,...,T-1$$
$$d_1 = \tau$$

where the maximum is taken over all non-anticipative strategies $j = j(t, d_t)$. Here U stands for a given saver's utility function of his wealth representing saver's preferences. Using the law of iterated expectations we have

$$E(U(d_T)) = E(E(U(d_T) \mid I_t)) = E(E(U(d_T) \mid d_t))$$

and we can conclude that the conditional expectation $E(U(d_T) \mid d_t)$ should be maximal. Let us denote

$$V_t(d) = \max_j E(U(d_T) \mid d_t = d).$$

Again by using the law of iterated expectations

$$E(U(d_T) \mid d_t) = E(E(U(d_T) \mid d_{t+1}) \mid d_t)$$

we obtain the Bellman equation

$$V_t(d) = \max_{j \in \{1,2,...,m\}} E[V_{t+1}(F_t(d, j))] = E[V_{t+1}(F_t(d, j(t, d)))]$$

for $t = 1, 2,...,T-1$ where $V_T(d) = U(d)$. Using the last equation, the optimal feedback strategy $j(t, d_t)$ can be found by backward calculations. This strategy provides the decision for the optimal fund for each time t and level of savings d_t.

2.2. The Constant Relative Risk Aversion (CRRA) Utility Function

An important part of the model consists in a suitable choice of the utility function U. The utility function varies across savers and represents their attitude to risk. A key role in determining the utility function is played by the coefficient of relative risk aversion $C(d) = -dU''(d)/U'(d)$. A constant relative risk aversion $C(d) \equiv a > 0$ for every $d > 0$ implies that a saver has tendency to hold a constant proportion of his wealth in any class of risky assets as the wealth varies (see e.g. Friend and Blume, 1975; Pratt, 1964 and Young, 1990). In this case the utility function is uniquely given by

$$
\begin{aligned}
U(d) &= -Ad^{1-a} + B \quad \text{if } a > 1 \\
U(d) &= A\ln(d) + B \quad \text{if } a = 1 \\
U(d) &= Ad^{1-a} + B \quad \text{if } a < 1
\end{aligned}
$$

where A, B are constants, $A > 0$. This class of functions is referred to as Constant Relative Risk Aversion (CRRA) functions. The coefficient a of relative risk aversion plays an important role in many fields of economics. There is a consensus today, that the value should be less than 10 (see e.g. Friend and Blume, 1975; Mehra and Prescott, 1985; Pratt, 1964; Young, 1990). In our numerical experiments we considered values of a close to 9. It could be lower for lower equity premium. It is worth to note that the CRRA function is a smooth, increasing and strictly concave function for $d > 0$.

3. Sensitivity Analysis of the Model in the Case of the Slovak Pension System

The proposed model was tested in Kilianová et al. (2006). A basic sensitivity analysis with respect to asset returns, wage growth and risk aversion was also performed. The authors assumed the highest possible stocks proportions in the funds, i.e. 80%, 50%, 0% in the Growth, Balanced and Conservative Funds respectively. As we already mentioned in Introduction, the reality is different. Pension asset managers are very cautious and hold significantly lower proportions of stocks in their funds. In this section we discuss the sensitivity of the level of savings to the proportions of stocks in managed portfolios. Furthermore, we perform sensitivity analysis with respect to the correlation between stocks and bonds returns and the coefficient of risk aversion. At the end of this section we rigorously prove linearity of the level of savings with respect to the contribution rate denoted by τ.

3.1. Slovak Pension System and Calibration of the Model Parameters

According to Slovak legislature the percentage of salary transferred each year to a pension fund is τ =9%. We have assumed the period $T = 40$ of saving. The forecast for the expected wage growth ρ_t in Slovakia has been taken from Kvetan et al. (2007). The term structure of the wage growth $\{\rho_t,\ t = 1, \cdots, T\}$ from 2007 to 2046 is shown in Table 2.

Table 2
Wage Growth Prognosis in Slovakia

Period	Wage growth ρ_t (in %)
2007 – 2010	7.0
2011 – 2015	7.1
2016 – 2020	6.4
2021 – 2025	5.9
2026 – 2030	5.6
2031 – 2035	5.2
2036 – 2040	4.9
2041 – 2046	4.5

Source: Kvetan et al. (2007).

