P. Quittner and Ph. Souplet: Superlinear parabolic problems (2nd edition 2019)

December 1, 2025

Updates of references

[20] Monographs in Mathematics 106, Birkhäuser, Basel 2019. (But the results that we cite in Remarks 51.2 and 51.5 have not been included in the final version of that book. On the other hand, [20, Theorems VII.2.7.2 and VII.2.8.3] serve as an additional reference to the proof of Proposition 51.3.)

[96] Math. Ann. 378 (2020), 13–56.

[146] Anal. PDE 12 (2019), 1773–1804.

[149] Amer. J. Math. 142 (2020), 1439–1495.

[150] Dyn. Partial Differ. Equ. 16 (2019), 151–183.

[193] J. Amer. Math. Soc. 33 (2020), 527–607. The title "On strongly anisotropic type II blow up" has been modified to "Strongly anisotropic type II blow up at an isolated point".

[194] Mem. AMS 260, 1255 (2019).

[198] J. Eur. Math. Soc. 22 (2019), 283–344.

[221] J. Math. Pures Appl. 128 (2019), 339–378.

[225] Anal. PDE 13 (2020), 215–274.

[226] J. Funct. Anal. 280 (2021), 108788.

[227] Acta Math. Sinica 35 (2019), 1027–1042.

[229] Ann. Scuola Norm. Sup. Pisa Cl. Sci. (5) 21 (2020), 569–641.

[262] J. Math. Pures Appl. 137 (2020), 143–177.

[269] Adv. Nonlinear Anal. 9 (2020), 479–495.

[273] (An improvement of) this preprint appeared as: K. Fellner, J. Morgan, B.-Q. Tang, Global classical solutions to quadratic systems with mass control in arbitrary dimensions, Ann. Inst. H. Poincaré Anal. Non Linéaire 37 (2020) 281–307. Please see the comments below concerning the open problems OP 3.3.

[290] Math. Ann. 377 (2020), 317–333.

[568] Int. Math. Res. Notices 2020 (2020), 541–606.

[595] Adv. Math. 355 (2019), 106763.

[658] Proc. Amer. Math. Soc. 148 (2020) 2997–3008.

[662] Discrete Contin. Dyn. Syst. 41 (2021), 413–438.

[663] Nonlinear Anal. 191 (2020), 111639.

[677] J. Math. Pures Appl. 133 (2020) 66–117.

[773] SIAM J. Math. Anal. 51 (2019), 991–1013.

[808] Trans. Amer. Math. Soc. 371 (2019), 5899–5972.

Updates of Appendix I (Selection of open problems)

OP1.3: Partially solved (the answer is no for solutions that are bounded in every finite strip) in L. Dupaigne, B. Sirakov and Ph. Souplet, Int. Math. Res. Notices 2022 (2022), 9024–9043.

OP2.1: Completely solved (the answer is no) in H. Miura and J. Takahashi, J. Eur. Math. Soc. (2024), DOI 10.4171/JEMS/1551 (Preprint arXiv:2206.10790) and Preprint arXiv:2310.09750.

OP2.3: Completely solved (the answer is yes) in P. Quittner, Duke Math. J. 170 (2021), 1113–1136 (cf. also P. Quittner, Partial Differ. Equations Appl. 3 (2022), 26).

OP2.5: Partially solved (the answer is yes for positive solutions) in P. Quittner, Duke Math. J. 170 (2021), 1113–1136, and completely solved (the answer is yes) in H. Miura, J. Takahashi and E. Zhanpeisov, Preprint arXiv:2510.17229.

OP2.6: Partially solved (the answer is yes for positive solutions), as a consequence of [442] and P. Quittner, Duke Math. J. 170 (2021), 1113–1136.

OP2.8: Remarkable progress in the study of type II blow-up in the critical case $p = p_S$ has been achieved in M. del Pino, M. Musso, J. Wei, Q. Zhang and Y. Zhou, Preprint arXiv:2002.05765; M. del Pino, M. Musso, J. Wei and Y. Zhou, Discr. Contin. Dyn. Sys. 40 (2020), 3327–3355; T. Li, L. Sun and S. Wang, Preprint arXiv:2204.11201; J. Harada, Ann. Inst. H. Poincaré, Anal. Non Linéaire 37 (2020), 309–341; L. Zhang and J. Zhao, J. Fixed Point Theory Appl. 25 (2023), 67; J. Harada, Annals of PDE 6 (2020), 13; K. Wang and J. Wei, Preprint arXiv:2101.07186.
A new type of type II blow-up in the supercritical case $p = 3$, $5 \leq n \leq 7$, has been proved in M. del Pino, C.-C. Lai, M. Musso, J. Wei and Y. Zhou, Preprint arXiv:2006.00716.

OP2.9: Partially solved in M. del Pino, M. Musso and J. Wei, Anal. PDE 13 (2020), 215–274; J. Wei, Q. Zhang and Y. Zhou, J. Differential Equations 398 (2024), 38–140; Z. Li, J. Wei, Q. Zhang and Y. Zhou, Nonlinear Anal. 247 (2024), 113594; J. Wei and Y. Zhou, J. Elliptic Parabol. Equ. (2025), DOI 10.1007/s41808-025-00356-1.

OP3.1: For problem (31.21), new sufficient conditions for the nonexistence in the case $n \geq 5$ were obtained in K. Li, M. Li and J. Wei, Preprint arXiv:2510.06613. For problem (31.22) with any $p, q > 1$, the result on nonexistence of bounded solutions from [178] (cf. Remark 31.11(i)) has been extended to solutions that are bounded in every finite strip in Y. Li and Ph. Souplet, J. Funct. Anal. 289 (2025), 111107.

