Dynamic and Static Strategies for the Funded Pillar of the Slovak Pension System

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Abstract

Since January 2005, pensions in Slovakia are operated by a three-pillar system proposed by the World Bank. The paper discusses and mathematically captures principles of the pension reform in Slovakia. We also discuss the impact of the reform on the deficit of the pension system. We mainly focus on the mandatory, fully funded second pillar. We present a dynamic decision model for determining the optimal switching strategy between pension funds with different risk profiles. The resulting strategies depend on individual risk preferences of future pensioners. Our results illustrated the fact that gradual decreasing of the risk during the saving for future pension is rational. Moreover, it is shown that dynamic decision strategies outperform the static ones.

1. Introduction

Before January 2005, pensions in Slovakia were operated by the unfunded pay-as-you-go system. Mainly because of high unemployment and low contributions paid on behalf of unemployed by the government as well as high contribution evasions, the system generated deficits. The demography crisis was supposed to generate further pressure on the balance of the pay-as-you-go system. In April 2003 the government passed the Principles of the Pension Reform in the Slovak Republic. The goals of the pension reform were to secure a stable flow of high pensions to the beneficiaries, and sustainability and overall stability of the system. The corresponding legislation was passed in December 2003. Three important elements of the pension reform were:

1. change of the indexation of pensions
2. increasing the retirement age
3. launching a funded pillar

The former indexation by wage growth was replaced by swiss indexation (average of wage growth and inflation). The former retirement age for men 60 and for women 54 was increased to 62 for both sexes with gradual increase 9 months every year. The reform established a system based on three pillars:

1. the mandatory non-funded first pillar (pay-as-you-go pillar)
2. the mandatory fully funded second pillar
3. the voluntary fully funded third pillar

The contribution rates were set for the first pillar at 19.75% (old age 9%, disability and survival 6% and reserve fund 4.75%) and for the second pillar 9%. The total rate is about 0.75% higher than the old one.

Fund type	Stocks	Bonds and money market instruments
Growth Fund	up to 80%	at least 20%
Balanced Fund	up to 50%	at least 50%
Conservative Fund	no stocks	100%

Table 1: Limits for investment for the pension funds (second pillar)

The new system is obligatory for those entering the labor market, and optional for existing contributors of age below 52 years, who therefore would loose option to return to the old system, but would keep benefits acquired in the old system (they will receive full pension for years participated in the old system, and half a pension corresponding to their participation in the new system).

A thorough description of the Slovak pension reform with calculations of the balance of the pension system and expected level of pensions in the new system could be found in (Goliaš, 2003), (Melicherčík and Ungvarský 2004), (Thomay 2002).

Compared to Poland and Hungary, the Slovak second pillar is more substantial. Contribution rates are higher in Slovakia - compared to 7.3% in Poland and 6% (with possible future increase to 8%) in Hungary. A thorough description of the pension reforms in Hungary and Poland could be found in (Benczúr 1999), (Chlon et al. 1999), (Fultz 2002), (Palacios & Rocha 1998) and (Simonovits 2000).

The savings in the second pillar are managed by pension asset administrators. Each pension administrator manages three funds: Growth Fund, Balanced Fund and Conservative fund, each of them with different limits for investment (see Tab. 1). At the same time instant savers may hold assets in one fund only. Up to 15 years before retirement, the saver may not hold assets in the Growth Fund and up to 7 years all assets must be in the Conservative Fund. The main goal of this paper is to study the level of pensions in the new system. We present a dynamic accumulation model in which the contributor at each time instant using his/her risk preferences and the information about past behaviour of the financial markets chooses one of the possible funds (Growth, Balanced and Conservative). We have illustrated that adopted pension fund regulations can be supported by means of this model.

The paper is organized as follows: In Section 2 we briefly present the impact of the pension reform on the deficit of the pension system. Section 3 contains the formulation of the dynamic accumulation model. In Section 4 we compare dynamic and static strategies. We also discuss sensitivity of dynamic fund switching strategies with respect to various scenarios of development of financial markets, wage growth development as well as individual risk preferences. The last section contains final remarks and conclusions.

2. Pension reform and the balance of the new pension system

As a consequence of the negative demographic development, in the last time the pay-as-you-go pension systems started to generate deficits. The ratio of a number of men older than 60

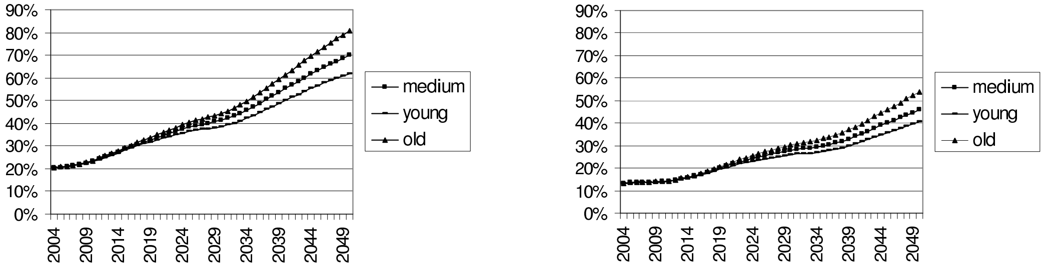


Figure 1: The ratio of the number of men older than 60 to those 18-60 years old (left) and older than 65 to those 18-65 years old (right) according to the young, medium and old scenarios of the demography development in Slovakia. Source: Melicherčík and Ungvarský 2004.