Following a usual assumption on the log-normal behavior of asset prices we shall assume that asset returns are normally distributed, i.e. $r = \mu + \sigma Z$ where μ and σ are the mean value and volatility of asset returns and $Z \sim N(0, 1)$ distributed random variable. Stocks have been represented by the S&P500 Index. For the calibration we have taken the same time period (January 1996 – June 2002) as in Kilianová et al. (2006) with average return $\mu^S = 10.28\%$ and standard deviation $\sigma^S = 16.90\%$. Bonds have been represented by ten years US governmental bonds (January 1996 – June 2002) with average return $r^B = 5.16\%$ and standard deviation $\sigma^B = 0.82\%$. According to empiric studies (see David and Veronesi, 2008; Li, 2002) the correlation between stocks and bonds is influenced by the correlation determined by the uncertainty about the expected inflation together with unexpected inflation and the real interest rate on the market. This paper has no intention to predict these macroeconomic variables and therefore we have used the correlation determined from the historical data $corr = -0.1151$. Each pension asset manager has in its portfolio three funds: F_1 – Growth Fund, F_2 – Balanced Fund, F_3 – Conservative Fund. These funds consist of an α-part stocks and correspondingly $(1 - \alpha)$-part of bonds. Then returns and variance can be calculated from formulae

$$r_j = \alpha r^S + (1-\alpha) r^B, \quad \sigma_j^2 = \alpha^2 (\sigma^S)^2 + 2\alpha(1-\alpha)\sigma^S \sigma^B corr + (1-\alpha)^2 (\sigma^B)^2$$

for $j = 1, \ldots, m$.

Finally, let us mention that pension asset management managers are charging the following fees:

- Management fee (1% from the monthly contribution),
- Fee for the administration of the account (0.07% from the average monthly net asset value of the fund).

Therefore we have considered the effective contribution rate τ = 8.91% (= 9%*0.99). The fee for the administration of the account (0.84% p.a.) has been subtracted from the yearly asset returns. Our goal is to estimate the mean value $E(d_t)$ and the risk represented by the standard deviation $\sigma(d_t)$ during the period $t = 1, 2, ..., T$.

3.2. Different Stock Proportions for the Growth and Balanced Funds

The legislation restrictions shown in Table 1 are not completely exploited by pension asset managers. In Table 3 we show an overview of the current state of the funds, regarding the stock proportion in each of them.[2]

Table 3

Structure of Stocks and Bonds Proportions (in %)

	AXA		CSOB		AEGON		ING		ALIANZ		VUB	
	S	B	S	B	S	B	S	B	S	B	S	B
F_1	22.40	77.60	21.05	78.05	27.29	72.71	20.20	79.80	22.70	77.30	22.27	77.73
F_2	17.40	82.60	17.14	82.86	23.34	76.66	14.80	85.20	17.40	82.60	16.99	83.01
F_3	0	100	0	100	0	100	0	100	0	100	0	100

Note: F_1 – Growth fund, F_2 – Balanced fund, F_3 – Conservative fund, S – % representation of stocks, B – % representation of bonds, money market instruments and other assets.

Source: Monthly reports of pension asset management companies for 31. 01. 2008

Returns and the average structure of these funds can be seen in Tables 4 and 5.

Table 4

Overview of the Returns for the Period from February 2, 2007 to February 2, 2008 (in %)

	AXA	CSOB	AEGON	ING	ALIANZ	VUB
F_1	0.676	0.464	1.634	2.549	0.410	–1.175
F_2	1.555	0.857	2.246	2.920	1.247	0.305
F_3	3.800	4.611	3.988	3.469	4.279	3.927

Source: National bank of Slovakia (2008).

Table 5

Average Proportions of Asset Classes

Type of Fund	Stocks (in %)	Bonds and Money Market Instruments (in %)
Growth	22.65	77.35
Balanced	17.85	82.15
Conservative	–	100

Note: Calculated as the average of values in Table 3.

Source: Our calculations.

[2] The real distribution of savings to the Growth, Balanced and Conservative funds and the performance analysis of the funds in first years after the pension reform can also be found in Miťková et al. (2007).

