MORREY SPACES AND BOUNDEDNESS AND DECAY OF NONRADIAL GLOBAL SOLUTIONS FOR A SUPERCRITICAL SEMILINEAR HEAT EQUATION IN $\mathbb{R}^n$

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Abstract: We prove the boundedness of global classical solutions for the semilinear heat equation $u_t - \Delta u = |u|^{p-1}u$ in the whole space $\mathbb{R}^n$, with $n \geq 3$ and supercritical power $p > (n+2)/(n-2)$. This is proved without any radial symmetry or sign assumptions, unlike in all the previously known results for the Cauchy problem, and under spatial decay assumptions on the initial data (roughly speaking $u = o(|x|^{-2/(p-1)})$), that are essentially optimal in view of the known counter-examples. Moreover, we show that any global classical solution has to decay in time faster than $t^{-1/(p-1)}$, which is also optimal and in contrast with the subcritical case.

The proof relies on nontrivial modifications of techniques developed by Chou-Du-Zheng and by Blatt-Struwe for the case of convex bounded domains. They are based on weighted energy estimates of Giga-Kohn type, combined with an analysis of the equation in a suitable Morrey space. This method actually works for any convex, bounded or unbounded, smooth domain, but at the same time captures some of the specific behaviors associated with the case of the whole space $\mathbb{R}^n$.

As a consequence we also prove that the set of initial data producing global solutions is open in suitable topologies, and we show that the so-called "borderline" global weak solutions blow up in finite time and then become classical again and decay as $t \to \infty$. These results confirm the key role of Morrey spaces in the understanding of the structure of the set of global solutions for $p > p_S$.