Stability and instability of asymptotic profiles for fast diffusion

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Abstract

This talk is concerned with asymptotic profiles for solutions to the Cauchy-Dirichlet problem for the Fast Diffusion equation (FD) in smooth bounded domains under the so-called Sobolev subcritical condition. It is well-known that every solution of (FD) vanishes in finite time with a power rate; more precisely, it asymptotically approaches to a separable solution (Berryman and Holland '80). Then the asymptotic profile for each vanishing solution can be characterized as a non-trivial solution of the Emden-Fowler equation (EF). The stability of asymptotic profiles has been discussed for the case that (EF) has a unique positive solution; on the other hand, the case that (EF) may have multiple (positive) solutions had not been studied for many years.

In this talk, we shall first see how to formulate notions of stability and instability of asymptotic profiles, and then, we shall discuss criteria to distinguish the stability of each asymptotic profile. Moreover, we shall focus on how to treat non-isolated asymptotic profiles; indeed, (EF) may admit a one-parameter family of positive solutions, e.g., for sufficiently thin annular domains. In particular, for thin annular domain cases, each non-radial asymptotic profile belonging to a one-parameter family turns out to be stable and the radial positive profile turns out to be unstable. The method of analysis relies on variational method, uniform extinction estimates for solutions to (FD), the Lojasiewicz-Simon inequality and energy techniques. Furthermore, we shall also discuss other related issues.

References

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