In our simulations we examine the path of the mean wealth $E(d_t)$ and its standard deviation $\sigma(d_t)$ in three scenarios:

a) Stocks representing the upper limits shown in Table 1,

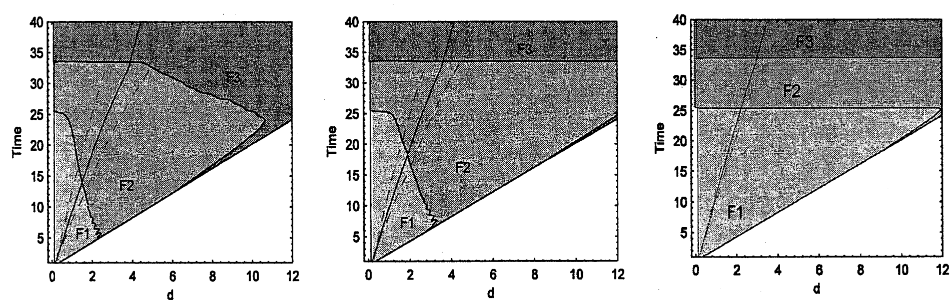
b) Lower proportions of stocks comparing to the limits shown in Table 1,

c) Current state of stocks proportions observed from the monthly reports of the pension asset management companies summarized in Table 5.

In Figure 1 and Table 6 we present results of simulations for the scenarios. The curvilinear solid line in Figure 1 represents the path of mean wealth $E(d_t)$ obtained by 10 000 simulations and here we use the risk aversion parameter $a = 9$. The dashed curvilinear lines correspond to $E(d_t) \pm \sigma(d_t)$ intervals where $\sigma(d_t)$ is the standard deviation of the random variable d_t. For scenario 1, savers mean wealth at the end of the saving period ($T = 40$) is 4.33 times his last year gross salary. The interval of one standard deviation is [3.27, 5.39]. In this case the optimal strategy for a saver is to use all three types of funds. One can conclude, that with decreasing stock proportions in both funds (Growth, Balanced) saver's mean wealth $E(d_T)$ and its standard deviation $\sigma(d_t)$ has a decreasing tendency (scenario 2, scenario 3), despite the longer period during which he stays in the Growth Fund. Scenario 3 with contemporary average proportions of stocks in the pension funds leads to substantially lower level of final mean wealth $E(d_T)$. Taking into account the current financial crisis, these cautious strategies are understandable. On the other hand, the stock proportions in the pension funds should be inevitably increased in the future. From Figure 1 one can observe that results for scenario 3 represent a strategy where the saver uses very simple decision rule that is independent of the development of asset returns (Growth Fund in the first 25 years, Balanced Fund in the next 8 years and Conservative Fund in the last 7 years) and it is following only prescribed law regulations.

Figure 1

Level of Savings for Different Stocks to Bonds Proportions in Portfolios



Scenario 1 Scenario 2 Scenario 3

Source: Our calculations.

Table 6

Results of Simulations for Different Stocks to Bonds Proportions in Portfolios

Scenario	Mean $E(d_T)$	StdDev $\sigma(d_t)$	Switch F_1-F_2	Switch F_2-F_3	F_1		F_2		F_3	
					S	B	S	B	S	B
1	4.33	1.06	14	33	80	20	50	50	–	100
2	4.04	0.79	18	33	60	40	37.5	62.5	–	100
3	3.43	0.31	25	33	22.65	77.35	17.85	82.15	–	100

Source: Our calculations.

3.3. Correlation between Stocks and Bonds Returns

From historical data, the estimated correlation *corr* = –0.1151 shows very small dependence of the price movements of stocks compared to bonds. In Table 7 we recall the classification of the correlation according to Cohen (1988).

Table 7

Types of Correlation

Correlation	Negative	Positive
Small	[–0.29, –0.10]	[0.10, 0.29]
Middle	[–0.49, –0.30]	[0.30, 0.49]
Large	[–1, –0.5]	[0.5, 1]

Source: Our calculations.

