SELF-SIMILAR EXTINCTION FOR A DIFFUSIVE HAMILTON-JACOBI EQUATION WITH CRITICAL ABSORPTION

PHILIPPE LAURENÇOT

ABSTRACT. The behavior near the extinction time is identified for non-negative solutions to the diffusive Hamilton-Jacobi equation with critical gradient absorption

$$\partial_t u - \Delta_p u + |\nabla u|^{p-1} = 0 \quad \text{in } (0,\infty) \times \mathbb{R}^N \ ,$$

and fast diffusion $2N/(N+1) < p < 2$. More precisely, given a non-negative and radially symmetric initial condition with a non-increasing profile which decays sufficiently fast as $|x| \to \infty$, it is known that the corresponding solution u vanishes identically after a finite time $T_e > 0$, called the extinction time. More precise information is available on the dynamics near the extinction time. In fact, the solution u to the above equation approaches a uniquely determined *separate variable solution* of the form

$$U(t,x) = (T_e - t)^{1/(2-p)} f_*(|x|), \quad (t,x) \in (0,T_e) \times \mathbb{R}^N \ ,$$

as $t \to T_e$. A cornerstone of the convergence proof is an underlying variational structure of the equation. Also, the selected profile f_* is the unique non-negative solution to a second order ordinary differential equation which decays exponentially at infinity. A complete classification of solutions to this equation is provided and relies on different arguments depending whether $N = 1$ or $N \geq 2$. For the former, the second order ordinary differential equation is transformed to a first order one with some monotonicity properties with respect to the shooting parameter. As for the latter, no monotonicity argument seems to be available and it is overcome by the construction of an appropriate *Pohozaev functional*. These are joint works with Razvan Gabriel Iagar (Madrid).

INSTITUT DE MATHÉMATIQUES DE TOULOUSE, UMR 5219, UNIVERSITÉ DE TOULOUSE, CNRS, F–31062 TOULOUSE CEDEX 9, FRANCE

E-mail address: laurenco@math.univ-toulouse.fr