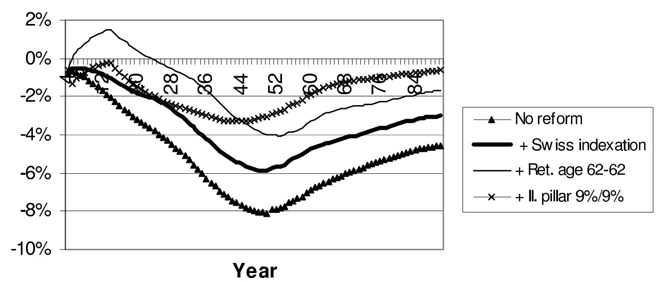


Figure 2: Impact of important steps of the pension reform on the balance of the pension system. The balance of the pension system (in years 2006-2090) is depicted in percentage of GDP. Source: Melicherčík and Ungvarský 2004.

to those 18-60 years old and the ratio of a number of men older than 65 to those 18-65 years old (see Fig. 1) clearly indicates that the deficit of the pension system could be significantly decreased by a higher retirement age.

The balance of the pension system also depends on a method of indexation of pensions. In the former system, the pensions were formally indexed by the wage growth. This indexation was never fulfilled completely because of a big pressure on a public finance. Since in the future a positive real wage growth is supposed, the indexation by the inflation could decrease the deficit of the pension system. Finally, the compromise was chosen: the Swiss indexation (the average of the wage growth and the inflation).

The most important step of the pension reform was the launch of the second (funded) pillar. This creates deficit pressures, because some contributors switch part of their contributions from the first (pay-as-you-go) pillar to the second pillar. However, once pensions will be paid from the second pillar, expenditures of the first pillar will decrease, as those who switched will receive lower pensions from the first pillar. According to the law, people older than 52 will remain in the old system. The other (that have already entered the labor market) have a choice to switch. In the calculations we assume that all between 18 and 25 years old will switch to the new system. Then the percentage of those who switch decline linearly, and only 5% of 52 year old switch.

In Fig. 2 we can see the impact of the important steps of the pension reform on the deficit

Year	Wage growth (real)	Infl. rate	Unemploy-ment rate	GDP growth (real)	Year	Wage growth (real)	Infl. rate	Unemploy-ment rate	GDP growth (real)
2006	4.0	3.5	14.4	5.5	2016	3.5	3.0	11.2	4.0
2007	4.0	3.5	14.0	4.5	2017	3.5	3.0	11.0	4.0
2008	4.0	3.5	13.6	4.5	2018	3.5	3.0	10.8	4.0
2009	3.5	3.5	13.2	4.5	2019	3.5	3.0	10.6	3.5
2010	3.5	3.5	12.8	4.5	2020	3.5	3.0	10.3	3.5
2011	3.5	3.5	12.5	4.0	2021	3.5	3.0	10.0	3.5
2012	3.5	3.5	12.2	4.0	2022	3.5	2.5	9.5	3.5
2013	3.5	3.5	11.9	4.0	2023	3.5	2.5	9.0	3.5
2014	3.5	3.5	11.6	4.0	2024	3.5	2.5	8.5	3.5
2015	3.5	3.0	11.4	4.0	2025-90	3.0	2.0	8.0	3.0

Table 2: Macroeconomic forecasts. Source: Slovak Savings Bank (SLSP)

of the pension system. We start with no reform deficit development and then we gradually add Swiss indexation, increasing the retirement age to 62 for both sexes and finally the launch of the second pillar. It is clear that the change of indexation and increase of the retirement age have positive impact on the balance of the pension system. Concerning the launch of the second pillar, the positive impact could be seen later. After the year 2044 the deficit of the two-pillar system is lower than the deficit of the pure pay-as-you-go system. The macroeconomic forecasts used for the calculation are in Tab. 2.

Calculations for different scenarios of demographic development, different indexations of pensions and different retirement age could be found in (Melicherčík and Ungvarský 2004).

3. The dynamic accumulation model

Pension funds usually hold portfolio consisting of bonds and equities. Limits for their weights in the portfolio may differ across the countries. In Slovakia, each pension company manages three funds: Growth Fund, Balanced Fund and Conservative fund, each of them with different limits for investment (see Tab. 1). As it was already mentioned in Introduction, instant savers may hold assets in one fund only and they may not hold assets in the Growth Fund up to 15 years before retirement. Moreover, all assets should be held in the Conservative Fund up to 7 years before retirement. The intention of these restrictions and government regulations was to lower the risk of the value of savings to fall substantially shortly before retirement. Intuitively it is clear that e.g. a 20% fall of asset values after the first year of saving affects the level of future pension less than a 20% fall in the last year of saving leading to a 20% fall of the pension benefits. In (Kilianová et al. 2006) we give more precise mathematical arguments to justify the idea of gradual switching to the less risky funds as the retirement age approaches.

One can think about static strategies where the time instants where a contributor switches between the funds are determined at the beginning of the saving. The most risk averse contributor deposits the savings all the time to the Conservative Fund. The least risk averse investor contributes to the risky funds as long as the law permits it: up to 15 to the retirement age years to the Growth Fund, next 8 years to the Balanced Fund and the last 7 years to the Conservative Fund.

On the other hand, it makes sense to postpone the decision about switching between the funds to later times. It is, e.g. possible that in the first 10 years of saving the stock markets have excellent returns and even a risk less averse contributor decides to secure the saved amount by switching to the Balanced or Conservative Fund. In the next part we present a dynamic accumulation model, where the contributor at each time, having an information about the saved amount and past development of financial markets, decides what fund is for him/her optimal. In Section 4 we compare the performance of dynamic and static strategies.

