

One of the fundamental problem in the spectral theory (or functional analysis) can be stated as: *find self-adjoint extensions of a densely defined closed symmetric operator*. We present our recent results concerning a solution of this problem in connection with the (time reversed) discrete symplectic system

$$z_k(\lambda) = \mathbb{S}_k(\lambda) z_{k+1}(\lambda), \quad \mathbb{S}_k(\lambda) := \mathcal{S}_k + \lambda \mathcal{V}_k, \quad (\text{S}_\lambda)$$

where  $\lambda \in \mathbb{C}$  is the spectral parameter and the  $2n \times 2n$  complex-valued matrices  $\mathcal{S}_k$  and  $\mathcal{V}_k$  are such that

$$\mathcal{S}_k^* \mathcal{J} \mathcal{S}_k = \mathcal{J}, \quad \mathcal{V}_k^* \mathcal{J} \mathcal{S}_k \text{ is Hermitian}, \quad \mathcal{V}_k^* \mathcal{J} \mathcal{V}_k = 0, \quad \Psi_k := \mathcal{J} \mathcal{S}_k \mathcal{J} \mathcal{V}_k^* \mathcal{J} \geq 0$$

with the skew-symmetric  $2n \times 2n$  matrix  $\mathcal{J} := \begin{pmatrix} 0 & I \\ -I & 0 \end{pmatrix}$ , the superscript  $*$  denoting the conjugate transpose, and  $k$  belonging to a discrete interval  $\mathcal{I}$ , which is finite or unbounded above. In contrast to the classical case, it is worth noticing that system  $(\text{S}_\lambda)$  does not produce a densely defined operator and so we need to employ the theory of linear relations. In addition, we discuss also the theory of square summable solutions of system  $(\text{S}_\lambda)$ , i.e., solutions of  $(\text{S}_\lambda)$  satisfying  $\sum_{k \in \mathcal{I}} z_k^*(\lambda) \Psi_k z_k(\lambda) < \infty$ , which are utilized in the given characterization.