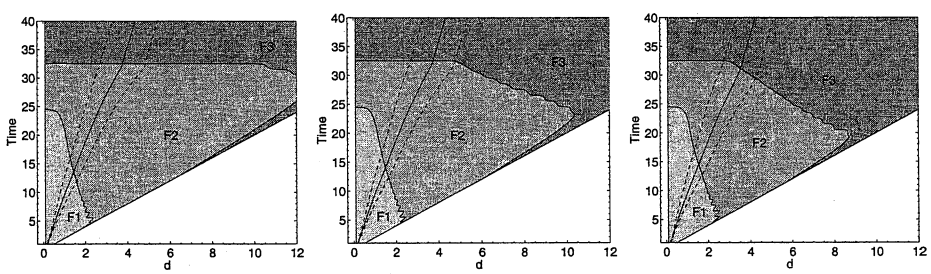
In our simulations we examine three different examples with the following correlation for all types of funds

- Negative (*corr* = –0.8),
- Zero (*corr* = 0),
- Positive (*corr* = 0.8).

The results of calculations with the risk aversion coefficient a = 9 and scenario 1 form Table 6 are summarized in Figure 2 and Table 8.

Figure 2

Results of Simulations for Different Correlations between Stocks and Bonds Returns



corr = –0.8 *corr* = 0 *corr* = +0.8

Source: Our calculations.

Table 8
Level of Savings and Switching Times for Different Correlation

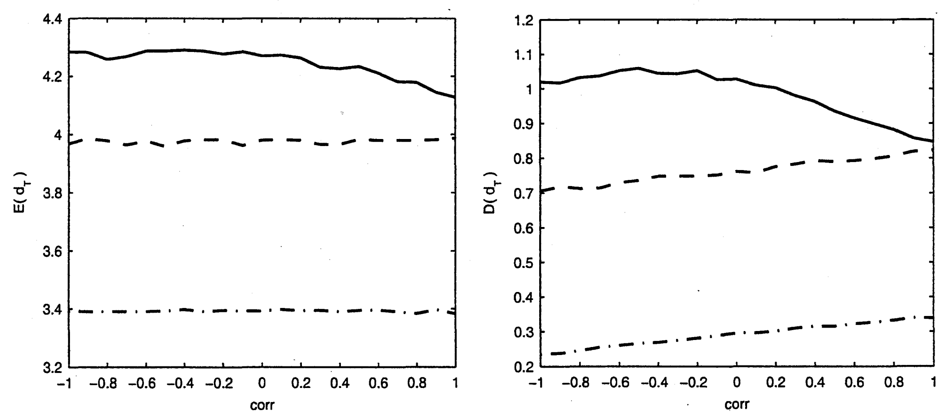
Correlation		$E(d_T)$	$\sigma(d_T)$	Switch F_1-F_2	Switch F_2-F_3
Negative	–0.8	4.29	1.05	14	32
Zero	0	4.27	1.03	14	32
Positive	0.8	4.17	0.88	15	31

Source: Our calculations.

One can see that results of our simulations are not very sensitive with respect to the correlation between stock and bond returns. The highest expected level of savings is achieved with negative correlation. This is in accord with economic intuition because negative correlation leads to better diversification of the portfolio implying thus reduction of the risk.

It is interesting to run a mean variance analysis for the possible outcome of $E(d_T)$ and $\sigma(d_T)$ in order to examine how these variables are influenced by the correlation between stock and bond returns. For three different scenarios (see Tab. 6) we examine the path of $E(d_T)$ and $\sigma(d_T)$.

Figure 3
Mean and Standard Deviation of d_T as Functions of Stocks to Bonds Correlation



Note: Mean value $E(d_T)$ and standard deviation $\sigma(d_T)$ (labelled by $D(d_T)$) of the terminal value d_T as functions of correlation between stocks and bonds returns for various scenarios as described in Table 6. Solid line corresponds to scenario 1, dashed line to scenario 2 and dashed-dotted line to scenario 3.

Source: Our calculations.

The results are depicted in Figure 3. For scenario 1 it is clear that for increasing positive correlation between stock and bond returns, the expected saver's wealth $E(d_T)$ and its standard deviation $\sigma(d_T)$ are decreasing. The reason for such a behavior is that the funds became more risky and hence the saver tends to switch earlier to a less risky fund because of his aversion to risk. Scenarios 2 and

