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CALIBRATION OF ONE FACTOR INTEREST RATE MODELS

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In this paper we introduce a two step optimization method for calibration of one factor interest rate models, in particular of Cox, Ingersoll, and Ross model (CIR) and Vašiček model. In the first optimization step we minimize the sum of squares of differences of theoretical yield curve computed from given models and real market yield curve. For this computation we use an evolution strategy algorithm. The minimum is achieved on a one dimensional curve. In the second step we find the global maximum of the likelihood function over this curve. We introduce the results of calibration for stable western European financial markets and their comparison to emerging economies like, e.g., Slovakia, the Czech Republic and Poland.

K e y w o r d s: interest rate model, parameter estimation

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1 INTRODUCTION

There are many attempts to calibrate one factor interest rate models. Classical approaches are based on Generalized Method of Moments [1], Gaussian estimation method [5] or Monte Carlo filtering approach [8]. In all these cases the purpose is to estimate the model parameters for various western financial markets. For Central European financial markets Vojtek (see [9]) estimated conditional volatilities by using various types of GARCH models. Less attention is however put on one factor model parameters estimation and its possible application to Central European countries. The main goal of this paper is to make a contribution to this field and to estimate and compare one factor model parameters for several Central European financial markets.

In this paper we propose and analyze a new method for calibration of one factor interest rate models to estimate model parameters. Two models are discussed in this paper — the CIR and Vašiček interest rate models. Both of them have explicit solutions. We present a min-max optimization method for calibration of considered models. In the first step we minimize the sum of squares of differences of theoretical yield curve computed from given models and real market yield curve. The minimum is attained on a one dimensional curve in four dimensional model parameter space. Then by maximization of the likelihood function on this curve we obtain the four investigated parameters. The novelty of the method, to our best knowledge, consists in the second step in which we maximize likelihood function over restricted parameter space. Finally, our method is applied to real market term structure data from Euro-zone markets like EURO-LIBOR, and emerging economies like Poland (WIBOR), Slovakia (BRIBOR) and the Czech Republic (PRIBOR).

The paper is organized as follows: in Section 2 we briefly review the one factor interest rate models, in particular CIR and Vašiček models. We also present the transformation of parameters of these models and the optimal choice of some of these parameters. Section 3 describes the two step optimization method. We also discuss a numerical method for finding the minimum of the cost functional. In Section 4 we present the results of calibration for above mentioned real market data and Section 5 concludes the paper.

2 ONE FACTOR INTEREST RATE MODEL

One factor interest rate models are derived from an assumption made on the behavior of a short interest rate. We assume that the short rate follows the mean reverting process of the form:

$$\mathrm{d}r_t = \kappa(\theta - r_t)\mathrm{d}t + \sigma r_t^{\gamma}\mathrm{d}w_t \tag{1}$$

where $\{w_t, t \geq 0\}$ denotes the standard Wiener process and κ, θ, σ are positive constants. Parameter κ is the speed of reversion, σ is the volatility of the process and θ is the limiting interest rate. The parameter γ is determining the type of the model. If $\gamma = \frac{1}{2}$ then the model derived from (1) is referred to as the Cox, Ingersoll, and Ross (CIR) model [3, 4]. If $\gamma = 0$ then it is called the Vašiček model [3, 4]. The main step of deriving a one factor model is a construction of risk-less portfolio of two bonds with different maturities. Applying the Itô lemma we next obtain a parabolic partial differential equation for the price of the zero coupon bond $P = P(t, T, r)$

$$\frac{\partial P}{\partial t} + (\kappa(\theta - r) - \tilde{\lambda}(r)\sigma r^{\gamma})\frac{\partial P}{\partial r} + \frac{1}{2}\sigma^2 r \frac{\partial^2 P}{\partial r^2} - rP = 0\,, \tag{2}$$

where $t \in (0, T)$ and $r > 0$. The parameter $\tilde{\lambda}$ is different for the two discussed models. For the CIR model we take $\tilde{\lambda}(r) = \lambda r^{\frac{1}{2}}/\sigma$ whereas for Vašiček model we