3.1. Mathematical formulation of the dynamic accumulation model

Suppose that the future pensioner deposits once a year a τ-part of his/her yearly salary w_t to a pension fund $j \in \{1, 2, \ldots, m\}$. Denote by s_t, $t = 1, 2, \ldots T$ the accumulated sum at time t where T is the expected retirement time. Then the budget-constraint equations read as follows:

$$\begin{aligned} s_{t+1} &= s_t(1 + r_t^j) + w_{t+1}\tau \, , \; t = 1, 2, \ldots, T-1 \, , \\ s_1 &= w_1\tau \end{aligned} \tag{1}$$

where r_t^j is the return of the fund j in the time period $[t, t+1)$. When retiring the pensioner will strive to maintain his/her living standard in the level of the last salary. From this point of view, the saved sum s_T at the time of retirement T is not precisely what the future pensioner cares about. More important is the ratio of the cumulative sum s_T and the yearly salary w_T, i.e. $d_T = s_T/w_T$. Using the quantity $d_t = s_t/w_t$ one can reformulate the budget-constraint equation (1):

$$\begin{aligned} d_{t+1} &= F_t(d_t, j) \, , \; t = 1, 2, \ldots, T-1 \, , \\ d_1 &= \tau \end{aligned} \tag{2}$$

where

$$F_t(d, j) = d \, \frac{1 + r_t^j}{1 + \varrho_t} + \tau \, , \; t = 1, 2, \ldots, T-1 \tag{3}$$

and ϱ_t denotes the wage growth defined by the equation

$$w_{t+1} = w_t(1 + \varrho_t) \, .$$

Suppose that each year the saver has the possibility to choose a fund $j(t, I_t) \in \{1, 2, \ldots, m\}$, where I_t denotes the information consisted of the history of returns $r_{t'}^j$, $t' = 1, 2, \ldots, t-1$, $j \in \{1, 2, \ldots, m\}$ and the wage growth $\varrho_{t'}$, $t' = 1, 2, \ldots, t-1$. Now suppose that the history of the wage growth ϱ_t, $t = 1, 2, \ldots, T-1$ is deterministic and the returns r_t^j are assumed to be random and they are independent for different times $t = 1, 2, \ldots, T-1$. The relevant information is then the quantity d_t only. Hence $j(t, I_t) \equiv j(t, d_t)$. One can formulate a problem of dynamic stochastic programming:

$$\max_j E(U(d_T)) \tag{4}$$

with the following recurrent budget constraint:

$$\begin{aligned} d_{t+1} &= F_t(d_t, j(t, d_t)) \, , \; t = 1, 2, \ldots, T-1 \, , \\ d_1 &= \tau \end{aligned} \tag{5}$$

where the maximum is taken over all non-anticipative strategies $j = j(t, d_t)$. Here U stands for a given preferred utility function of wealth of the saver. Using the tower law for the conditional expectation

$$E(U(d_T)) = E(E(U(d_T)|I_t)) = E(E(U(d_T)|d_t))$$

we conclude that $E(U(d_T)|d_t)$ should be maximal. Let us denote

$$V_t(d) = \max_j E(U(d_T)|d_t = d) \, . \tag{6}$$

Then by using the tower law

$$E(U(d_T)|d_t) = E(E(U(d_T)|d_{t+1})|d_t)$$

we obtain the Bellman equation

$$V_t(d) = \max_{j \in \{1,2,\ldots,m\}} E[V_{t+1}(F_t(d, j))] = E[V_{t+1}(F_t(d, j(t, d)))] \, , \tag{7}$$

for $t = 1, 2, \ldots, T-1$, where $V_T(d) = U(d)$. Using (7) the optimal feedback strategy $j(t, d_t)$ can be found backwards. This strategy gives the saver the decision for the optimal fund for each time t and level of savings d_t. Details of the numerical procedure can be found in (Kilianová et al. 2006).

3.2. The iso-elastic utility function

An important part of the problem (4)-(5) is the choice of the utility function U. The utility function varies across the investors and represents their attitude to the risk. A key role in defining the utility function plays the coefficient of proportional risk aversion $C(x) = -xU''(x)/U'(x)$. Constant proportional risk aversion implies that people hold a constant proportion of their wealth in any class of risky assets as their wealth varies (see e.g. (Friend & Blume 1975), (Pratt 1964) and (Young 1990)). In this case the utility function is of the form

$$\begin{aligned} U(x) &= -Ax^{1-C} + B \quad \text{if } C > 1, \\ U(x) &= A\ln(x) + B \quad \text{if } C = 1 \, , \\ U(x) &= Ax^{1-C} + B \quad \text{if } C < 1 \end{aligned} \tag{8}$$

where A, B are constants and $A > 0$. One can easily prove that, concerning the problem (4)-(5), the utility functions U and $A.U + B$ are equivalent.

In our case constant risk aversion implies that the utility functions $U(d)$ and $U(\kappa d)$ where κ is a constant lead to the same strategies. We use the iso-elastic utility function

$$U(d) = \frac{1}{1-a}\left((\kappa d)^{1-a} - 1\right) \tag{9}$$

where $a > 0$ is the constant coefficient of proportional risk aversion. Using $\kappa = 1/12$ the utility function is "steeper" for reasonable values and the numerical procedure is more stable. Problem (4)-(5) then maximizes the expected utility of savings (compared to the last yearly salary) appointed for 1/12 of the yearly benefits (i.e. the benefits for 1 month). It is clear that maximizing monthly benefits or yearly benefits should lead to the same strategy and therefore the constant proportional risk aversion makes sense in our case.

The coefficient of proportional risk aversion a has been in the literature discussed for several years (and has not been resolved yet). There is a consensus today, that the value should be less than 10 (see e.g (Mehra and Prescott 1985)). In our typical results we considered values close to 17. It should be probably lower for lower stock returns. However, our goal was to formulate the mathematical model and to manage the numerical procedure. The reader can change the parameters and calibrate the model in his/her own way.

4. Pension portfolio simulations. Dynamic and static strategies

The purpose of this section is to present results of pension portfolio simulations for dynamic and static strategies. The results concerning the dynamic strategies will be summarized in graphical plots of the so-called optimal choice function $j = j(t, d)$ as well as several tables discussing computed results of optimization. The role of the optimal choice function $j = j(t, d)$ consists in providing an information when to switch between different funds for a given level of the ratio d of saved money and wage. We focus our attention to two basic questions and problems: 1) what are the regions of constant values of $j(t, d)$; 2) what is the path of expected values of d_t.

