

# Parental bonus in pension systems: The case of Slovakia

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## Abstract

Population aging and low birth rates are linked to the problem of unsustainability of ongoing pension systems. As demographic predictions follow unfavorable developments, adjusting such pension systems is inevitable. This paper discusses introducing child-related benefits into pension system models and their advantages and disadvantages. The model with child-related pension benefits dependent on the average wage is examined concerning the effects of the child factor on individual fertility and private savings. Subsequently, we estimate the size of the child factor in the current setting of Slovakia's pension system and several other alternatives. Finally, the optimal setting of the above pension system model is presented and compared with the presented alternatives. We show that the current setting of the pension system can be brought closer to the optimum by, for example, more generous awarding of personal wage points for raising children.

## Introduction

The long-term trend of population aging comes with wide-ranging economic implications. The progress and availability of health care for the general population increased the average life expectancy. However, the level of total fertility rate is far from 2.1, which is, according to [Craig \(1994\)](#), the level necessary to maintain the population in developed countries. Combined, these factors lead to several socio-economic problems, one of which is the sustainability of the pension systems.

Throughout history, resources were often reallocated from the active population to the elderly and children. In the past, the family held the role of the distributor ([Lee \(1994\)](#)). If an individual had no children to take care of her in her old age, she had to find the means to provide for herself, and children thus represented a form of pension security.

With the introduction of the state-managed pension systems, in which, if at all, the number of children is only minimally taken into account, the need to have children receded. In the literature, e.g., [Cigno and Werding \(2007\)](#); [Sinn \(2005\)](#); [van Groezen et al. \(2003\)](#), we even come across the idea that the very setting of the pension system can have an anti-population effect. Popular PAYG pension schemes are often dependent on an individual's working career. Numerous studies, e.g., [Bonnet and Rapoport \(2020\)](#); [Meurs et al. \(2010\)](#); [Németh and Szabó-Bakos \(2021\)](#), conclude that it is often more advantageous for women not to have children. The upbringing of children leads to a withdrawal from a work life, which leads to a lower income and consequently to a lower pension claim. We recommend studies [Möhring \(2015\)](#); [Neels et al. \(2018\)](#); [Peeters and De Tavernier \(2015\)](#); [Rutledge et al. \(2017\)](#) for more information on the topic.

Introducing the child allowances should at least partially cover these losses, but as stated in [Jankowski \(2011\)](#), the effectiveness of the caregiver programs is still in question, and the studies that have been done in this regard show mixed results (see [d'Addio \(2013\)](#); [Stahlberg et al. \(2006\)](#)). Still,

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fertility remains very low, which is not only an individual concern but affects the whole population as PAYG pension schemes are strongly linked to fertility.

The pension system's sustainability depends on its stability and the financial balance between contributions and expenditures. The primary financial source of the pension system is individual contributions in the form of regularly paid financial contributions from earned wages. Therefore the stability of such a pension scheme lies within a continuous replacement of the generations. That way, the new contributors can cover the retirees' costs. The upbringing of an offspring thus becomes as essential of a contribution to the pension system as the payment of contributions itself. Both forms of contributions to the pension scheme should then be adequately assessed when calculating an individual's pension benefit. This idea brings us to the so-called child pension models as mentioned in [Sinn \(2005\)](#). This type of pension scheme introduces child-related benefits in various ways. It seems to positively affect fertility, considering several empirical studies [Gábos et al. \(2009\)](#); [Giday and Szegő \(2018\)](#). Another study [Thiemann \(2015\)](#) also shows that introducing child benefits increases maternal old-age income without a negative effect on employment. According to [Fenge and Weizsäcker \(2010\)](#), the child pension scheme should not wholly replace the PAYG pension systems, but their combination may be the way to solve the sustainability problem.

As of January 2023, Slovakia introduced parental pension into the pension benefit scheme, see [Law no. 461/2003 Coll. on social insurance as amended](#). Children can contribute 3% of their average monthly wage to increase their parent's pension benefits (the benefits are equally divided between both parents). Such change may be considered as a try to implement child related pension scheme to compensate parents for the costs of bringing up a child. However, the question of whether it is a meaningful change remains.

The paper is structured as follows: in the first part, two simple models of the pension system with child benefits are presented. We then look at the properties of the average wage benefit model. Finally, we will estimate the child factor for the current setting of Slovakia's first pillar of the pension system and its various adjustments. The results are compared with the optimal child factor calculated using the presented model.

## Two pension scheme models with child factor

Child pension models can be designed differently, including introducing a parental bonus into the pension scheme. This paper considers models directly implementing the child factor into the pension formula.

Let's start with a simple pension model with three overlapping generations. For simplicity, we consider a one-sex generation consisting of women. The first period represents childhood when the individual is inactive and dependent on parental care. In the second period,  $t$ , she works for a wage of  $w_t$  and contributes  $\tau_t w_t$  to the pension system, where  $\tau_t$  is the contribution rate. In the third period, she receives a pension. Assuming the pension system, where raising children has a direct impact on the final pension income, we will set the pension benefit in the following form:

$$p_{t+1} = B_{t+1} \frac{w_t}{\bar{w}_t} + \alpha \tau_{t+1} W_{t+1}, \quad (1)$$

where  $W_{t+1}$  represents the wages of all children of the individual. The benefit thus consists of two parts. The first part  $B_{t+1} \frac{w_t}{\bar{w}_t}$  is based on the ratio of the individual's wage  $w_t$  to the average wage  $\bar{w}_t$  of a given generation at time  $t$ . It reflects the workload of an individual in an economically active period.

