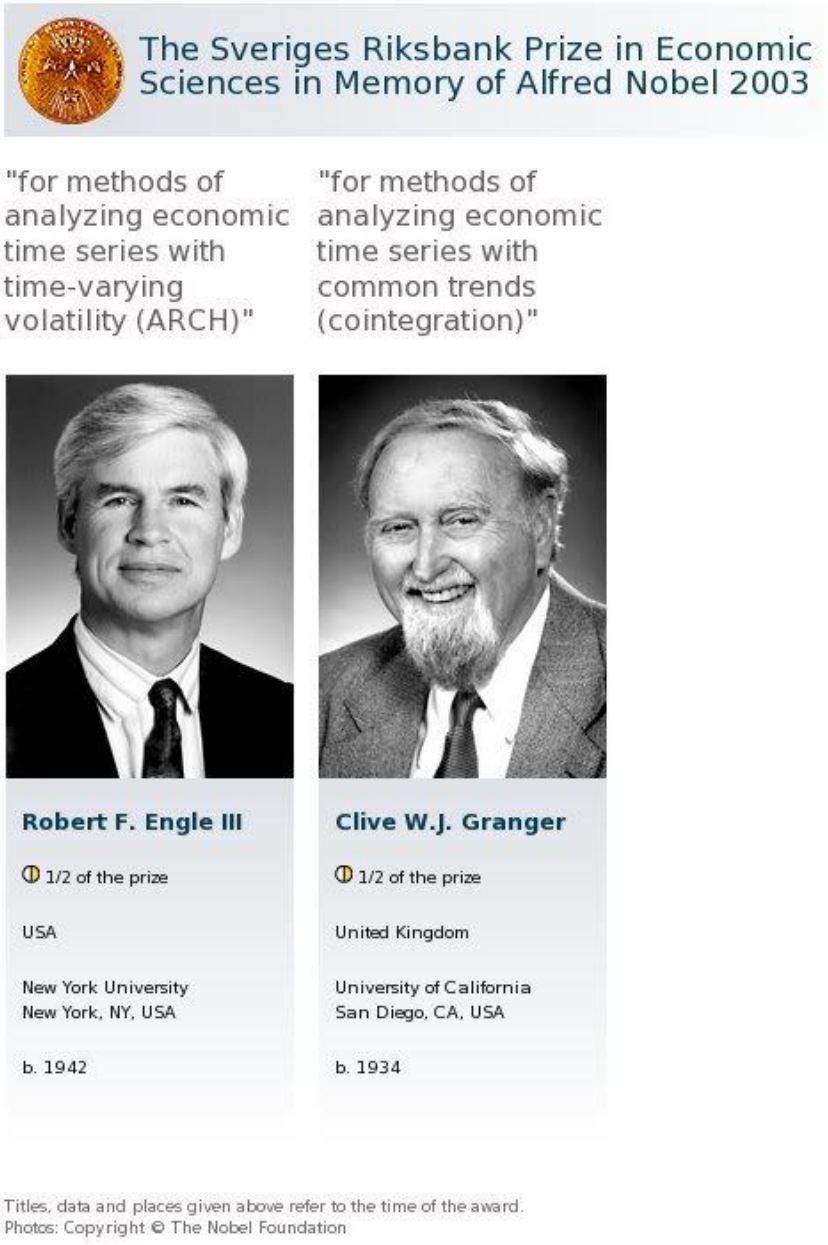
Cvičenie 3: Modelovanie volatility (ARCH, GARCH, ...)

:: ARCH a GARCH modely ::

- **ARCH -** autoregressive conditional heteroskedasticity **-** ide teda o modelovanie nekonštantnej disperzie
- **GARCH -** generalized ARCH
- Referencie na prvé články s týmito modelmi:
  - ARCH model:
    - *Robert F. Engle, **Autoregressive Conditional Heteroskedasticity With Estimates of the Variance of U.K. Inflation**, Econometrica 50 (1982), pp. 987 - 1008.*
  - GARCH model:
    - *Tim Bollerslev, **Generalized Autoregressive Conditional Heteroskedasticity**, Journal of Econometrics 31 (1986), pp. 307 - 327*
    - *Stephen J. Taylor, **Modelling Financial Time Series**, John Wiley, Chichester, 1986*

