Comparison of rescaled energy
for a supercritical nonlinear heat equation

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In this talk I will discuss blow-up of radially symmetric solutions of the nonlinear heat equation with power nonlinearity, whose exponent is above the Sobolev critical exponent. Our first goal is to show that the so-called type II blow-up is a rather exceptional phenomenon, and almost all blow-ups are of type I. Our method is based on a combination of the zero-number principle and energy estimates.

The second goal of my talk is to show that the constant positive solution attains the smallest rescaled energy among all non-zero stationary solutions of the rescaled equation, including the singular one. This means, among other things, that the flat self-similar solution is most stable among all self-similar blow-up solutions. This leads to a sharp criterion for no blow-up. Interestingly, these two apparently different results are closely related to each other and can be derived from the same argument.

This is joint work with Frank Merle.