The second component containing the child factor  $\alpha$  is the entitlement to a pension benefit depending on the sum of wages of the offspring of an individual  $W_{t+1}$ . Such a setting reflects the newly defined parental pension in the pension system in Slovakia, where the child-related part of the pension benefit is linked to the wages of the individual's children.

The contribution of these children to the pension system is  $\tau_{t+1} W_{t+1}$ . Comparing the mean values of contributions to the pension system  $\tau_{t+1} \bar{W}_{t+1}$  and expenses  $B_{t+1} \frac{\bar{w}_t}{\bar{w}_t} + \alpha \tau_{t+1} \bar{W}_{t+1}$ , the value of the

balance coefficient  $B_{t+1}$  is obtained

$$B_{t+1} = (1 - \alpha) \tau_{t+1} \overline{W}_{t+1}.$$

Pension benefit (1) is then reformulated as

$$p_{t+1} = \tau_{t+1} \overline{W}_{t+1} \left[ (1 - \alpha) \frac{w_t}{\overline{w}_t} + \alpha \frac{W_{t+1}}{\overline{W}_{t+1}} \right], \quad (2)$$

and the effect of the child factor  $\alpha$  on the pension benefit is described by the expression

$$\frac{\partial p_{t+1}}{\partial \alpha} = \tau_{t+1} \overline{W}_{t+1} \left[ \frac{W_{t+1}}{\overline{W}_{t+1}} - \frac{w_t}{\overline{w}_t} \right].$$

Hence the increase of child factor  $\alpha$  reduces the pension benefit claim if  $\frac{W_{t+1}}{\overline{W}_{t+1}} < \frac{w_t}{\overline{w}_t}$ , i.e. when the total wage of the children in relation to its average value is lower than the ratio of the parent's salary to its generational average. This is necessarily fulfilled if the individual has no child, if the child has died, or when her children are unemployed or have gone abroad and therefore do not report any income. Thus, Slovakia's newly introduced parental pension may lead to unequal claiming rights for parents falling under the mentioned categories.

To suppress these negative consequences, the pension benefit formula is modified as follows

$$p_{t+1} = B_{t+1} \frac{w_t}{\overline{w}_t} + \alpha \tau_{t+1} n_{t+1} \overline{w}_{t+1}, \quad (3)$$

where  $n_{t+1}$  is the number of individual's children. The first part of the pension benefit formula remained the same as before. On the other hand, the second component containing the child factor  $\alpha$  implies the entitlement to a pension benefit solely depending on the individual's fertility  $n_{t+1}$  and the contribution rate  $\tau_{t+1}$  from the average wage of overall offspring generation  $\overline{w}_{t+1}$ . Therefore, the child-related part of the pension benefit is not totally individually based but uses the average wage.

According to (3), the pension benefit could indicate the child factor to be favorable for anyone with at least one child. However, given the condition of a balanced pension system, we will see that this advantage will not be so apparent. Taking the mean values of the contributions of the offspring generation and the pension benefits (3), the balance coefficient may be expressed as

$$B_{t+1} = \tau_{t+1} \overline{W}_{t+1} - \alpha \tau_{t+1} \overline{n}_{t+1} \overline{w}_{t+1}, \quad (4)$$

Let us denote  $N_t$  the absolute number of members of the generation  $t$ . Then the average fertility of the generation  $t$  can be expressed as the ratio of members of the offspring generation to the number of the members of the parental generation, i.e.,  $\overline{n}_{t+1} = N_{t+1}/N_t$ . Thus, the following holds  $\overline{W}_{t+1} = \frac{N_{t+1} \overline{w}_{t+1}}{N_t} = \overline{n}_{t+1} \overline{w}_{t+1}$  and using (4) the value of the coefficient  $B_{t+1}$  may be expressed as

$$B_{t+1} = (1 - \alpha) \tau_{t+1} \overline{n}_{t+1} \overline{w}_{t+1}.$$

For the pension benefit (3) then one has

$$p_{t+1} = \tau_{t+1} \overline{w}_{t+1} \left[ (1 - \alpha) \overline{n}_{t+1} \frac{w_t}{\overline{w}_t} + \alpha n_{t+1} \right]. \quad (5)$$

The total effect of the child factor  $\alpha$  on the pension benefit can be expressed as

$$\frac{\partial p_{t+1}}{\partial \alpha} = \tau_{t+1} \overline{w}_{t+1} \left[ -\overline{n}_{t+1} \frac{w_t}{\overline{w}_t} + n_{t+1} \right] = \tau_{t+1} \overline{n}_{t+1} \overline{w}_{t+1} \left[ \frac{n_{t+1}}{\overline{n}_{t+1}} - \frac{w_t}{\overline{w}_t} \right].$$

The effect of the child factor on the pension benefit depends on the sign of the expression  $\frac{n_{t+1}}{\overline{n}_{t+1}} - \frac{w_t}{\overline{w}_t}$ . The child factor thus positively affects the pension benefit of individuals whose individual fertility is higher than the average, but at the same time, their salary does not exceed the average value of their generation. Similarly, more than high fertility may be required for the child factor to increase

the value of the pension benefit if the individual is a high-earner. One can also observe that if an individual's wage is  $k$ -times the average wage of a given generation, then her fertility must be more than  $k$ -times the average fertility for the child factor to have a positive effect on her pension benefit.