Before presenting results of simulation we have to discuss input data like e.g. fund structures and characteristics, the wage growth ϱ. Concerning the structure of funds we consider the present situation in Slovak Republic. According to the adopted government regulation there are three funds (i.e. $m = 3$). Namely, the Growth, Balanced and Conservative fund (see Tab. 1). Hereafter, we shall suppose that these three funds are constructed from stocks (S) and secure bonds (B) where stocks are represented by S&P Poor's Index (Jan 1981 - June 2003) with average return $r_s = 0.1329$ and standard deviation $\sigma_s = 0.1558$ whereas the secure bonds are represented by 1 year US government bonds (Jan 1996 - June 2002) with the average return $r_b = 0.0516$ and standard deviation $\sigma_b = 0.0082$. We assumed that there is no correlation between stock and bond returns.

We shall suppose that the structure of funds (F_1 = Growth fund, F_2 = Balanced fund,

Fund	Return	StdDev
F_1	$r_1 = 0.1166$	$\sigma_1 = 0.1247$
F_2	$r_2 = 0.0923$	$\sigma_2 = 0.0780$
F_3	$r_3 = 0.0516$	$\sigma_3 = 0.0082$

Period	wage growth $(1 + \varrho_t)$
2006-2008	1.075
2009-2014	1.070
2015-2021	1.065
2022-2024	1.060
2025-2050	1.050

Table 3: Data used for computation. Fund returns and their standard deviations (left), expected wage growth for the period 2006–2050 (right).

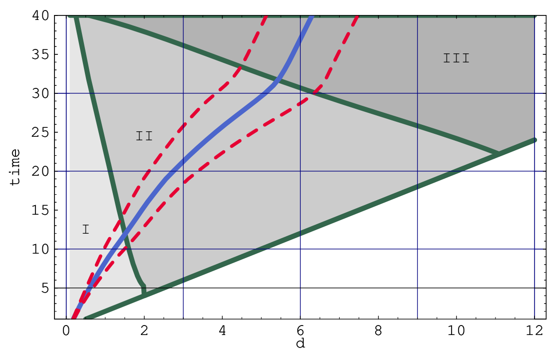


Figure 3: Regions of optimal choice and the path of average saved money to wage ratio.

F_3 = Conservative fund) of the second pension pillar in Slovak Republic is as follows:

$$\begin{aligned} F_1 &= 0.8 \times S + 0.2 \times B \\ F_2 &= 0.5 \times S + 0.5 \times B \\ F_3 &= B \end{aligned} \tag{10}$$

Both returns r_i and standard deviations $\sigma_i, i = 1, 2, 3$, of the above funds can be easily calculated from returns r_s, r_b and standard deviations σ_s, σ_b (see Tab. 3).

According to Slovak legislature the percentage of salary transferred each year to a pension fund is $\tau = 9\%$. We assumed the period of saving to be $T = 40$ years. The data for the expected wage growth ϱ are taken from Slovak Savings Bank (SLSP). The values are shown in Tab. 3.

4.1. Description of computed results and simulations

In Fig. 3 we present a typical result of our analysis. It contains three distinct regions in the (d, t) plane determining the optimal choice $j = j(d, t)$ of a fund depending on time $t \in [1, T - 1]$ and the average saved money to wage ratio $d \in [d_{min}, d^t_{max}]$. For practical purposes we chose $d_{min} = 0.09$ and $d^t_{max} = t/2$ for $t \geq 1$. In each year $t = 1, ..., T - 1$ we invest the saved amount of money s_t resp. d_t to one of the funds $j = 1, 2, 3$ depending on the computed optimal value $j = j(d, t)$. In the first year of saving we take $d_1 = d_{min}$.

mean $E(d_T)$	switch 1-2	switch 2-3
6.30	12 (11-14)	32 (30-34)

Table 4: Summary of computation of the averaged saved money to wage ratio d_T and switching times.

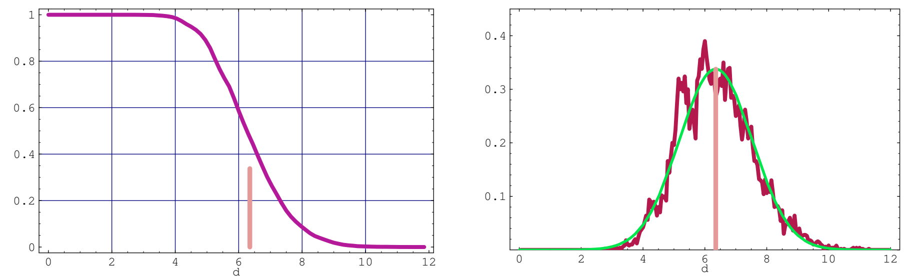


Figure 4: Cumulative distribution function $1 - F$ (left). Histogram of simulations and density function (right). $E(d_T) = 6.35$, $\sqrt{D(d_T)} = 1.18$

The curvilinear solid line in Fig. 3 represents the path of the average wealth $E(d_t)$, obtained by 10000 simulations.[1] Notice that, for $t > 1$, the ratio d_t is a random variable depending on (in our case normally distributed) random returns of the funds and on the computed optimal fund choice matrix $j(d, t'), t' < t$. The dashed curvilinear lines correspond to $E(d_t) \pm \sigma_t$ intervals where σ_t is the standard deviation of the random variable d_t.