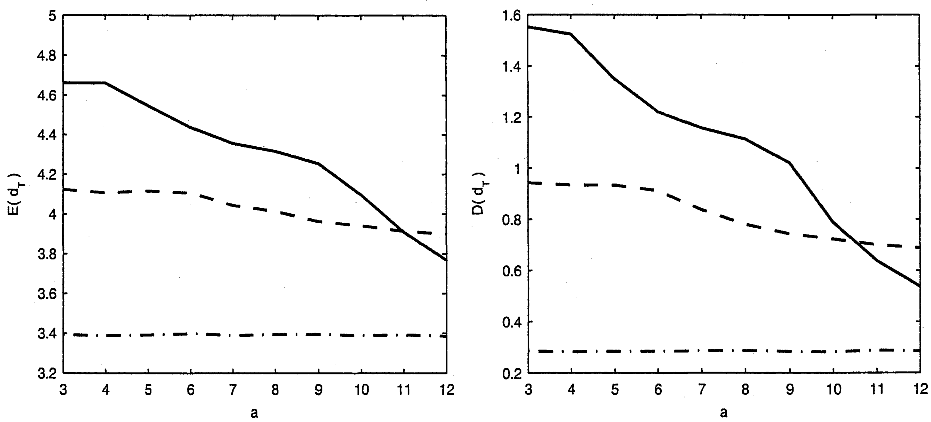
3 show a weak relationship between the correlation and $E(d_T)$. The reason for such a behavior is a small stock to bond proportion for corresponding funds. Consequently, the funds are less risky and thus the saver has no tendency to switch between them. If there are no switches between funds, the mean wealth is independent of the correlation.

3.4. Sensitivity to the Risk Aversion

The risk profile of the saver (characterized by his risk aversion coefficient a) can significantly influence the mean wealth during the saving period. With increasing risk aversion, the saver prefers a less risky fund and thus he reduces the potential growth. In this section we present a brief overview of results for the mean wealth $E(d_T)$ and its standard deviation $\sigma(d_T)$ as a function of the savers risk profile. The results are depicted in Figure 4. We use the same scenarios of the stock proportions as in the previous section (see Tab. 6).

Figure 4

Mean and Standard Deviation of d_T as Functions of the Risk Aversion Parameter



Note: Mean value $E(d_T)$ and standard deviation $\sigma(d_T)$ (labeled by $D(d_T)$) of the terminal value d_T as functions of correlation between stocks and bonds returns for various scenarios as described in Table 6. Solid line corresponds to scenario 1, dashed line to scenario 2 and dashed-dotted line to scenario 3.

Source: Our calculations.

Comparing scenarios 1 and 2 it is interesting to note that for a saver with a higher risk aversion $a > 11$ the stock structure according to scenario 2 is more suitable. The reason is that the saver can decide between all three funds. The structure of the funds in scenario 1 is not suitable for his risk profile because the saver is not willing to take the risk associated with the Growth Fund. This fact has an effect on his final mean wealth $E(d_T)$. On the other hand a risk-loving saver would prefer the structure of the funds according to scenario 1.

From scenario 3 it is clear, that saver's risk profile has almost no impact on the decision process. In such a scenario, the optimal choice is to stay in the Growth Fund and switch to the Balanced/Conservative Fund only because of legislation restrictions (i. e. 15 years before retirement, the saver can't stay in the Growth Fund and 7 years before the retirement, he must be in the Conservative Fund).

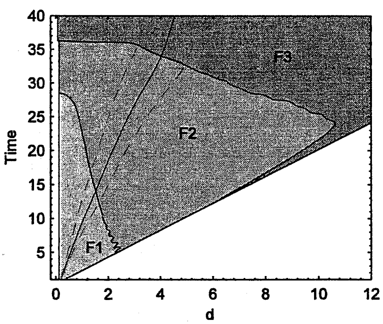
Another way how to take into account the risk aversion is to increase/decrease the limits for compulsory switch to less risky funds. In what follows, we present an example of two alternatives of possible modifications of prescribed switching limits:

a) The alternative 12/4. In this example the saver must not be in the Growth Fund 12 years before the end of the saving period, and he/she must be in the Conservative Fund 4 years prior the end of the saving period.

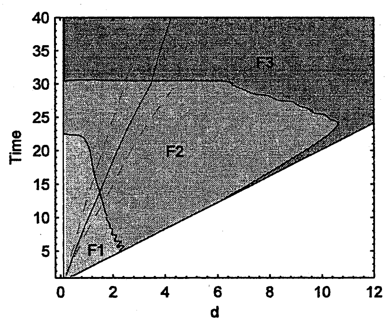
b) The alternative 18/10. The saver must not be the Growth Fund 18 years before the end of the saving period, and he/she must be in the Conservative Fund 10 years prior the end of the saving period.

