Blowup in chemotaxis systems with nonlocal diffusion

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We study radially symmetric solutions of the Keller-Segel model in $d \geq 2$ dimensions with the diffusion described by fractional powers of the Laplacian

$$u_t + (-\Delta)^{\alpha/2} u + \nabla \cdot (u \nabla v) = 0,$$

$$\Delta v + u = 0, \quad x \in \mathbb{R}^d, \ t > 0.$$

In particular, global in time radially symmetric solutions are constructed for the initial data satisfying estimates like $0 \leq u_0(x) \leq u_C(x) \equiv s(\alpha, d)|x|^{-\alpha}$ with some explicit constant $s(\alpha, d)$. u_C is, for certain $d \geq 2$ and $\alpha \in (0, 2)$, a singular solution. Moreover, a finite time blowup is shown for solutions with "big" initial data. Criteria for this dichotomous behavior of solutions can be expressed in terms of the norms of suitable Morrey spaces $M^{d/\alpha}(\mathbb{R}^d)$.

References

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