In comparison to pension benefit (2), the latter defined pension benefit (5) consists of the parental allowance based on individual fertility. However, its final value depends on the average wage of the offspring generation. It, therefore, does not have the shortcomings of the model where the pension benefit depends on the wages of one's children. Likewise, the average wage-scaled pension benefit is close to the setting of the pension system of the Slovakia, where during parental leave, the parent is allocated a constant 0.6 personal wage point (the ratio of the individual's and average salary) as the compensation benefit. In January 2023, a parental pension was added to this compensation, which can be modeled using equation (2).

## Properties of the model with average wage related pension benefit

In this section, we follow up on the results of [Fenge and Meier \(2005\)](#). The model framework consists of three periods of overlapping generations, assuming the identity of all individuals in a given generation. In the first period of life, the individual depends on parental care.

Following the transition to an economically active period, one earns wage  $\tilde{w}_t$ . The wage  $\tilde{w}_t$  is reduced by pension contribution rate  $\tau_t$ . If the individual in the second period decides to raise  $n_{t+1}$  children, the wage is further reduced by the factor  $1 - f(n_{t+1})$ , where  $f(n_{t+1})$  is the loss function representing the lost wage due to the raising  $n_{t+1}$  children, that satisfies  $f(0) = 0$ ,  $f'(n_{t+1}) > 0$  and  $f''(n_{t+1}) \geq 0$ . In the first period, individuals' disposable income is divided between consumption  $c_t$  and personal savings  $s_t$ .

In the last period of life, the individual no longer works and receives the pension benefit

$$p_{t+1}^* = \tau_{t+1} \tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1 - f(n_{t+1})}{1 - f(\bar{n}_{t+1})} + \alpha n_{t+1} \right]. \quad (6)$$

Therefore in the old age period, the consumption  $z_{t+1}$  of an individual consists of the pension benefit  $p_{t+1}^*$  and the individual savings multiplied by the interest rate factor  $R_{t+1}$ .

Supposing that the individual maximizes her lifetime utility  $U$ , the pension model in [Fenge and Meier \(2005\)](#) then considers the optimization problem

$$\begin{aligned} \max_{s_t, n_{t+1}} \quad & U(c_t, z_{t+1}, n_{t+1}) \\ \text{s.t.} \quad & c_t + s_t = (1 - \tau_t) [1 - f(n_{t+1})] \tilde{w}_t, \\ & z_{t+1} = R_{t+1} s_t + p_{t+1}^*, \end{aligned} \quad (7)$$

where  $\tilde{w}_t$  represents the wage of a childless individual. The lifetime utility function  $U(c_t, z_{t+1}, n_{t+1})$  is considered to be continuous, strictly increasing, and concave in each of the underlying arguments. Moreover, we assume the lifetime utility function to exhibit additive separability.

Using the results of [Fenge and Meier \(2005\)](#), we will present the properties of a model based on the pension benefit (5) linked through the child factor to the average wage of the offspring generation. According to our framework  $w_t = \tilde{w}_t [1 - f(n_{t+1})]$ . Assuming the homogenous wage in the population, i.e.  $\tilde{w}_t = \bar{w}_t$ , it follows  $\overline{\tilde{w}_t [1 - f(n_{t+1})]} = \bar{w}_t [1 - \overline{f(n_{t+1})}]$  and  $\overline{\tilde{w}_{t+1} [1 - f(n_{t+2})]} = \bar{w}_{t+1} [1 - \overline{f(n_{t+2})}]$ . Then the pension benefit declared in (5) may be rewritten as

$$p_{t+1} = \tau_{t+1} \bar{w}_{t+1} [1 - \overline{f(n_{t+2})}] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1 - \overline{f(n_{t+1})}}{1 - \overline{f(\bar{n}_{t+1})}} + \alpha n_{t+1} \right].$$

According to the Jensen's inequality for the convex function,  $f(\bar{n}_{t+1}) \leq \overline{f(n_{t+1})}$  holds true for any loss function  $f(n_{t+1})$ . Assuming a homogeneous population with  $n_t = \bar{n}_t$  or linear form of the loss function, we obtain equality between  $f(\bar{n}_{t+1})$  and  $\overline{f(n_{t+1})}$ . In such a case, the average wage-related

pension benefit (5) can be rewritten in the form of the pension benefit from [Fenge and Meier \(2005\)](#). That way the pension benefit consists of the individual fertility related benefit with the share of the child factor  $\alpha$ . The other part with the share  $1 - \alpha$  represents the individual work related claim on the pension system with respect to the average state of the economy.

Since our average wage-related pension benefit model is consistent with the Fenge-Meier model under the stated assumptions, we can use results from [Fenge and Meier \(2005\)](#). These results may be obtained by analyzing the optimization problem (7). First, we consider a homogeneous population with  $n_t = \bar{n}_t$ .

Analysis of the necessary optimality conditions leads to the following effect of the child factor on individual fertility:  $\frac{\partial n_{t+1}}{\partial \alpha} > 0$ . As expected, an increase in the child factor  $\alpha$  leads to a structural change in the pension system with a greater emphasis on the part dependent on individual fertility. This ultimately leads to fertility growth. On the other hand, higher fertility and subsequent care for the offspring inevitably reduce an individual's overall wage.

The relationship between private savings and child factor is characterized by the derivative  $\frac{\partial s_{t+1}}{\partial \alpha}$ . Considering the properties of the utility function, the sign of the said derivative of private savings, according to the child factor, depends only on the value of  $U_{ns}$ . The personal savings are thus negatively affected by the increase of the child factor if the offspring and the savings are substitutes, i.e., when  $U_{ns} < 0$ . If a reduction in the part of the pension benefit dependent on a working career would sufficiently exceed the increase in the pension benefit linked to individual fertility, i.e., when

$$\alpha < (1 - \alpha)\bar{n}_{t+1} \frac{f'(n_{t+1})}{1 - f(\bar{n}_{t+1})},$$

then it may lead to an increase in private savings. Otherwise  $\frac{\partial s_{t+1}}{\partial \alpha} < 0$ .