In Figure 5 and Table 9 we present results of our calculations for alternatives a) and b).

Figure 5
Results of Simulations for Different Limits as Described in Alternatives 12/4 and 18/10



The alternative 12/4



The alternative 18/10

Source: Our calculations.

Table 9
The Level of Savings and Switching Times with Respect to Switching Limits

	$E(d_T)$	$\sigma(d_T)$	Switch $F_1 - F_2$	Switch $F_2 - F_3$
Alternative 12/4	4.44	1.04	14	34
Alternative 18/10	4.14	0.95	14	30
Original 15/7	4.33	1.06	14	33

Source: Our calculations.

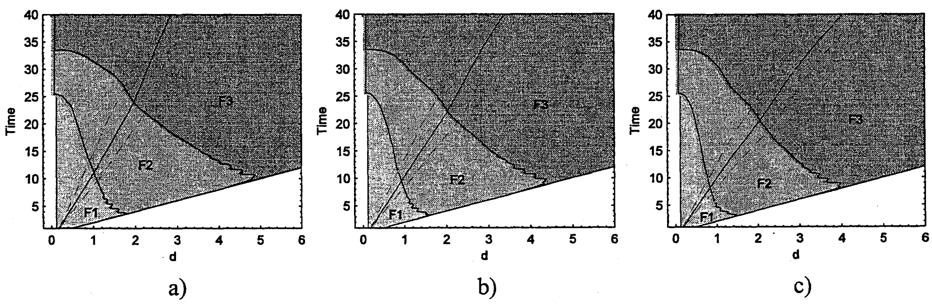
For both alternatives we use the same risk aversion coefficient a = 9 and stock to bond proportions as in scenario 1 from Table 6. Our computational results show that, independently of whether restriction for the Growth Fund increases from 15 to 18 or decreases from 15 to 12 years, the optimal strategy is to stay in the Growth Fund "just" first 14 years of saving. On the other hand, modification of the second restriction for staying in the Conservative Fund leads to different results. Increasing the restriction from 7 to 10 years leads to activation of the switch between F_2 and F_3 funds. Moreover, increasing the restriction from 7 to 10 years yields lower expected level of savings $E(d_T) = 4.14$ compared to the original restriction 15/7.

3.5. Adjusted Asset Returns and Wage Growth

Due the impact of the global financial crisis and the change in global economy we present results of our model for reduced growth potential for the stock return for S&P 500 Index by 4 percentage points and the bond return for the 10 year USA Government bonds by 2 percentage points. Because the Slovak economy is an open and export oriented economy, we have also considered reduction of the wage growth potential for Slovakia as well. Reducing the wage growth in Table 2 by 1, 2 and 3 percentage points for each corresponding period, we show the effect of reduction on the savers mean wealth $E(d_t)$ and its standard deviation $\sigma(d_t)$ during and at the end of the saving period. Results are shown in Figure 6 and Table 10.

Figure 6

Results of Simulations for Decreased Stock and Bond Returns with Adjusted Wage Growth



Source: Our calculations.

Table 10

Level of Savings and Switching Times for Different Wage Growth Scenarios

Scenario	$E(d_T)$	$\sigma(d_T)$	Switch $F_1 - F_2$	Switch $F_2 - F_3$	Wage Growth
a)	2.82	0.32	11	24	–1 %
b)	3.30	0.38	10	22	–2 %
c)	3.92	0.44	8	21	–3 %

Source: Our calculations.

In our calculations we use the same risk aversion coefficient $a = 9$ and stock to bond proportions as in scenario 1 from Table 6. The results are in correspondence with an economic intuition stating that decreased stock and bond returns imply decreased expected levels of savers mean wealth $E(d_T)$. However the decreased wage growth increases the savers mean wealth $E(d_T)$.