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take $\tilde{\lambda}(r) = \lambda$. The parameter λ represents the so-called market price of risk. A solution P to (2) must satisfy the terminal condition $P(T,T,r) = 1$ for any $r > 0$. There is an explicit solution of PDE (2) for both models and it can be written as: $P(T-\tau,T,r) = A(\tau)e^{-B(\tau)r}$, $\tau = T - t \in [0,T]$, where the functions A, B satisfy

$$B(\tau) = \frac{1-e^{-\kappa\tau}}{\kappa}$$
$$A(\tau) = \exp\Big[(B-\tau)\Big(\theta - \frac{\sigma^2}{2\kappa^2} - \frac{\sigma\lambda}{\kappa}\Big) - \frac{\sigma^2 B^2}{4\kappa}\Big]$$

($B = B(\tau)$) for the Vašiček model, and

$$B(\tau) = \frac{2(e^{\eta\tau}-1)}{(\kappa+\lambda+\eta)(e^{\eta\tau}-1)+2\eta}$$
$$A(\tau) = \Big(\frac{\eta e^{(\kappa+\lambda+\eta)\tau/2}}{e^{\eta\tau}-1}B(\tau)\Big)^{\frac{2\kappa\theta}{\sigma^2}}$$

and $\eta = \sqrt{(\kappa+\lambda)^2 + 2\sigma^2}$ for the CIR model.

2.1 Parameter reduction

The idea of reducing the four dimensional parameter space into three parameters is possible in the case of CIR and Vašiček model, too. The parameter reduction for the CIR model consists in introduction the following new variables:

$$\beta = e^{-\eta}, \quad \xi = \frac{\kappa+\lambda+\eta}{2\eta}, \quad \varrho = \frac{2\kappa\theta}{\sigma^2} \tag{3}$$

where $\eta = \sqrt{(\kappa+\lambda)^2 + 2\sigma^2}$. Returning back to the original CIR parameters we have

$$\kappa = \eta(2\xi - 1) - \lambda, \quad \sigma = \eta\sqrt{2\xi(1-\xi)}, \quad \theta = \frac{\varrho\sigma^2}{2\kappa}.$$

where $\eta = -\ln\beta$. Functions $A(\tau), B(\tau)$ can be expressed in terms of new variables β, ξ, ϱ as follows:

$$B(\tau) = -\frac{1}{\ln\beta}\frac{1-\beta^\tau}{\xi(1-\beta^\tau)+\beta^\tau},$$
$$A(\tau) = \Big(\frac{\beta^{(1-\xi)\tau}}{\xi(1-\beta^\tau)+\beta^\tau}\Big)^{\varrho}.$$

As far as the Vašiček model is concerned we put

$$\beta = e^{-\kappa}, \quad \xi = \theta - \frac{\sigma^2}{2\kappa^2} - \frac{\sigma\lambda}{\kappa}, \quad \varrho = \frac{\sigma^2}{4\kappa}. \tag{4}$$

Then for the original Vašiček parameters we have:

$$\kappa = -\ln\beta, \quad \sigma = 2\sqrt{\rho\kappa}, \quad \theta = \xi + \frac{\sigma^2}{2\kappa^2} + \frac{\sigma\lambda}{\kappa}.$$

and, consequently,

$$B(\tau) = -\frac{1-\beta^\tau}{\ln\beta},$$
$$A(\tau) = \exp\Big(\xi(B(\tau)-\tau) - \varrho B^2(\tau)\Big).$$

Summarizing, in both studied one factor models the yield curve depends only on three transformed parameters β, ξ and ϱ defined in (3) and (4), respectively.

3 TWO STEP OPTIMIZATION METHOD FOR CALIBRATION OF ONE FACTOR INTEREST RATE MODELS

In this section the calibration method for estimation of one factor models parameters is discussed. The main principles of the calibration of the CIR and Vašiček model parameters are the same.

3.1 Minimization of the cost functional

In order to measure the quality of approximation of the set of real market yield curves by computed yield curves from each model corresponding to present values of the short interest rate value we consider the cost functional: $U(\beta,\xi,\varrho) = \frac{1}{m}\sum_{j=1}^m \frac{1}{n}\sum_{i=1}^n (R_j^i - \bar{R}_j^i)^2\tau_j^2$. It measures the time-weighted distance of the real market yield curves $\{R_j^i, j = 1,\dots,m\}$ and the set of computed yield curves $\{\bar{R}_j^i, j = 1,\dots,m\}$ at time $i = 1,\dots,n$, determined from the bond price — yield curve relationship $A_j e^{-B_j R_0^i} = e^{-\bar{R}_j^i \tau_j}$, where $r^i = R_0^i$ is the overnight interest rate at time $i = 1,\dots,n$, $A_j = A(\tau_j)$ and $B_j = B(\tau_j)$ where $\tau_1 < \tau_2 < \dots < \tau_m$ stand for maturities of bonds forming the yield curve. We put $\tau_0 = 0$. Expression of the cost functional can be rewritten in form:

$$U(\beta,\xi,\varrho) = \frac{1}{m}\sum_{j=1}^m \Big((\tau_j E(R_j) - B_j E(R_0) + \ln A_j)^2 + D(\tau_j R_j - B_j R_0)\Big) \tag{6}$$

where $E(X_j)$ and $D(X_j)$ denote the mean value and dispersion of the vector $X_j = \{X_j^i, i = 1,\dots,n\}$. So in the first step we find the minimum of the cost functional (6). We obtain the vector of (β,ξ,ϱ) for any given λ.

The parameter reduction described in the previous section can be followed by optimal selection of some of the parameters. Given β and ξ we can pick an optimal value for ϱ parameter in the CIR model; $\varrho_c^{opt} = \varrho_c^{opt}(\beta,\xi)$. Indeed, solving the first order optimality condition $\frac{\partial U}{\partial \varrho} = 0$ yields:

$$\sum_{j=1}^m (\ln A_j)^2 = -\sum_{j=1}^m (\tau_j E(R_j) - B_j E(R_0))\ln A_j$$

and so the optimal ϱ_c can be determined as follows:

$$\varrho_c^{opt} = -\frac{\sum_{j=1}^m (\tau_j E(R_j) - B_j E(R_0))\ln A_j(\beta,\xi,1)}{\sum_{j=1}^m (\ln A_j(\beta,\xi,1))^2}.$$

For Vašiček model it can be derived in a similar way by solving the equation $\frac{\partial U}{\partial \varrho} = 0$. We have

$$0 = \sum_{j=1}^{m} (\tau_j E(R_j) - B_j E(R_0) + \xi(B_j - \tau_j) - \varrho B_j^2) B_j^2.$$

It implies that for the optimal ϱ_v^{opt} for Vašiček model we have

$$\varrho_v^{opt} = \frac{\sum_{j=1}^{m} (\tau_j E(R_j) - B_j E(R_0) + \xi(B_j - \tau_j)) B_j^2}{\sum_{j=1}^{m} B_j^4}.$$

In the case of Vašiček model we can furthermore pick up the optimal value for the parameter ξ_v^{opt} by solving $\frac{\partial U}{\partial \xi} = 0$. Then

$$0 = \sum_{j=1}^{m} (\tau_j E(R_j) - B_j E(R_0) + \xi(B_j - \tau_j) - \varrho B_j^2)(B_j - \tau_j)$$

and the optimum parameter ξ_v^{opt} is:

$$\xi_v^{opt} = -\frac{\sum_{j=1}^{m} (\tau_j E(R_j) - B_j E(R_0) - \varrho B_j^2)(B_j - \tau_j)}{\sum_{j=1}^{m} (B_j - \tau_j)^2}.$$

Solving the above two linear equations for optimal values ξ_v^{opt} and ϱ_v^{opt} we are yet able to conclude that ξ_v^{opt} and ϱ_v^{opt} depend on β only.

Summarizing, for the CIR as well as for the Vašiček model we have first order necessary conditions for the minimizer of the cost functional. These conditions can be used either for further parameter reduction of the problem (2D problem for the CIR model and even 1D problem for the Vašiček model) or for testing whether a numerical approximation is close to a minimizer. The latter property has been used in practical implementation of the minimization method.

We briefly discuss an optimization method for finding the minimum of the cost functional U. The function U need not be necessarily convex and therefore gradient method like, e.g., Newton-Kantorovich method may capture a local minimum only. This is why we used a robust numerical method generically converging to the global minimum of U. There is a wide class of optimization methods based on stochastic algorithms. These methods are referred to as Evolution strategies (ES) (see, e.g., [6, 7]). In our case we used a slight modification of the well known $(p+c)$ ES. Recall that the $(p+c)$ ES has p parents and c children (offsprings) per population among which the p best individuals are selected to be next generation parents by their fitness value. The procedure is repeated until some termination criterion is satisfied.

