On the behavior of eigenfunctions for the linearized Gel'fand problem

Hiroshi Ohtsuka
Kanazawa University
ohtsuka@se.kanazawa-u.ac.jp

We consider the Gel'fand problem $-\Delta u = \lambda e^u$ and its linearized eigenvalue problem $-\Delta v = \mu\lambda e^u v$ in a two-dimensional bounded domain Ω under the Dirichlet boundary condition. Here λ is a positive small parameter. The Gel'fand problem or its variant is known to appear in a wide variety of areas of mathematical science such as conformal deformations of surfaces, self-dual gauge field theories, equilibrium states of vortices, stationary states of chemotactic motions, and so forth. It is now well understood that a solution u for the Gel'fand problem may blow up on some distinct finite points as $\lambda \downarrow 0$. In this talk, we are interested in the effect of the blowing-up behavior of u on the eigenfunction v for the linearized problem. We first discuss the asymptotic behavior of some of eigenvalues (μ) as $\lambda \downarrow 0$ in terms of the blow up points where the solution of the Gel'fand problem (u) blows up as $\lambda \downarrow 0$. As a consequence, we see the asymptotic shapes of some of eigenfunctions (v). We also would like to discuss the usage of the generalized Rellich identity in the sketch of the proof.

This is based on the joint works with Francesca Gladiali (Univ. Sassari), Massimo Grossi (Univ. Roma "La Sapienza"), and Takashi Suzuki (Osaka Univ.).