3.6. Contributions

In what follows, we shall assume saver's utility function is the CRRA function having the form $U(d) = -d^{1-a}$ where $a > 1$. The aim of this section is to prove that the level of savings is proportional to the contribution rate in the proposed model.[3]

By a backward mathematical induction for $t = T, T-1, \ldots, 1$, we shall prove that the value function $V_t(d)$ satisfies $V_t^{\lambda\tau}(\lambda d) = \lambda^{1-a} V_t^{\tau}(d)$ for any t, d and $\lambda > 0$. Since $U(d) = -d^{1-a}$ the statement is obvious for $t = T$. Now suppose that $V_{t+1}^{\lambda\tau}(\lambda d) = \lambda^{1-a} V_{t+1}^{\tau}(d)$. As

$$F_t^{\lambda\tau}(\lambda d, j) = \lambda d \frac{1 + r_t^j}{1 + \rho_t} + \lambda\tau = \lambda F_t^{\tau}(d, j)$$

we have

$$\begin{aligned} V_t^{\lambda\tau}(\lambda d) &= \max_{j \in \{1,2,\ldots,m\}} E\left[V_{t+1}^{\lambda\tau}(F_t^{\lambda\tau}(\lambda d, j))\right] \\ &= \max_{j \in \{1,2,\ldots,m\}} E\left[V_{t+1}^{\lambda\tau}(\lambda F_t^{\tau}(d, j))\right] \\ &= \max_{j \in \{1,2,\ldots,m\}} E\left[\lambda^{1-a} V_{t+1}^{\tau}(F_t^{\tau}(d, j))\right] \\ &= \lambda^{1-a} V_t^{\tau}(d) \end{aligned}$$

and statement follows for the time t. It is worth to note, that the argument $j^{\lambda\tau}(t, \lambda d)$ of the maximum of the expected value

$$\max_{j \in \{1,2,\ldots,m\}} E\left[V_{t+1}^{\lambda\tau}(F_t^{\lambda\tau}(\lambda d, j))\right] = \lambda^{1-a} \max_{j \in \{1,2,\ldots,m\}} E\left[V_{t+1}^{\tau}(F_t^{\tau}(d, j))\right]$$

is independent of $\lambda > 0$, i. e. $j^{\lambda\tau}(t, \lambda d) = j^{\tau}(t, d)$. Again, by a forward mathematical induction for $t = 1, \ldots, T$, the stochastic variable d_t^{τ} defined recursively

[3] This statement holds also for $0 < a \leq 1$. However, in our calculations only values $a > 1$ have been used. To simplify the proof, we have considered only these values.

$$d_{t+1}^{\tau} = F_t^{\tau}(d_t^{\tau}, j^{\tau}(t, d_t^{\tau})) = d_t^{\tau}\frac{1+r_t^{j^{\tau}(t,d_t^{\tau})}}{1+\rho_t} + \tau, \quad t = 1, 2, \ldots, T-1$$

$$d_1^{\tau} = \tau$$

satisfies

$$d_t^{\lambda\tau} = \lambda d_t^{\tau}$$

Therefore, the level of savings considered as a stochastic variable is proportional to the contribution rate. As a consequence we obtain, in particular, the mean value $E(d_t^{\tau})$ of the saved sum d_t^{τ} is a linear function of the parameter $\lambda > 0$, i.e.

$$E(d_t^{\lambda\tau}) = \lambda E(d_t^{\tau})$$

for any $t = 1, \ldots, T$ and $\alpha > 0$. It means that the averaged saved sum is always a linear function of the contribution rate $\tau > 0$ as

$$E(d_t^{\tau}) = \frac{\tau}{0.09} E(d_t^{0.09})$$

Conclusions

We have performed a qualitative and quantitative analysis of the dynamic stochastic accumulation model proposed in Kilianová et al. (2006). It turned out that pension asset managers are very cautious and hold the stocks proportions in pension portfolios substantially below the law limits. Therefore, a new sensitivity analysis was necessary. We have performed the sensitivity analysis for various stocks proportions in the corresponding funds including contemporary situation influenced by financial crisis. We have concluded that stocks proportions should be increased in the future.

The level of savings is not very sensitive with respect to correlation between stocks and bonds returns. The best results were achieved for negative correlation.

Using the contemporary proportions of stocks in the portfolios, one can conclude, that future pensioner tends to stay in the Growth and Balanced Funds as long as possible (respecting the law limits) regardless of his/her risk profile. For higher stocks proportions the expected level of savings as well as risk (represented by the standard deviation) increases with decreasing aversion to risk.