Based on previous results, linking pension benefits to an individual fertility in the form of the child factor may increase the individual fertility itself. From a demographic point of view, this is a positive consequence. This leads to the question about the optimal value of the child factor  $\alpha$ . By analyzing the indirect utility function

$$V(\alpha) = U \left\{ (1 - \tau_t) [1 - f(n_{t+1}(\alpha))] w_t - s_t(\alpha), R_{t+1} s_t(\alpha) \right. \\ \left. + \tau_{t+1} w_{t+1} [1 - f(\bar{n}_{t+2}(\alpha))] \left[ (1 - \alpha) \bar{n}_{t+1}(\alpha) \frac{1 - f(n_{t+1}(\alpha))}{1 - f(\bar{n}_{t+1}(\alpha))} + \alpha n_{t+1}(\alpha) \right], n_{t+1}(\alpha) \right\}$$

[Fenge and Meier \(2005\)](#) conclude that setting the  $\alpha = 1$  is not optimal and does not maximize the utility of the individuals. On the other hand, in a steady state equilibrium with stationary sequence  $\{\tau_t, \alpha_t, w_t, R_t\}$  and  $n = n_{t+1} = \bar{n}_{t+1} = \bar{n}_{t+2}$ , if a given pension system does not involve any fertility-related pensions, then the introduction of the child factor leads to an increase in overall utility. At the same time, if there is an internal solution, the optimal value of the child factor meets the following condition

$$\alpha^* = \frac{1 - f(n(\alpha^*))}{1 - f(n(\alpha^*)) + n(\alpha^*) f'(n(\alpha^*))}. \quad (8)$$

Since the lefthand side of the previous expression (8) is increasing in  $\alpha^*$  while the right-hand side decreases in  $\alpha^*$ , the optimal  $\alpha^*$  is unique.

## Child factor in Slovakia

The Slovak pension system is based on two pillars. The first pillar is Pay-As-You-Go and defined benefit, the second pillar is savings, defined contribution. Pension system participants can use only the first pillar, or both. In the second case, the pension from the first pillar is reduced proportionally. For simplicity, we assume the existence of only one pillar throughout this article. Even in such a setting, we can make relevant conclusions regarding the child factor in the pension system in Slovakia.

A parental pension has been implemented in Slovakia since January 2023. However, even before that, this pension system had some form of parental bonus. The monthly pension benefit in Slovakia

is calculated using the following formula

$$D = APWP * CP * CPV. \quad (9)$$

The variable  $CPV$  corresponds to the current pension value. Next, the  $CP$  stands for the length of the pension contribution period. This also includes the period of parental leave. Finally, the variable  $APWP$  represents the average personal wage point, which is calculated as follows

$$APWP = \frac{\text{sum of the personal wage points in the reference period}}{\text{number of years of the pension insurance period}}.$$

Personal wage points ( $PWP$ ) are further given as a ratio of the personal wage assessment base and the general assessment base in a given year.

To calculate the implied child factor in the Slovak pension system, we assume the pension benefit is consistent with the pension benefit formula (6). Therefore the assumption about the homogenous wage  $\tilde{w}_t$  is used. Compared to the actual setting of the pension system in Slovakia, which is the combination of the pension system models presented in the first section, this is a simplification. Still, the results can give us an insight into how the modification of the pension system can impact the child factor, fertility, and consumption. We calculate the implied values of the child factor for several settings of the pension system in Slovakia.

#### Case 1

To compensate for the lost wage due to a child's upbringing, the Slovak pension system includes an artificial assignment of a constant personal wage point, namely 0.6 per year of parental leave. The bonus may be claimed till the child reaches the age of six.

Let two individuals have the same value of  $CP = 40$  years, corresponding to the number of years of pension insurance in the reference period. Further, we assume that each year both individuals have the same personal wage assessment base equal to the general assessment basis. If the first person does not have any child, then her average personal wage point is equal to 1. Therefore this person is entitled to a pension benefit equal to

$$D_1 = APWP_1 * CP * CPV = 40 * CPV. \quad (10)$$

Let the other person has  $n_{t+1}$  children, spending the maximum possible period of six years on parental leave with each child. Let us also assume that these periods do not overlap. Then the personal wage points of this person are equal to 1 while active working life and 0.6 while on parental leave. The average personal wage point then satisfies

$$APWP_2 = \frac{\sum_{i=1}^{40-6n_{t+1}} 1 + \sum_{i=1}^{6n_{t+1}} 0.6}{40} = \frac{40 - 6n_{t+1} + 3.6n_{t+1}}{40} = \frac{40 - 2.4n_{t+1}}{40},$$

and the pension benefit equals to

$$D_2 = APWP_2 * CP * CPV = (40 - 2.4n_{t+1}) * CPV. \quad (11)$$

Comparing (10) and (11) with the Fenge-Meier equivalent of the formula for the pension benefit (6) gives

$$M * 40 * CPV = \tau_{t+1} \tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1 - f(0)}{1 - f(\bar{n}_{t+1})} + \alpha 0 \right], \quad (12)$$

$$M * (40 - 2.4n_{t+1}) * CPV = \tau_{t+1} \tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1 - f(n_{t+1})}{1 - f(\bar{n}_{t+1})} + \alpha n_{t+1} \right]. \quad (13)$$