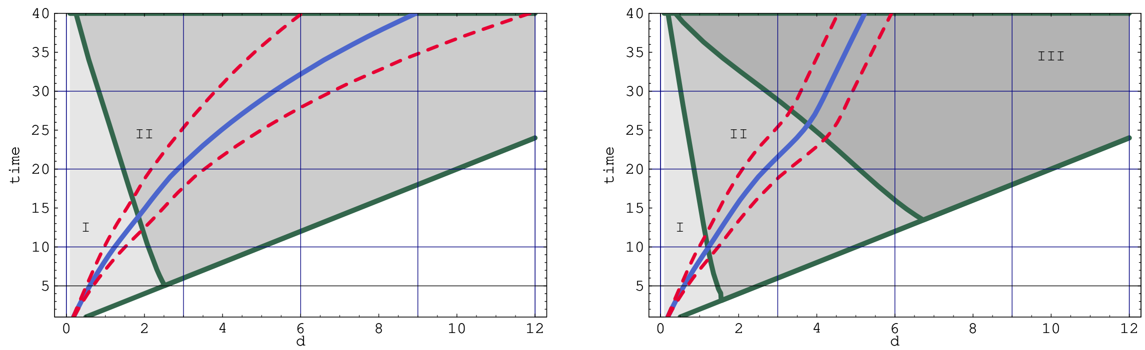
In Tab. 4 we present the average final wealth $E(d_T)$ as well as the so-called *switching-times* for averaged path $E(d_t), t \in [1, T-1]$, and the intervals (in brackets) of switching times for one standard deviation of the average path. The normalized histogram resp. distribution function of the simulated final wealth is very similar to a normal distribution, as can be seen in Fig. 4.

In the next sections we pay our attention to the sensitivity of results when some parameters are changing.

4.2. Sensitivity analysis for varying risk aversions

Let us consider different risk aversion parameters a in the utility function: $a = 14, 17, 21$. It should be obvious that increasing risk aversion leads to a choice of less risky fund. Indeed, based on our computations, one can observe that increasing a (increasing risk aversion) causes that the switches between funds are shifted to an earlier time, i.e. we switch from fund 1 to fund 2 sooner, as well as from fund 2 to fund 3. Obviously, for higher values of the risk aversion parameter a we obtain lower levels of the final wealth. Results for the experiments are displayed in Fig. 5, Fig. 3 and Tab. 5.

[1] One can derive a formula for mean and standard deviation of d_t. However this formula is complicated and therefore we (because of simplicity) have decided to perform simulations.



$a = 14$ $a = 21$

Figure 5: Sensitivity of regions of optimal choice with respect to different risk aversion values of the parameter $a = 14, 21$.

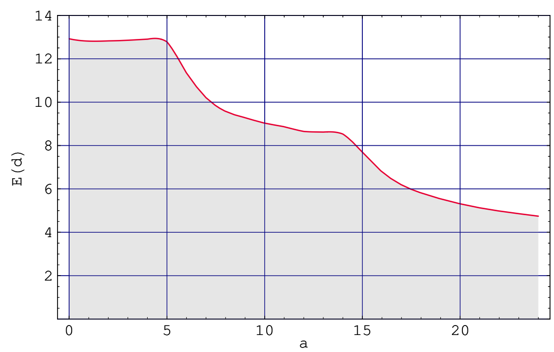


Figure 6: Relation between the risk aversion parameter a and the level of saved averaged wealth $E(d_T)$.

The relation between different values of risk aversion parameter $a \in (0, 25)$ and the final wealth (saved d) is shown in Fig. 6. We can see that the curve can be divided into three parts, where the "breakpoints" are for those a where there are *no* switches, *one* switch, and *two* switches between funds in the optimal strategy.

4.3. Pension levels from one-pillar and two-pillar systems

The two-pillar system is optional for those entering the labor market before January 2005. According to the law, the initial monthly pension for those who remained in the one-pillar system is:

$$P = APWP.N.APV$$

where APV (actual pension value) was set by law at 183.58 (in January 2004) to provide a 50% replacement rate (average initial pension/average gross wage). The law assumes automatic annual valorization of ADH by the nominal gross wage growth. $APWP$ (Average Personal Wage Point) represents the average of the ratio of individual gross wage to average

a	mean $E(d_T)$	switch 1-2	switch 2-3
14	8.95	14 (12-16)	never
17	6.30	12 (11-14)	32 (30-34)
21	5.22	10 (8-12)	26 (24-27)
23	4.96	9 (8-11)	23 (22-26)

Table 5: Results for fixed wage growths, fixed returns and standard deviations (see values in Tab. 3), different risk aversion parameter a.

gross wage. N stands for the number of years, in which pension contributions were paid. The average gross wage in the Slovak Republic in 2003 was SKK 14 365 (source: Statistical Office of the Slovak Republic). Therefore, the average gross wage received for 40 years implied the initial pension of roughly 50% of the gross wage in January 2004 (1*40*183.58 = 7343.2). Since ADH is indexed by the nominal gross wage growth, the 50% replacement rate (average initial pension/average gross wage) should be preserved.

Participants of the two-pillar system will receive full pension for the time they participated in the old system and half a pension for the time they participated in the new system. Therefore, the worker who will participate in the two-pillar system only will achieve a 25% replacement rate from the pay-as-you-go pillar. This replacement rate serves as a benchmark when deciding whether to switch to the two-pillar system: if the second-pillar will earn more than 25% replacement, then the switch is optimal. However, 25% replacement of the first pillar is unfair compared to the second pillar, because the latter does not create deficits to be covered by public finance. The former, based on 62-62 retirement age and assuming zero deficit of the pension system, will lead to deterioration of replacement rate (17% in 2054, see (Melicherčík and Ungvarský 2004)) and therefore will have to be subsidized by public finance.

