A priori estimates and singularities for the viscous parabolic Hamilton-Jacobi equation

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Here we consider the nonnegative weak solutions of the parabolic problem

$$u_t + \mathcal{A}u + |\nabla u|^q = 0 \quad \text{in } \Omega \times (0,T) \tag{1}$$

where Ω is a domain of $\mathbb{R}^N$, $q > 1$, and the model case of operator is $\mathcal{A} = -\varepsilon\Delta_p$, $p > 1, \varepsilon > 0$.

We show that a regularizing effect occurs for the function u in the case of the Dirichlet problem with bounded boundary data. We deduce a universal estimate, available for the function u, independent of p and ε, which is not obtained by using Bernstein techniques.

In the usual case $p = 2$, $\mathcal{A} = -\varepsilon\Delta$, we deduce the non existence of very singular solutions of equation (1) for $q \geq \frac{N+2}{N+1}$, showing that the well known existence results for $q < \frac{N+2}{N+1}$ are optimal. We also prove that any punctual singularity at $(x,t) = (0,0)$ is removable.