Linear Schrödinger operators and non-linear PDE

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We discuss a new approach to boundary value problems for semi-linear PDE with measure data in a bounded domain D in R^N. This employs techniques of linear potential theory involving the Schrodinger operator $-\Delta + V$ where V is positive and satisfies a certain growth condition at the boundary. As application we obtain a complete classification of the positive solutions of the equation $-\Delta u + u^q = 0$, for any $q > 1$.

In the subcritical case $1 < q < (N+1)/(N-1)$ such a classification was obtained by Le Gall (1993) for $q = 2$, $N = 2$ and by Marcus and Veron (1996) in arbitrary dimension. In the supercritical case a complete classification was previously obtained by M'selati (2002) for $q = 2$ and Dynkin (2004) for $q \leq 2$ employing probabilistic methods.