Now we describe our modification of $(p+c)$ ES called $(p+c+d)$ ES henceforth. In the start-up of the $(p+c+d)$ ES we randomly generate an initial population of p parents consisting of vectors (β, ξ, ϱ). There are prescribed lower and upper bounds for components β, ξ and ϱ of each vector in the starting population. In each step of the ES algorithm we generate a set of c offsprings from the parent population ($c \le p$). c individuals from the parent generation are perturbed by the Gaussian noise with a zero mean and a fixed dispersion (0.01 in our case). The modification $(p + c + d)$ ES comprise selection on wider set. It means that we include a randomly generated set of d wild type individuals forming the so-called wild population. The procedure of generation of the wild type population is the same as for generation of the initial population. Notice that we construct offsprings and wild type individuals in such a way that the prescribed bounds for β, ξ and ϱ of each vector in the set are satisfied. To each of the c offsprings together with the d individuals of the wild population we assign a fitness value representing the value of the cost functional U. Next we include a corrector step consisting of improving the set of p parents by NK iterates of the Newton-Kantorovich gradient minimization method. As a result we obtain a set of p improved parents. The best p individuals from the set of p parents, p improved parents, c offsprings and d wild type individuals are selected to be the next generation of parents. We repeat this procedure until the overall number of steps is less than N. We also perform the first order necessity test as described above. In our computations we chose $N = 300$, $p = c = d = 10^5$ and $NK = 30$. For further details concerning ES based stochastic algorithms and their convergence properties we refer to [6, 7].

Similarly as in the case of gradient optimization methods, for a general minimized function, an ES based stochastic algorithm need not necessarily converge to the global minimum. Additional assumptions like, e.g., convexity made on a minimized function are required. We are unable to verify these conditions in our particular case. Nevertheless, our numerical experience based on repeated experiments with different numerical constants indicates that the ES algorithm described above indeed converges to a global minimum of the cost functional U. Moreover, an important question concerning existence and uniqueness of a global minimum of U on Ω arises. Notice that data vectors R_j, $j = 0, \dots, m$, enter expression for U in terms of their means and covariances. Now if a global minimum of U is attained at several minimizers then one can perturb input data vectors R_j slightly in order to destroy their multiplicity achieving thus a unique global minimizer of U. Therefore, for generic data vectors R_j, $j = 0, \dots, m$, there exists a unique global minimizer of U.

Finding the global unique minimum of U completes the first step of the two step optimization procedure.

3.2 Maximization of a restricted likelihood function

Now we proceed with a second step of our method. Notice that the aim of the first "minimization" step of the method described in Section 3.1 was to find a point $(\breve{\beta}, \breve{\xi}, \breve{\varrho})$ — a unique global minimum of the cost functional $U = U(\beta, \xi, \varrho)$. Bearing in mind parameter

Table 1. Numerical results of calibration for short term structures for BRIBOR, WIBOR, PRIBOR and EURO-LIBOR (EULIB).

BRIBOR	κ	σ	θ	λ	MLR
1/4 2003	688.298	8.960	0.0025	-658.6	0.528
2/4 2003	38.467	1.509	0.0458	-6.3	0.719
3/4 2003	598.875	8.276	0.0031	-568.8	0.536
4/4 2003	793.487	9.396	0.0022	-764.1	0.551
WIBOR					
1/4 2003	10.103	0.622	0.0564	-0.362	0.702
2/4 2003	7.459	0.877	0.0204	-4.457	0.519
3/4 2003	193.565	6.097	0.0029	-189.9	0.371
4/4 2003	2.910	0.842	0.0004	-3.09	0.388
PRIBOR					
1/4 2003	0.098	0.007	0.0248	0.092	0.904
2/4 2003	36.934	0.728	0.0209	-4.714	0.514
3/4 2003	2.823	0.060	0.0201	-0.200	0.814
4/4 2003	3.385	0.097	0.0187	-0.626	0.685
EULIB					
1/4 2003	34.118	0.689	0.0241	-1.897	0.712
2/4 2003	0.734	0.024	0.0244	0.276	0.864
3/4 2003	40.018	0.699	0.0175	-7.797	0.703
4/4 2003	9.217	0.286	0.0178	-2.243	0.758

reduction described in Section 2.1 there exists a C^∞ smooth one dimensional curve of original model parameters $(\kappa_\lambda, \theta_\lambda, \sigma_\lambda, \lambda) \in R^4$ parameterized by $\lambda \in \check{J}$ corresponding to the same transformed triple $(\check{\beta}, \check{\xi}, \check{\varrho})$ for which the minimum of U (in terms of transformed variables β, ξ, ϱ) is attained.