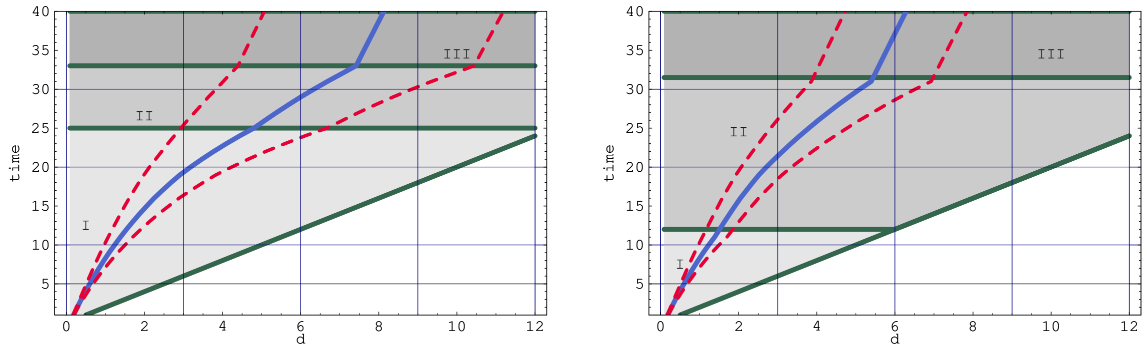
Assume that a retired person (in the two-pillar system) buys an annuity for a pension indexed by the level of interest rates. In this case, the initial replacement rate is d_T/L, where L stands for the expected time of receiving the pension (in years). According to the medium option of the demographic scenario (source: INFOSTAT), the life expectancy of a person reaching the age of 62 was 75 for men and 85 for women. These figures are likely to increase in the next decades. Setting $L = 20$ and using the mean values from Tab. 5, one can conclude that a two-pillar system would probably outperform the one-pillar system.

Calculations with static strategies and different asset returns could be found in (Melicherčík and Ungvarský 2004)).

4.4. The comparison of dynamic and static strategies

To compare the performance of dynamic and static strategies we have chosen two representatives of the static ones:

1. The most risky (accepting the law regulations) strategy with switching times 1-2 after 25 years and 2-3 after 33 years.



a) switching times 25, 33 b) switching times 12, 32

Figure 7: Static strategies - the development of savings.

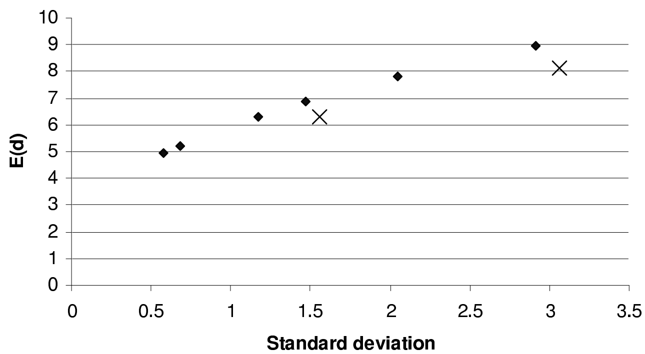


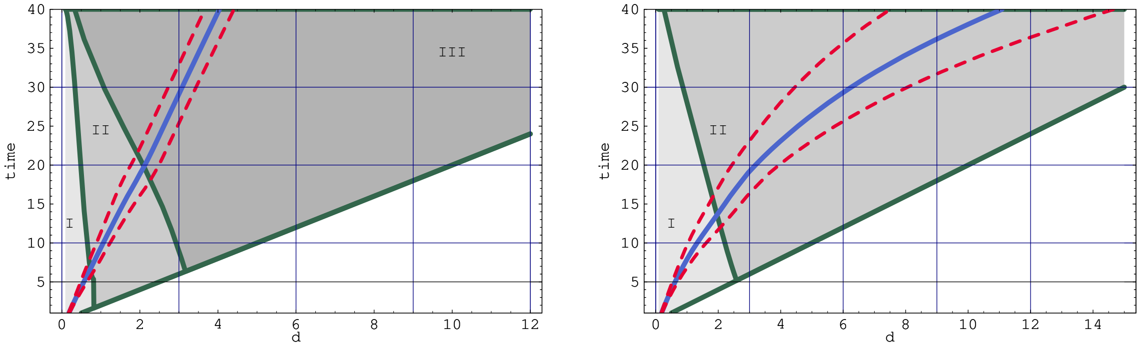
Figure 8: Mean-variance analysis of dynamic and strategic strategies .

2. The strategy with switching times 12 and 32 similar to a typical representative of dynamic strategies with the risk aversion parameter $a = 17$.

In Fig. 7 one can see the average d_t development and $E(d_t) \pm \sigma_t$ intervals for chosen static strategies. The strategy with switching times 12 and 32 has slightly lower $E(d_T)$ comparing to a dynamic one with $a = 17$ but significantly lower the standard deviation of d_T, $\sigma_T = 1.56$. A mean-variance analysis of dynamic strategies with different risk aversion ($a = 14, 15, 16, 17, 21, 23$) and the two static ones (crosses) is depicted in Fig. 8. The static strategies are clearly inefficient.

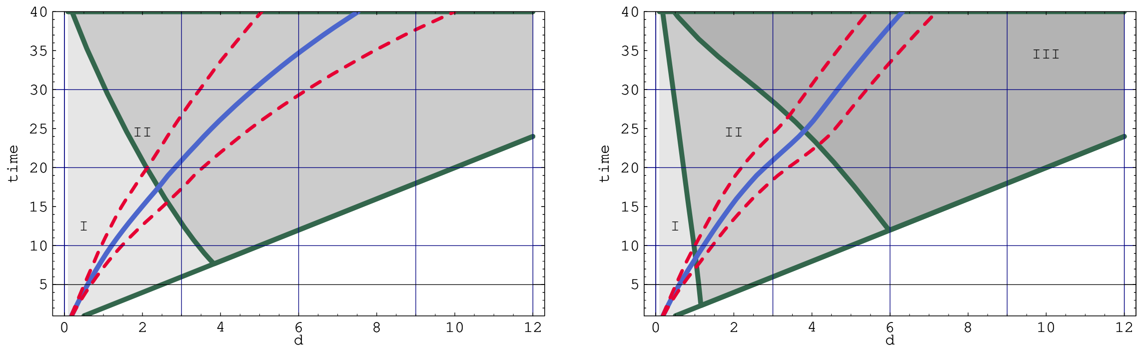
4.5. Sensitivity analysis for various stock and bond returns