Parameter  $M$  stands for the number of months of pension benefits payments. If 40 years represent the maximum number of working years, then 40 years correspond to one unit of working time. When raising  $n_{t+1}$  offspring, the individual will be left with  $\frac{40-6n_{t+1}}{40}$  units of work, which corresponds to the value of  $f(n_{t+1}) = 6n_{t+1}/40$ . We obtain the  $CPV$  from (12) as

$$CPV = \frac{1}{40M} \tau_{t+1} \tilde{w}_{t+1} (1 - \alpha) \bar{n}_{t+1} \frac{40 - 6\bar{n}_{t+2}}{40 - 6\bar{n}_{t+1}}. \quad (14)$$

According to the (13) and (14) the implied child factor  $\alpha$  in the current Slovak pension system is then equal to

$$\alpha = \frac{3.6\bar{n}_{t+1}}{40 - 2.4\bar{n}_{t+1}}. \quad (15)$$

At the level of the total fertility rate of 1.59 (corresponds to the value for the Slovak Republic in 2020 according to Eurostat), i.e., when the individual fertility rate is approximated as  $1.59/2 = 0.795$ , then the value of the child factor in the given pension system is 7.5134%.

Now let's assume a stationary state with time-invariant individual fertility, i.e.,  $\bar{n}_{t+1} = \bar{n}_{t+2} = n$ . Equation (14) then takes the form

$$CPV = \frac{1}{40M} \tau_{t+1} \tilde{w}_{t+1} \left( 1 - \frac{3.6n}{40 - 2.4n} \right) n.$$

Setting the  $M = 240$ ,  $\tau_{t+1} = 22.75\%$ ,  $n = 1.59/2$  and  $\tilde{w}_{t+1} = 40 * 12 * 1296\text{€}$  (1296€ corresponds to the Slovakian average monthly gross wage in September 2022, CEIC) gives the current pension value of 10.8393€. This value is implied by the balance of the pension system budget. If we refer to the current pension value set to 15.1300€ in 2022 in Slovakia according to Social Insurance, the stationary state value obtained from our model is significantly lower. The reason is low fertility and a non-zero child factor. This can indicate the unsustainability of the Slovak pension scheme. We introduce several modifications to the Slovak pension system by explicitly involving the child factor in the pension benefit.

### Case 2

Let the policy change so that an additional personal pension point is equally redistributed between the parents until the child is 25. The average personal wage point of a person with an individual fertility level  $n_{t+1}$  then satisfies

$$APWP_2 = \frac{\sum_{i=1}^{40-6n_{t+1}} 1 + 25n_{t+1}}{40} = \frac{40 + 19n_{t+1}}{40}.$$

Following the previous procedure the child factor is now

$$\alpha = \frac{25\bar{n}_{t+1}}{40 + 19\bar{n}_{t+1}}. \quad (16)$$

For the assumed individual fertility 1.59/2, the child factor is  $\alpha = 39.4854\%$ . This modification rapidly increased the implied child factor compared to the original case with no explicit fertility-related pension benefit. The subsequent current pension value in the stationary fertility state is 7.0922€ implying that introducing such a child factor reduces pension value even more.

### Case 3

Let us look at more subtle ways of modifying the Slovak pension benefit. Assume that the working child redirects 5% of their monthly gross wage  $G_{t+1}$  to directly increase their parents' pension benefit. This bonus will be redistributed equally between two parents. Analogically to the unmodified case, we compare an individual without any child, consistent with equation (12) and one whose individual fertility is  $n_{t+1}$  children who satisfies

$$M [(40 - 2.4n_{t+1})CPV + 0.05n_{t+1}G_{t+1}] = \tau_{t+1} \tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1 - f(n_{t+1})}{1 - f(\bar{n}_{t+1})} + \alpha n_{t+1} \right].$$

Substituting for the loss function  $f(n_{t+1}) = \frac{6n_{t+1}}{40}$  and using the expression for the  $CPV$ , which is the same as in (14), we have the formula for the child factor

$$\alpha = \frac{2MG_{t+1} + 3.6\tau_{t+1}\tilde{w}_{t+1}\bar{n}_{t+1}\frac{40-6\bar{n}_{t+2}}{40-6\bar{n}_{t+1}}}{\tau_{t+1}\tilde{w}_{t+1}(40-6\bar{n}_{t+2}) + 3.6\tau_{t+1}\tilde{w}_{t+1}\bar{n}_{t+1}\frac{40-6\bar{n}_{t+2}}{40-6\bar{n}_{t+1}}}. \quad (17)$$

In the fertility stationary state, i.e.  $\bar{n}_{t+2} = \bar{n}_{t+1} = n$ , the expression (17) is simplified to

$$\alpha = \frac{\frac{2MG_{t+1}}{\tau_{t+1}\tilde{w}_{t+1}} + 3.6n}{40 - 2.4n}.$$

Let  $M = 240$  months. For simplicity, let's further assume that the monthly gross wage, with an expected career of 40 years, can be approximated as

$$G_{t+1} = \frac{\tilde{w}_{t+1}(1 - 6n/40)}{40 * 12}.$$

Then the child factor can be expressed as

$$\alpha = \frac{\frac{40-6n}{40\tau_{t+1}} + 3.6n}{40 - 2.4n}. \quad (18)$$

With the overall value of the pension contribution rate set to 22.75% and for  $n = 1.59/2$ , the child factor  $\alpha$  is 17.6768%. The underlying *CPV* in the stationary fertility state is 9.6482€.