In order to construct estimation of the model parameters $\kappa, \theta, \sigma, \lambda$ we proceed with the second optimization step in which we find the global maximum of the standard Gaussian likelihood function (LF) over the above mentioned λ parameterized curve representing of global minimizers of the cost functional U. The two step optimization method combines the maximum likelihood estimation with minimization of the cost functional U. In the case of parameter estimation of a stand-alone short rate process having the form (1) the LF is:

$$\ln L(\kappa, \sigma, \theta) = -\frac{1}{2} \sum_{t=2}^{n} \Big(\ln v_t^2 + \frac{\varepsilon_t^2}{v_t^2} \Big)$$

where $v_t^2 = \frac{\sigma^2}{2\kappa}\left(1 - e^{-2\kappa}\right) r_{t-1}^{2\gamma}$, $\varepsilon_t = r_t - e^{-\kappa} r_{t-1} - \theta(1 - e^{-\kappa})$ (see [2]). If estimation of model parameters (κ, σ, θ) is realized by maximization of likelihood function over the whole set R_+^3 then the maximum is unrestricted. The value of the unrestricted maximum likelihood function is:

$$\ln L^u = \ln L(\kappa^u, \sigma^u, \theta^u) = \max_{\kappa, \sigma, \theta > 0} \ln L(\kappa, \sigma, \theta).$$

In our approach we make use of restricted maximization of $\ln L$ over the λ parameterized curve $\{(\kappa_\lambda, \theta_\lambda, \sigma_\lambda), \lambda \in \check{J}\}$. This can be expressed in original model parameters as follows:

$$\ln L^r = \ln L(\kappa_{\bar\lambda}, \sigma_{\bar\lambda}, \theta_{\bar\lambda}) = \max_{\lambda \in \check{J}} \ln L(\kappa_\lambda, \sigma_\lambda, \theta_\lambda),$$

where $\check{J} = (-\infty, \check{\eta}(2\check{\xi} - 1))$ in the case of the CIR model and $\check{J} = R$ for Vašiček model. The argument $\bar\kappa = \kappa_{\bar\lambda}, \bar\sigma = \sigma_{\bar\lambda}, \bar\theta = \theta_{\bar\lambda}$ of the maximum of the restricted likelihood function $\ln L^r$ is adopted as a result of two step optimization method for calibrating the model parameters. A global maximizer of the unrestricted likelihood function $\ln L^u$ has been computed by the same variant of the ES algorithm described in the previous section. Since maximization of the restricted likelihood function $\ln L^r$ is performed over one dimensional parameter λ and the function $\lambda \mapsto \ln L(\kappa_\lambda, \sigma_\lambda, \theta_\lambda)$ is smooth we could apply a standard optimization software package Mathematica in order to find a global maximizer of the restricted likelihood function.

For measuring the accuracy of calibration we introduce the maximum likelihood ratio (MLR) as a ratio of the maximum values of the restricted $\ln L^r$ and unrestricted $\ln L^u$ likelihood functions. We have $MLR \leq 1$ and if MLR is close to 1 then the restricted maximum likelihood value is close to the unrestricted one. In this case one can therefore expect that the estimated values $(\bar\kappa, \bar\sigma, \bar\theta)$ of the model parameters are close to the argument $(\kappa^u, \sigma^u, \theta^u)$ of the unique global maximum of the unrestricted likelihood function. It may indicate that a simple estimation of parameters based on the mean reversion equation (1) for the short rate process r_t is also suitable for estimation of the whole term structure.

4 RESULTS OF CALIBRATION

In this section we present the results of calibration for various European financial markets data like BRIBOR (Slovakia), PRIBOR (the Czech Republic),WIBOR (Poland) and EURO-LIBOR (Euro-zone data). In Table 1 there are quarterly results of calibration for BRIBOR, WIBOR, PRIBOR and EURO-LIBOR for year 2003. There are presented estimated parameters $\kappa, \theta, \sigma, \lambda$, and, the value of the Maximum likelihood ratio.