Now, let us examine the impact of the change in returns of funds on the optimal strategy and results. One can expect that if for example the return of stocks becomes higher, it will be more favorable to "stay" in the first resp. the second fund for a longer period. In our computations, we first fix the bond return and increase/decrease the stock return ($a = 17$ and other parameters are fixed). This change mirrors in the returns of fund 1 and 2. The



a) lower stock return $r_s = 0.10$ b) higher stock return $r_s = 0.15$

and fixed bond return $r_b = 0.0516$



c) lower bond return $r_b = 0.03$ d) higher bond return $r_b = 0.065$

and fixed stock return $r_s = 0.1329$

Figure 9: Sensitivity of regions of optimal choice for various expected values of asset and bond returns.

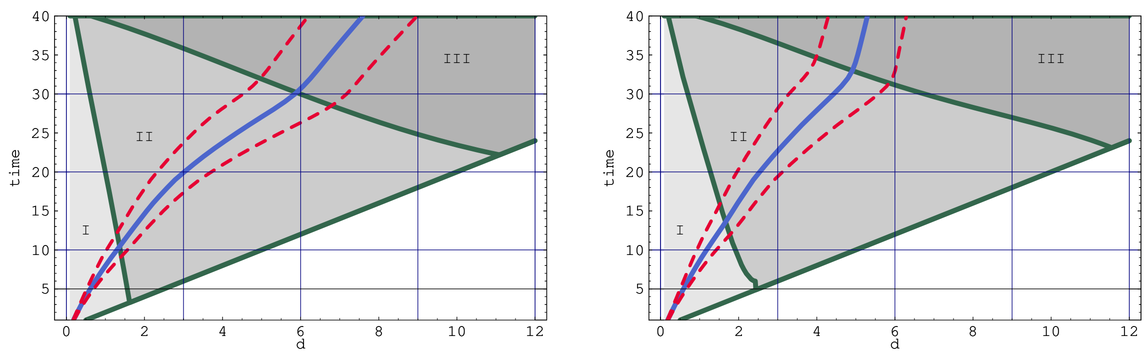
obtained results show that a higher return of stocks implies a later switch from more risky to less risky funds. The wealth in the final period of savings is higher too. Secondly, we fix the stock return and increase/decrease the bond return. A higher return of bonds implies an earlier switch from more risky to less risky funds. For overview of all results see Fig. 9 and Tab. 6. Interestingly enough, comparing the original bond returns we obtained the same expected wealth $E(d_T)$ assuming lower risk (see Fig. 5 ($a = 17$) and Fig. 9, part d).

4.6. Sensitivity analysis with respect to varying wages

Finally, we consider different wage growth rates. The intuition says that one can expect lower saved money to wage ratio d_t for higher wage growth ϱ. To examine the influence of this parameter on results, we considered the wage growth being raised (uniformly for all time periods) resp. lowered by 1%. We denote by $\varrho^{(+1\%)}$ ($\varrho^{(-1\%)}$) the wage growth development derived from Tab. 3 where ϱ_t has been increased by 1% (decreased by 1%) for each of five periods in Tab. 3. As we can see in Fig. 10 and Tab. 7, a higher wage growth leads to a lower wealth, guided by shifting the switch-times to later moments.

Stock & Bond returns	Fund returns	mean $E(d_T)$	switch 1-2	switch 2-3
$r_s = 0.1329$ $r_b = 0.0516$	$r_1 = 0.1166$ $r_2 = 0.0923$ $r_3 = 0.0516$	6.30	12 (11-14)	32 (30-34)
$r_s = 0.10$ $r_b = 0.0516$	$r_1 = 0.09032$ $r_2 = 0.0758$ $r_3 = 0.0516$	4.03	7 (6-8)	20 (18-22)
$r_s = 0.15$ $r_b = 0.0516$	$r_1 = 0.13032$ $r_2 = 0.1008$ $r_3 = 0.0516$	11.08	14 (12-16)	never
$r_s = 0.1329$ $r_b = 0.03$	$r_1 = 0.11232$ $r_2 = 0.08145$ $r_3 = 0.03$	7.52	18 (16-20)	never
$r_s = 0.1329$ $r_b = 0.065$	$r_1 = 0.1193$ $r_2 = 0.099$ $r_3 = 0.065$	6.32	9 (8-10)	25 (23-27)

Table 6: Results for fixed wage growths, fixed $a = 17$, fixed standard deviations $\sigma_1 = 0.1247, \sigma_2 = 0.078, \sigma_3 = 0.0082$, and various bond and stocks returns r_b and r_s, resp.



lower wage growth $\varrho^{(-1\%)}$ higher wage growth $\varrho^{(+1\%)}$

Figure 10: Sensitivity of regions of optimal choice for various wage growth ϱ scenarios.

wage growth	mean $E(d_T)$	switch 1-2	switch 2-3
$\varrho^{(-1\%)}$	7.59	11 (9-13)	30 (28-32)
ϱ	6.30	12 (11-14)	32 (30-34)
$\varrho^{(+1\%)}$	5.30	14 (12-16)	33 (31-35)

Table 7: Results for fixed returns and standard deviations (see values in Tab.3), fixed $a = 17$, and different wage growths.

4.7. Interest rate targeting

However, it can hardly be expected that the returns of funds will remain constant over the whole period of saving. Based on the calibration of Cox-Ingersoll-Ross interest rate model it was shown in (Ševčovič and Urbánová 2005) that it is reasonable to expect the bond return to be decreasing to the value of approximately 2% in the time horizon of "some" years.