#### Case 4

According to [Law no. 461/2003 Coll. on social insurance as amended](#), the currently introduced parental pension in the Slovak republic pension scheme is 3% of one-twelfth of the total assessment base (gross wage) of the child for the calendar year two years preceding the relevant calendar year from which the pension contribution was paid. The parental bonus is evenly distributed between both parents. For simplicity, we will approximate this setting with the previous calculations by changing the percentage of the gross salary from 5% to 3%. The formula for the child factor in the stationary fertility state then changes to

$$\alpha = \frac{\frac{0.6(40-6n)}{40\tau_{t+1}} + 3.6n}{40 - 2.4n}, \quad (19)$$

In correspondence to  $n = 1.59/2$  and pension contributions of 22.75%, the child factor is at  $\alpha = 13.6114\%$ , and the current pension value, in this case, reaches the level of 10.1247€.

We have listed a few examples of the pension benefit settings and determined the implied child factor value. The question arises whether these values are optimal in terms of the assumed model or at least approaching the optimal ones. Therefore let us look at the optimal solution to the maximization problem (7).

Let the utility function be in the form of weighted sum of logarithms of its arguments, i.e.,

$$U(c_t, z_{t+1}, n_{t+1}) = \ln c_t + \delta \ln z_{t+1} + \gamma \ln n_{t+1},$$

where  $\delta$  is a discount factor representing a time preference of the consumption. According to [Buyse and Rapoport \(2014\)](#), the parameter  $\gamma$  states an individual's preferences or motivation to have a child.

Inspired by the form of the loss function from the previous part, where  $f(n_{t+1}) = \frac{6}{40}n_{t+1}$  was considered, we now choose  $f(n_{t+1}) = an_{t+1}$ , where  $a \in (0, 1)$ . The coefficient  $a$  represents the cost of bringing up one child. In such a case, where the loss function is linear, we have  $f(\bar{n}_{t+1}) = \overline{f(n_{t+1})}$ . The following first-order conditions determine the optimal decisions:

$$\begin{aligned} \frac{\partial U}{\partial s_t} &= -\frac{1}{c_t} + \frac{\delta R_{t+1}}{z_{t+1}} \\ &= \frac{-1}{(1 - \tau_t) [1 - f(n_{t+1})] \tilde{w}_t - s_t} \\ &+ \frac{\delta R_{t+1}}{R_{t+1}s_t + \tau_{t+1}\tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1-f(n_{t+1})}{1-f(\bar{n}_{t+1})} + \alpha n_{t+1} \right]} = 0, \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial U}{\partial n_{t+1}} &= \frac{-(1 - \tau_t) f'(n_{t+1}) \tilde{w}_t}{(1 - \tau_t) [1 - f(n_{t+1})] \tilde{w}_t - s_t} \\ &+ \frac{\delta \tau_{t+1} \tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ \alpha - (1 - \alpha) \bar{n}_{t+1} \frac{f'(n_{t+1})}{1-f(\bar{n}_{t+1})} \right]}{R_{t+1}s_t + \tau_{t+1}\tilde{w}_{t+1} [1 - f(\bar{n}_{t+2})] \left[ (1 - \alpha) \bar{n}_{t+1} \frac{1-f(n_{t+1})}{1-f(\bar{n}_{t+1})} + \alpha n_{t+1} \right]} + \frac{\gamma}{n_{t+1}} = 0. \end{aligned} \quad (21)$$

Consider the equilibrium state where the variables  $\tau_t$ ,  $R_t$ ,  $\tilde{w}_t$ ,  $s_t$ , and  $n_t$  are constants over time. In this case, equations (20) and (21) can be written as

$$\frac{\partial U}{\partial s} = \frac{-1}{(1-\tau)(1-an)\tilde{w}-s} + \frac{\delta R}{Rs + \tau\tilde{w}(1-an)n} = 0, \quad (22)$$

$$\frac{\partial U}{\partial n} = \frac{-(1-\tau)a\tilde{w}}{(1-\tau)(1-an)\tilde{w}-s} + \frac{\delta\tau\tilde{w}(\alpha-an)}{Rs + \tau\tilde{w}(1-an)n} + \frac{\gamma}{n} = 0. \quad (23)$$

Using equations (20)-(21), we have calculated the optimal values of  $n_{t+1}^*$ ,  $s_t^*$ , the implied values of  $p_{t+1}^*$ ,  $z_{t+1}^*$  and the total utility  $U$  for Cases 1-4 (see above) of the setting of the Slovak pension system. The results can be found in Table 1. Subsequently, we have substituted the formulas (15)-(16) and (18)-(19) into the equation (23), and by solving the equations (22)-(23), we have calculated the optimal values of  $n^*$ ,  $s^*$ , the implied values of  $p^*$ ,  $z^*$ ,  $U_e$  and the equilibrium value of  $\alpha = \alpha_e$  for the equilibrium states corresponding to Cases 1-4. The results are in Table 2.

Model calibration is as follows. The interest rate factor  $R$  was set to 2, which according to Černý and Melicherčík (2020) corresponds to the return of equal contributions over a lifetime working career with moderate risk aversion. At the same time, the wages of individuals were normalized to one unit, i.e.,  $\tilde{w}_t = \tilde{w}_{t+1} = \tilde{w} = 1$ . Therefore the optimal values of the individual savings correspond to the percentage of their wage. Since according to the equation (20), we have:  $z_{t+1} = \delta R c_t$ , we chose  $\delta = 1/4$ , because we assume about half the period of receiving the pension compared to the length of the working career, and therefore  $z_{t+1} = c_t/2$ . Since the Slovak pension system was set according to Case 1 for many years, we set the value  $\gamma = 0.1581$ , at which in this case the implied value  $\alpha$  merges with the equilibrium value  $\alpha_e$ . As in Cases 1-4, we set the values  $a = 6/40 = 0.15$  and  $\bar{n}_{t+1} = 1.59/2$ .