OP3.2: A very partial solution (nonexistence if $1 < p < p_S$, $|q - p| < \varepsilon$) follows from [334] and P. Quittner, Duke Math. J. 170 (2021), 1113–1136.

OP3.3: "What is the optimal growth condition guaranteeing global existence in systems with dissipation of mass ?": Partially solved in M. Pierre and D. Schmitt, Discrete Contin. Dyn. Syst. 43 (2023), 1113–1136. See also C. Sun, B.Q. Tang and J. Yang, J. Evol. Equ. 23 (2023), 44.

OP3.3: "Can one remove the entropy assumption for quadratic systems ? Is uniform boundedness true ?": Completely solved (both answers are yes) in K. Fellner, J. Morgan and B.Q. Tang, Ann. Inst. Henri Poincaré, Anal. Non Linéaire 37 (2020), 281–307 and K. Fellner, J. Morgan and B.Q. Tang, Discrete Contin. Dyn. Syst., Ser. S 14 (2021), 635–651.

OP4.4: Partially solved in R. Filippucci, P. Pucci and Ph. Souplet, Comm. Partial Differential Equations 45 (2020), 321–349 (the tangential profile is always more singular than the normal profile) and in N. Mizoguchi and Ph. Souplet, J. Geom. Anal. 33 (2023), 42 (complete classification of bubbling space-time behaviors in one space dimension).

OP4.6: Partially solved (the answer is yes for $p \leq 3$) in A. Attouchi and Ph. Souplet, Calc. Var. PDE 59 (2020), 153.

Updates of results

The updates of Appendix I above imply that many important results can be improved. For example, the update of OP2.3 implies that the assumptions $p < p_B$ or $p < p_{sg}$ or $p < \max(p_B, p_{sg})$ in Theorems 21.2, 21.2a, 21.8*, 26.8*, 26.9*, 38.1*, Remarks 20.12(ii), 21.9(b), 22.3, 23.3(a), 28.3, 28.8(i), the last paragraph before Proposition 28.4, the last sentence in Subsection 28.4 and the first paragraph of Remark 32.8a can be replaced by the assumption $p < p_S$.

In addition to the improvements following from the updates of Appendix I, several improvements have been achieved, for example:

p. 253: Formula (25.22) can be replaced by the simpler formula

$$u(x,t) = (\kappa + o(1))\left[T - t + \frac{p-1}{8p}\frac{|x|^2}{|\log|x||}\right]^{-\frac{1}{p-1}}$$

and the subsequent formula remains true without the assumption $t \geq T - |x|^2$. This follows from L. Chabi, Ph. Souplet, Math. Ann. 391 (2025), 4509-4554.

p. 308, Remark 28.8(i): For (possibly nonconvex) bounded domains and any $p > 1$, the decay of all subthreshold solutions was proved in P. Quittner and Ph. Souplet, Preprint arXiv:2504.11813 (Proc. London Math. Soc., to appear), which also contains new results for $\Omega = \mathbb{R}^n$.

p. 465, l. 6: For $\Omega = (0,1)$, further examples of solutions with more singular gradient blow-up rate have been given in A. Attouchi and Ph. Souplet, Calc. Var. PDE 59 (2020), 153, and a complete classification of gradient blow-up rates has then been obtained in N. Mizoguchi and Ph. Souplet, J. Geom. Anal. 33 (2023), 42.

p. 471, last sentence of Remark 40.24(c): A complete description of this behavior has been obtained in N. Mizoguchi and Ph. Souplet, Asympt. Anal. 133 (2023), 291-353.

Errata

p. 114, l. -1: The product $\chi_{\{R/2<|x|<R\}}\chi_{\{R^2/2<t-t_0<R^2\}}$ should be replaced with χ_{D_R}, where $D_R := \{R/2 < |x| < R,\ 0 < t - t_0 < R^2\} \cup \{|x| < R,\ R^2/2 < t - t_0 < R^2\}$.

p. 115, formula (18.4): The domains of integration in the last two integrals should be replaced with D_R (where D_R is as above).

p. 125, l. -7: The maximum principle used here is not covered by the results in Section 52, but can be derived from (1.3) and $\psi = \psi_+ - \psi_-$ by writing:

$$\lambda_1\|\psi_-\|_2^2 \leq \|\nabla\psi_-\|_2^2 = -\int_\Omega \nabla\psi\cdot\nabla\psi_- = \int_\Omega (\Delta\psi)\psi_- \leq -(\lambda_1-\varepsilon)\int_\Omega \psi\psi_- = (\lambda_1-\varepsilon)\|\psi_-\|_2^2.$$

p. 196, the proof of Proposition 22.5 for $p = 2$: $a = A/24$.

p. 369, l. 7: Citation [727, pp. 467–472] refers to the English translation of the book.

p. 451, Remark 40.6(ii) (Neumann problem): The case of higher dimensions is treated in S. Dabuleanu, J. Evol. Equ. 5 (2005), 35–60. See also J. Domínguez-de-Tena and Ph. Souplet, J. Elliptic Parabol. Equ. (2024), https://doi.org/10.1007/s41808-024-00311-6.

p. 620, l. 5 in the proof of Proposition 52.8: "a.e. in Q_T" should be replaced with "a.e. in $Q_T \cap \{z \geq 0\}$".

p. 621, Remark 52.9: For a more precise formulation and a detailed proof see J. Domínguez-de-Tena and Ph. Souplet, J. Elliptic Parabol. Equ. (2024), https://doi.org/10.1007/s41808-024-00311-6.

p. 687: Ref. [407] is not ordered alphabetically.

p. 697: The correct page numbers of ref. [565] are 63–71.