Global existence and boundedness of solutions to chemotaxis systems with general sensitivity

Takasi Senba (Fukuoka University, Japan)

This talk is based on the joint work with Prof. Kentarou Fujie.

In this talk, we consider solutions to the following chemotaxis system with general sensitivity.

$$\tau u_t = \Delta u - \nabla \cdot (u \nabla \chi(v)) \quad \text{in } \Omega \times (0, \infty),$$
$$\eta v_t = \Delta v - v + u \quad \text{in } \Omega \times (0, \infty)$$

with the Neumann boundary condition. Here, τ and η are non-negative constants, χ is a smooth function on $(0, \infty)$ satisfying $\chi'(\cdot) > 0$ and Ω is a bounded domain in $\mathbf{R}^2$.

It is well known that the chemotaxis system with direct sensitivity ($\chi(v) = \chi_0 v$, $\chi_0 > 0$) has blowup solutions in two dimensional case. On the other hand, there are many results of research on sufficient conditions for the system to have no blowup solutions. For example, in the case where $\chi(v) = \chi_0 \log v$ with $0 < \chi_0 \ll 1$ and $\frac{1}{(1+v)^k}$ with $k > 1$, any solution to the system exists globally in time and is bounded.

From these research, people expect that two dimensional systems have no blowup solutions, if $\lim_{v \to \infty} \chi'(v) = 0$. We talk some results on this conjecture.