Case	$\alpha$	$n_{t+1}^*$	$s_t^*$	$p_{t+1}^*$	$z_{t+1}^*$	$U$
1	7.51%	0.7951	0.0724	0.1593	0.3040	-0.8315
2	39.49%	1.0854	0.0586	0.1768	0.2941	-0.8239
3	17.68%	0.8698	0.0702	0.1603	0.3007	-0.8308
4	13.61%	0.8381	0.0713	0.1595	0.3020	-0.8312

**Table 1:** Optimal values of  $n_{t+1}^*$ ,  $s_t^*$ , the implied values of  $p_{t+1}^*$ ,  $z_{t+1}^*$  and the total utility  $U$  for Cases 1-4 of the setting of the Slovak pension system.

Case	$\gamma$	$\alpha_e$	$n^*$	$s^*$	$p^*$	$z^*$	$U_e$
1	0.1581	7.51%	0.7951	0.0723	0.1593	0.3040	-0.8315
2	0.1581	46.31%	1.1435	0.0418	0.2155	0.2991	-0.7944
3	0.1581	18.36%	0.8722	0.0653	0.1725	0.3031	-0.8207
4	0.1581	14.03%	0.8399	0.0682	0.1670	0.3035	-0.8250
Optimum	0.1581	75.88%	1.6083	0.0062	0.2776	0.2900	-0.7792
2	0.3	66.47%	2.1496	-0.0279	0.3313	0.2756	-0.6881
Optimum	0.3	67.48%	2.1683	-0.0289	0.3328	0.2750	-0.6881
3	1	47.34%	3.5667	-0.0791	0.3773	0.2191	0.0673
Optimum	1	46.65%	3.5568	-0.0789	0.3775	0.2197	0.0673

**Table 2:** Optimal values of  $n^*$ ,  $s^*$ , implied values of  $p^*$ ,  $z^*$ ,  $U_e$  and  $\alpha = \alpha_e$  for equilibrium states.

The results show that the equilibrium and implied values of  $\alpha$  are not very different from each other. The value of consumption in retirement is close to 0.3 in all cases. The optimal number of children varies more significantly. As the child factor  $\alpha$  increases, the optimal number of children and, of course, the pension benefit  $p$  increases, while savings  $s$  decrease. With approximately the

same consumption in retirement, the increasing number of children is thus financed by a reduction in savings. As the child factor increases, lifetime utility also increases. The highest is in Case 2, when the number of children is the highest linked to the pension benefit.

According to the presented model, the equation (8) for optimal equilibrium child factor  $\alpha^*$  can be simplified to the form  $\alpha^* = 1 - an(\alpha^*)$ . Inserting  $\alpha = 1 - an$  into (22) - (23) one can calculate the values of  $n^*$ ,  $s^*$ , the implied values of  $p^*$ ,  $z^*$ ,  $U_e$  and the equilibrium value of  $\alpha = \alpha_e$  for the optimal equilibrium state. One can observe (see Table 2) that  $\alpha_e = 75.88\%$  is optimal for  $\gamma = 0.1581$ . Table 1 shows Case 2 is closest to this with the lifetime utility  $U_e$  close to the optimal one. From January 1, 2023, the setting of the Slovak pension system is similar to Case 4. It can be seen that, compared to Case 1, fertility in the equilibrium state has increased slightly, but it still does not have a sufficient level. An acceptable value can be observed in Case 2. The current setting of the parental bonus in Slovakia can therefore be brought closer to the optimum, for example, by a more generous distribution of personal wage points for raising children. According to the presented model, we thereby create conditions for increasing fertility. The results from Table 2 confirm that for equilibrium states, the optimal number of children is increasing, while individual savings are decreasing as a function of the child factor. This is consistent with the theoretical conclusions of Section 2.

We have also calculated the corresponding values for  $\gamma = 0.3$  and  $\gamma = 1$ . Results in Table 2 show that for  $\gamma = 0.3$ , Case 2 is close to the optimum. Similarly, for  $\gamma = 1$ , the optimal equilibrium state can be well approximated by Case 3. However, it should be added that in these cases, unrealistically high numbers of children in the equilibrium state associated with negative savings values  $s^*$  (which means borrowing instead of saving) emerge. The values  $\gamma = 0.3$  and 1 are therefore not realistic.

The optimal solutions for several combinations of the parameters  $a$  and  $R$  are shown in Table 3. In the set scenarios, we fixed the value of the pension contribution rate at the level of 22.75%. For the parameter  $\gamma$ , as previously argued the value 0.1581 was used. The range for the interest rate factors  $R$  was drawn up from Černý and Melicherčík (2020). They reported values of  $R$  for lifetime equal contributions in the range of 1.8 – 3.7, where lower levels correspond to higher risk aversion. Finally, as before,  $\delta = 1/4$  and  $\tilde{w} = 1$  was considered.

It can be observed that raising the level of the penalty constant  $a$  increases individual savings and invokes the increasement of the child factor. The fertility level rapidly decreases with the upward movement of the penalty constant  $a$ . As for the individual fertility itself, it is, in each case, kept between 0.4574 and 3.4718, which is significantly below half of the maximum possible fertility level.

