A semilinear elliptic equation with a dynamical boundary condition

Tatsuki Kawakami

Department of Mathematical Sciences, Osaka Prefecture University, Japan

We consider the problem

$$(P) \qquad \begin{cases} -\Delta u = u^p, \quad u \geq 0, & x \in \Omega, \ \ t > 0, \\ \partial_t u + \partial_\nu u = 0, & x \in \partial\Omega, \ \ t > 0, \\ u(x,0) = \varphi(x) \geq 0, & x \in \partial\Omega, \end{cases}$$

where $N \geq 2$, Ω is a smooth domain of $\mathbb{R}^N$, Δ is the N-dimensional Laplacian (in x), ν is the exterior normal vector on $\partial\Omega$, $\partial_t := \partial/\partial t$, $\partial_\nu := \partial/\partial\nu$, $p > 1$ and φ is a nonnegative measurable function on $\partial\Omega$. In this talk we treat the two cases

(i) Ω is the half-space $\mathbb{R}^N_+$,

(ii) Ω is the the exterior of the unit ball $\{x \in \mathbb{R}^N : |x| > 1\}$,

and discuss results on existence, nonexistence and large-time behavior of small solutions. Furthermore, we show that local solvability of problem (P) is equivalent to global solvability of problem (P) and solvability of the stationary problem.

These are joint works with M. Fila and K. Ishige.