How can the blow up of the lunar perigee explain the instability of suspension bridges?

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In 1886, Hill introduced a class of linear second order ODE's with periodic coefficients, aiming to describe the lunar perigee; these equations generalize earlier equations introduced by Mathieu in 1868. In 1905, in his presentation of the mathematical works by Hill, Poincaré emphasizes that, among his discoveries, *...celle qui fera son nom immortel, c'est sa théorie de la Lune.* Poincaré was well aware that the Moon theory was not the only application of the Hill equation, his last comment on the equation reads *...quand elles s'étendront à un domaine plus vaste, on ne devra pas oublier que c'est à M. Hill que nous devons un instrument si précieux.* It is well-known that Poincaré was right, the very same equation is nowadays used to describe many different natural phenomena.

In this talk, we explain how the blow up for the Hill equations can be used to study the instability of suspension bridges. We start with a simple finite system of ODE's of Mathieu-type and we end up with infinite dimensional dynamical systems such as nonlinear beam and/or plate equations. The resulting PDE's are of fourth order in space, reflecting the bending energy appearing in these models.

The instability of suspension bridges may lead to unexpected torsional oscillations which were the main cause of some historical collapses. But to detect instabilities in an infinite dimensional dynamical system is a very difficult task. We show that this problem may be solved through a finite dimensional approximation with accurate estimates of the errors.

This talk is based on joint works with my co-authors:
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