Finally, we have proved that in the proposed model, the level of savings as a stochastic variable is proportional to the contribution rate.

References

BODIE, Z. – DETEMPLE, J. B. – OTRUBA, S. – WALTER, S. (2004): Optimal Consumption-portfolio Choices and Retirement Planning. Journal of Economic Dynamics & Control, *28*, No. 6, pp. 1115 – 1148.

BODIE, Z. – MERTON, R. – SAMUELSON, W. (1992): Labor Supply Flexibility and Portfolio Choice in a Life-cycle Model. Journal of Economic Dynamics & Control, *16*, No. 3 – 4., pp. 427 – 449.

COHEN, J. (1988): Statistical Power Analysis for the Behavioral Sciences. 2nd ed. Hillsdale, NJ: Lawrence Erlbaum Associates.

DAVID, A. – VERONESI, P. (2008): Inflation and Earnings Uncertainty and Volatility Forecasts: A Structural Form Approach (March 18, 2008). [Chicago GSB Research Paper; University of Calgary Haskyane School of Business Working Paper.] Available at SSRN: <ssrn.com/abstract=507122 >.

FRIEND, I. – BLUME, M. E. (1975): The Demand for Risky Assets. The American Economic Review, *65*, No. 5, pp. 900 – 922.

GOLIAŠ, P. (2003): Pension Calculations for the PAYG and the Funded Pension System in Slovakia. Bratislava: Academia Istropolitana Nova, Professional Programme in Applied Economics and Finance.

KILIANOVÁ, S. – MELICHERČÍK, I. – ŠEVČOVIČ, D. (2006): Dynamic Accumulation Model for the Second Pillar of the Slovak Pension System. Finance a úvěr – Czech Journal of Economics and Finance, *56*, No. 11 – 12, pp. 506 – 521.

KVETAN, V. – PÁLENÍK, V. – MLÝNEK, M. – RADVANSKÝ, M. (2007): Starnutie, zdravotný stav a determinanty výdavkov na zdravie v podmienkach Slovenska. [Výskumné štúdie 1.] Bratislava: Ekonomický ústav SAV. ISBN: 978-80-7144-160-1.

LI, L. (2002): Macroeconomic Factors and the Correlation of Stock and Bond Returns. [Working Paper, No. 02-46.] Yale: ICF.

MEHRA, R. – PRESCOTT, E. (1985): The Equity Premium: A puzzle. Journal of Monetary Economics, *15*, No. 2, pp. 145 – 161.

MELICHERČÍK, I. – UNGVARSKÝ, C. (2004): Pension Reform in Slovakia: Perspectives on the Fiscal Debt and Pension Level. Finance a úvěr – Czech Journal Of Economics and Finance, *54*, No. 9 – 10, pp. 391 – 404.

MERTON, R. (1969): Lifetime Portfolio Selection under Uncertainty: The Continuous-time Case. Review of Economics and Statistics, *51*, No. 3, pp. 247 – 257.

MIŤKOVÁ, V. – MLYNAROVIČ, V. – TUŠ, B. (2007): A Performance and Risk Analysis on the Slovak Private Pension Funds Market. Ekonomický časopis/Journal of Economics, *55*, No. 3, pp. 232 – 249.

NBS (2008): Available at: <http://www.nbs.sk/sk/dohlad-nad-financnym-trhom/dohlad-nad-dochodkovym-sporenim/ aktualna-hodnota-dochodkovej-jednotky-a-cista-hodnota-majetku-v-dochodkovom-fonde/vyvoj-aktualnej-hodnoty/aktualna-hodnota-dochodkovej-jednotky>.

PRATT, J. W. (1964): Risk Aversion in the Small and in the Large. Econometrica, *32*, No. 1 – 2, pp. 122 – 136.

SAMUELSON, P. (1969): Lifetime Portfolio Selection by Dynamic Stochastic Programming. The Review of Economics and Statistics, *51*, No. 3, pp. 239 – 246.

THOMAY, M. – ŠVEJNA, I. – ORAVEC, J. (2002): Koncepcia reformy dôchodkového systému. Bratislava: The F. A. Hayek Foundation.

YOUNG, H. P. (1990): Progressive Taxation an Equal Sacrifice. The American Economic Review, *80*, No. 1, pp. 253 – 266.