Slowly traveling waves, blow-up at spatial infinity and homoclinic orbits in nonlinear parabolic equations of fast diffusion type

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We consider the nonlinear diffusion equation $u_t = u^p u_{xx}$ for $p > 0$. We construct positive classical solutions which are of the form

$$u(x,t) = (-t)^{-\frac{1}{p}} F(x + \frac{1}{p\alpha} \ln(-t)), \qquad x \in \mathbb{R},\ t < 0,$$

with arbitrary $\alpha > 0$, by solving an associated ODE for F. These 'ancient slowly traveling wave solutions' have the following properties:

1.) If $p \leq 2$ then u blows up at time $t = 0$ with empty blow-up set.

2.) If $p > 2$ then for each $\tau > 0$, $u|_{\mathbb{R}\times(-\infty,-\tau)}$ can be extended so as to become an entire positive classical solution $\bar{u}$, defined on $\mathbb{R} \times \mathbb{R}$, such that $\bar{u}_x > 0$ on $\mathbb{R}$ and

$$\bar{u}(x,t) \to 0 \qquad \text{as } t \to \pm\infty,$$

locally uniformly with respect to $x \in \mathbb{R}$.