According to the increasing interest rate factor, as expected, individual fertility decreases while the child factor increases. As saving money and earning interest rate becomes more advantageous, the pension scheme must invoke a stronger linkage between pension benefits and individual fertility to sustain the individual fertility level. Personal savings do also increase considering higher interest rate factor.

Overall the optimal setting for the child factor is in the range of (65.28%, 86.28%), which is a very strong linkage between pension benefits and individual fertility. As the chosen preference parameter  $\gamma$  was considerably low, the motivation to have a child must be drawn from the pension system itself. Therefore the high values of the child factor. For individuals whose motivation to have children is minimal, only a substantial benefit in the form of a significant child pension may act as a persuasive mechanism to consider having offspring. Compared to the values obtained when examining the various settings of the pension benefit policy in the first part of this section, model optimal values are much higher. The considered policy settings may be insufficient to have any effect on compensating for the lost income due to raising the offspring.

From the perspective of individual savings, the values are kept between  $-17.41\%$  and  $11.53\%$ . It is worth noting the negative values of personal savings that occur mainly when the interest rate factor and the penalty constant are both sufficiently low. In an economic situation where the expected interest rate is low and having a child does not cost that much, one may be led to borrow money to cover the expenses of consumption and bringing up children.

Raising the penalty constant  $a$  results in the decreasement of the pension benefit and also decreases the old-age consumption, i.e. the cumulative value of the pension benefit and interest rate adjusted personal savings. The pension benefit varies from 8.98% to 51.56%, reaching the maxima for the

	$R = 1.50$						$R = 2.00$					
$a$	$n^*$	$s^*$	$\alpha^*$	$p^*$	$Rs^*$	$z^*$	$n^*$	$s^*$	$\alpha^*$	$p^*$	$Rs^*$	$z^*$
0.10	3.4718	-0.1741	0.6528	0.5156	-0.2612	0.2544	3.0320	-0.0846	0.6968	0.4806	-0.1692	0.3114
0.15	1.8977	-0.0542	0.7153	0.3088	-0.0813	0.2275	1.6083	0.0062	0.7588	0.2776	0.0124	0.2900
0.20	1.2063	0.0062	0.7587	0.2082	0.0093	0.2175	1.0238	0.0488	0.7952	0.1852	0.0976	0.2828
0.25	0.8487	0.0406	0.7878	0.1521	0.0609	0.2130	0.7312	0.0719	0.8172	0.1359	0.1438	0.2797
0.30	0.6416	0.0619	0.8075	0.1179	0.0928	0.2107	0.5624	0.0859	0.8313	0.1064	0.1718	0.2782
	$R = 3.00$						$R = 4.00$					
$a$	$n^*$	$s^*$	$\alpha^*$	$p^*$	$Rs^*$	$z^*$	$n^*$	$s^*$	$\alpha^*$	$p^*$	$Rs^*$	$z^*$
0.10	2.4125	0.0062	0.7588	0.4164	0.0186	0.4350	2.0476	0.0488	0.7952	0.3704	0.1952	0.5656
0.15	1.2832	0.0619	0.8075	0.2357	0.1857	0.4214	1.1248	0.0859	0.8313	0.2127	0.3436	0.5563
0.20	0.8436	0.0859	0.8313	0.1595	0.2577	0.4172	0.7611	0.1016	0.8478	0.1468	0.4064	0.5532
0.25	0.6216	0.0986	0.8446	0.1194	0.2958	0.4152	0.5721	0.1101	0.8570	0.1115	0.4404	0.5519
0.30	0.4901	0.1064	0.8530	0.0951	0.3192	0.4143	0.4574	0.1153	0.8628	0.0898	0.4612	0.5510

**Table 3:** Optimal values of the individual fertility rate  $n^*$ , personal savings  $s^*$ , and the child factor  $\alpha^*$  according to the selected levels of the parameter  $a$  and interest rate factor  $R$ . Underlying values of the first pillar pensions  $p^*$ , interest rate adjusted savings  $Rs^*$  and cumulative consumption in the old age  $z^*$  are also shown.

lowest interest rate factor and low-cost childrearing. As expected, the upward change in the interest rate factor puts more weight on personal savings, with interest rate adjusted values between  $-26.12\%$  and  $46.12\%$ , while dropping the pension benefit value.

Overall, old-age consumption increases with the increasement of the interest rate factor. There is a significant trade-off between pension benefits and personal savings as they offset each other, keeping the old-age consumption between  $21.07\%$  and  $56.56\%$ .

## Conclusions

We have presented two models of the pension system with a child factor. In the first model, pension benefits were tied to the wages of the descendants of the pension system participant. Such a setting is risky because of possible problems of descendants (unemployment, resettlement, etc.). In the second model, benefits were tied to the number of offspring and the average salary of all participants, removing the first setup's disadvantages.

The model with benefits tied to average wages has been rewritten into the formulation of the model of [Fenge and Meier \(2005\)](#). Our results have proved that (in the equilibrium state) as the child factor increases, so does fertility, while individual savings decrease.

We have calculated the implied size of the child factor for the current setting of the pension system in Slovakia as well as for its various modifications. Based on the Fenge-Meier model, we have calculated the value of the optimal child factor. The values of the implied child factor are much lower than the optimal ones. The system can be brought closer to the optimum by a more generous distribution of personal wage points for raising children. Increasing the cost of raising children reduces fertility, pension benefits, and pension consumption, and increasing the interest rate factor reduces fertility and pension benefits but increases total pension consumption.

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