Asymptotic soliton resolution for the critical nonlinear heat equation

Hiroshi Matano

University of Tokyo
matano@ms.u-tokyo.ac.jp

In this talk, I will discuss the asymptotic behavior of radially symmetric solutions of the nonlinear heat equation on $\mathbf{R}^N$ $(N > 2)$ with the Sobolev critical power nonlinearity.

In the case of time-global solutions, we show that the solution is asymptotically decomposed into a finite sum of rescaled ground states that hardly interact with each other because the ratio of the rescaling parameters goes to infinity. The total energy of the solution then converges to an integer multiple of the energy of the ground state. We call this behavior "soliton resolution" of the solution.

In the case where the solution blows up in finite time, and if the blow-up is of type II in a certain sense, we show that a similar soliton resolution occurs near the blow-up point.

The purpose of our results is to classify the asymptotic behavior of solutions; thus, whether multiple solitons really occur or not will not be discussed here. This is joint work with Frank Merle.