Let us investigate how the resulting strategy changes when we replace constant returns by returns decreasing monotonically to some target level. Let us assume that the return of each fund j decreases exponentially from the starting value r_0^j and in the infinite time horizon it converges to its target value r_∞^j. Then the rates in years $i = 1, 2, ..., T$, are given by formula

$$r_i^j = r_\infty^j + (r_0^j - r_\infty^j)\exp^{-K^j(i/T)} \tag{11}$$

for funds $j = 1, 2, 3$ and some depreciation coefficients K^j. Let r_0^j have the values given in Tab. 3, and let $r_\infty^j = r_0^j/2$ for all j. Thus, $r_0^1 = 0.1166$, $r_0^2 = 0.0923$, $r_0^3 = 0.0516$, and $r_\infty^1 = 0.0583$, $r_\infty^2 = 0.0462$, $r_\infty^3 = 0.0258$.

First, we investigate what happens if the returns of the first two funds remain constant, and the bond fund return alone decreases exponentially to the target value. For the constant rates r_{aver}^j we take the geometric average return obtained from (11) for years $i = 1, ..., T(= 40)$ and $K^1 = K^2 = K^3 = 2$. In the left bottom picture of Fig. 11 the rates of these constant returns are depicted and in the picture above it there are the optimal choice regions with the simulated wealth path. In the middle bottom picture of Fig. 11 there is the situation when only the returns of the first two funds are constant over time and the conservative fund return is exponentially decreasing. The corresponding optimal choice regions are depicted in the picture above. Although the final wealths in these two cases are not very different, we can observe the enlargement of the region II, i.e. a later switch to the conservative fund in case b). The border between regions II and III is more curved than the one corresponding to the averaged value of r_{aver}^3. This is also in accordance with our intuition.

Second, we answer the question what happens if *all* fund returns are exponentially decreasing according to (11). We cannot expect anything by intuition because everything depends on the concrete starting and target returns and on the speed of decrease. It thus remains a question how the resulting optimal strategy will change. The part c) of Fig. 11 gives the answer. The regions, the switching times, and the averaged final wealth do not change significantly compared to case a). One can observe only a slight deformation of the region borders. This gives us the experience that decreasing returns lead to the same strategy as returns fixed on the level of *geometric mean of the returns in the first situation*. However, it can be expected that if the level of fixed returns is much higher or much lower than the mentioned geometric mean, the resulting strategies will be more different from each other.

5. Conclusions

A pension reform was necessary if the country wanted to avoid high deficit of the pension system and ensure decent level of pensions. The second pillar will deepen the deficit in the

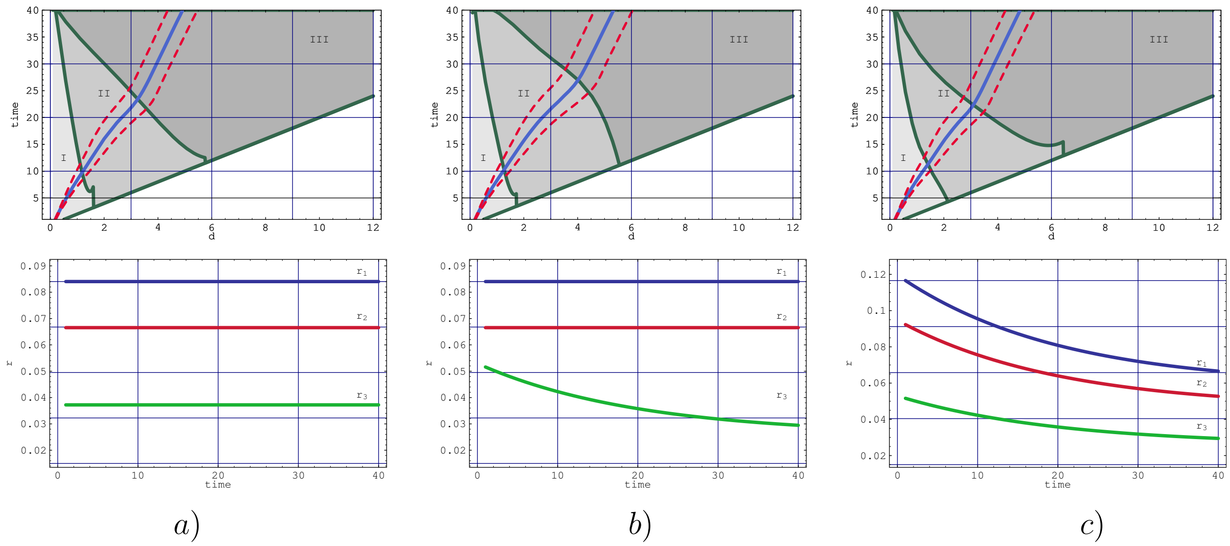


Figure 11: Regions of optimal choice and the path of average saved return. a) r_1, r_2, r_3 constant, b) r_1, r_2 constant, r_3 monotonically decreasing, c) r_1, r_2, r_3 monotonically decreasing.

first decades of the new pension system, but as more people will start receiving pensions from the funded pillar, the deficit will decline.

We have presented a dynamic accumulation model for determining optimal switching strategies for choosing pension funds with different risk profiles. Our results have proved that dynamic strategies outperform the static ones. The level of pensions in the two-pillar system should be higher than in the one-pillar system. The gradual decreasing of the risk (incorporated in the corresponding legislation) is reasonable and can be justified by means of a dynamic decision model. The resulting strategies depend on individual risk preferences of the future pensioners expressed by their individual utility functions. Moreover, accepting higher risk in the strategy the higher expected level of future pension can be achieved. In accord with common intuition, a higher wage growth implies lower performance of the funded pillar. Higher stock returns imply later switch to less risky funds accompanied with a higher risk. For higher bond returns an earlier switch to more conservative funds turned to be an optimal strategy. It is accompanied with a lower risk.

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