Behavior of the long term average interest rate θ is confirming the character of real interest rate of given data in a given period. The market price of risk λ is negative in most time periods. There are some short time periods, where the market price of risk is positive for EURO-LIBOR (2nd quartal). The volatility of the mean reverting process is very high for Slovak data. The same behavior can be observed for the parameter κ. On the other hand, results for the Czech data enables us to conclude that behavior of PRIBOR is very similar to EURO-LIBOR. The overall quality of calibration is better for

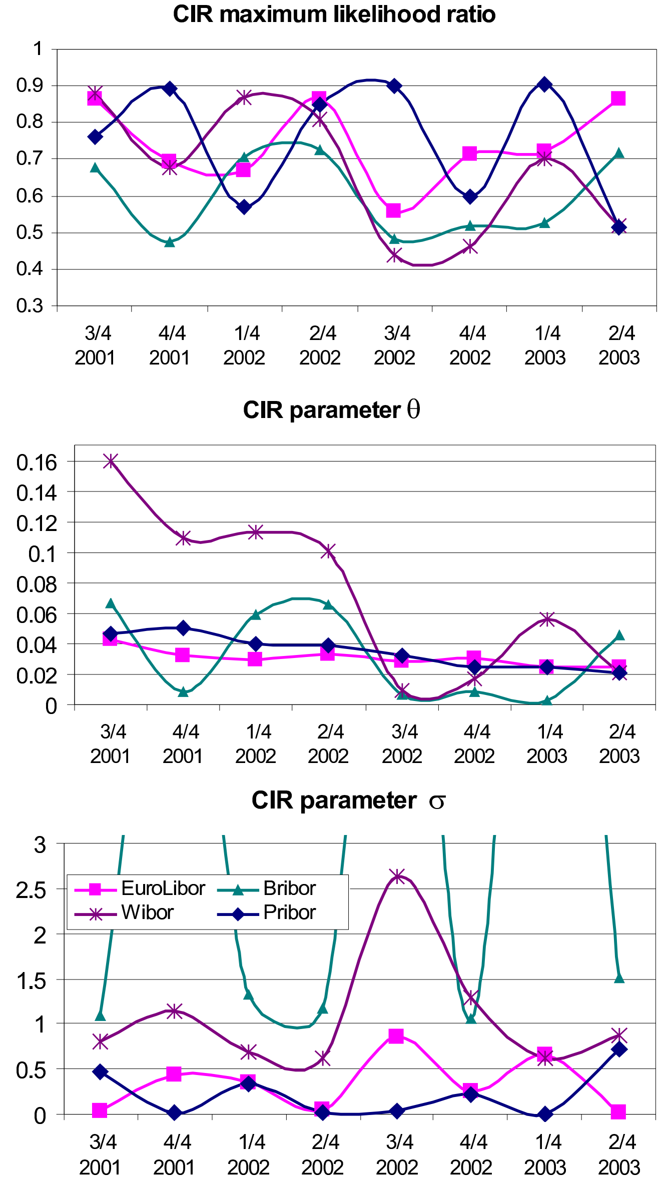

Fig. 1. Results of parameter estimation for various term structures.

PRIBOR and EURO-LIBOR. The maximum likelihood ratio (MLR), which measures the appropriateness of CIR model for the data, is higher for western European financial market data than for the emerging economies except of the Czech PRIBOR.

In Fig. 1 we compare the results of calibration for EURO-LIBOR, BRIBOR, PRIBOR and WIBOR. This graph displays MLR and estimated parameters θ and σ for the CIR model. Mostly, the MLR is better for EURO-LIBOR and worst for BRIBOR. For the last two samples (PRIBOR and WIBOR), it is varying. For PRIBOR and EURO-LIBOR the results are similar not only for θ but also for σ. The estimated volatility σ is considerably high for the Slovak term structure. The parameter θ is also quite volatile for the Slovak data.

5 DISCUSSION AND CONCLUSIONS

In this article we have presented a nonlinear regression method for calibration of well known Cox, Ingersoll, and Ross model and Vašiček model. In the first optimization step by ES, we have found the minimum of the cost functional for fixed λ. In the second optimization step we have maximized the maximum likelihood function over λ. We have calibrated real market term structures from various economies (stable western and emerging) for years 2001-2003. The accuracy of the calibration was tested with MLR tests. The MLR was in the range 0.7-0.9 for the western Europe markets data and it belonged to the range 0.4-0.6 for the emerging economies. According to our results we can state that the western European markets data are better described by CIR model. In emerging economies we can also use the CIR model but only for the Czech data.

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