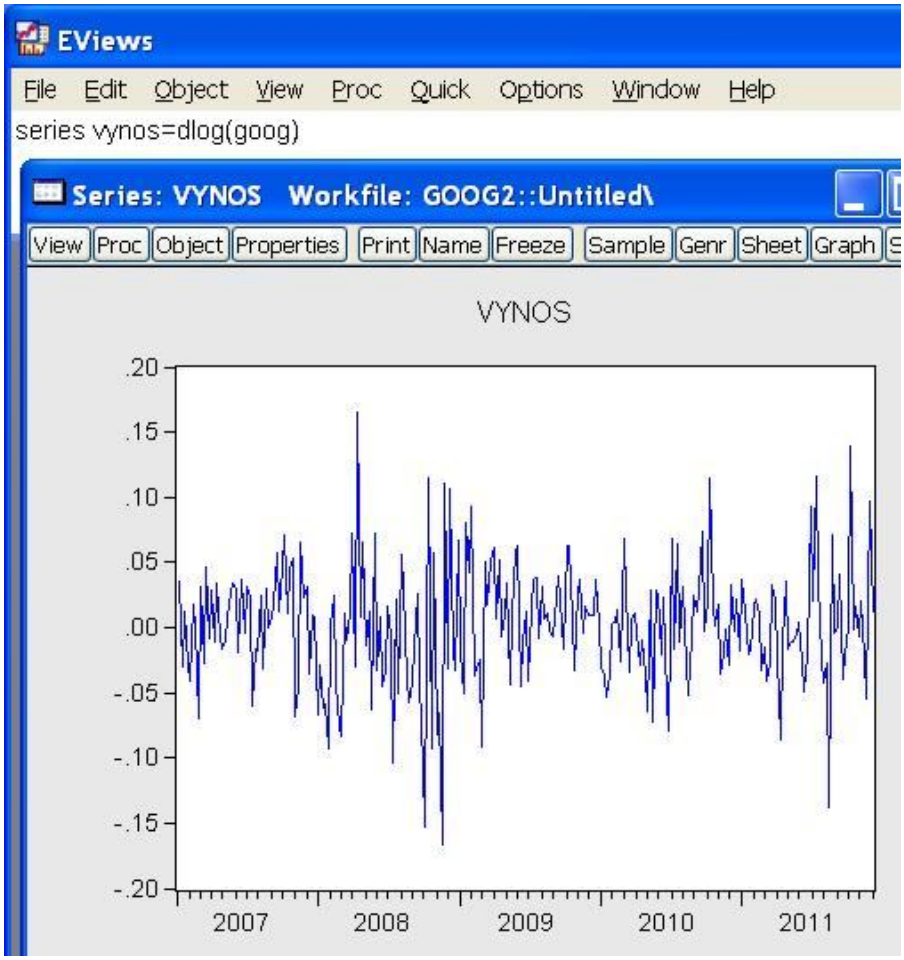


---

http://nobelprize.org/nobel_prizes/economics/laureates/2003/index.html

---

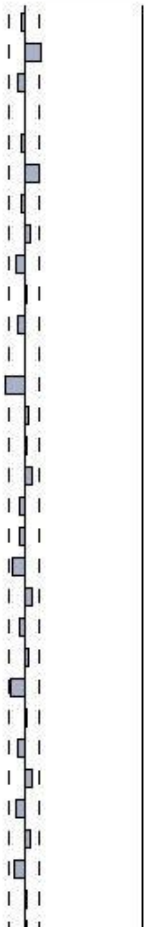
- Vytvoríme vektor výnosov a zobrazíme ich priebeh:



- Autokorelácie a parciálne autokorelácie nie sú signifikantné, Q štatistika sú tiež v poriadku. Zdá sa teda, že výnos je konštanta plus biely šum.

Correlogram of VYNOS

Date: 12/10/11 Time: 12:15
Sample: 1/03/2007 12/13/2011
Included observations: 257



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.022	-0.022	0.1204	0.729
		2	0.139	0.139	5.1993	0.074
		3	-0.064	-0.060	6.2864	0.098
		4	0.013	-0.008	6.3337	0.176
		5	-0.026	-0.009	6.5096	0.260
		6	0.125	0.123	10.630	0.101
		7	-0.021	-0.014	10.749	0.150
		8	0.063	0.027	11.793	0.161
		9	-0.070	-0.053	13.115	0.157
		10	0.028	0.015	13.326	0.206
		11	-0.050	-0.026	13.997	0.233
		12	0.006	-0.021	14.007	0.300
		13	-0.156	-0.147	20.657	0.080
		14	0.033	0.019	20.960	0.103
		15	0.020	0.079	21.071	0.135
		16	0.071	0.046	22.470	0.129
		17	-0.035	-0.040	22.816	0.155
		18	-0.045	-0.067	23.370	0.177
		19	-0.100	-0.048	26.184	0.125
		20	0.067	0.078	27.431	0.124
		21	-0.042	-0.026	27.933	0.142
		22	0.035	-0.031	28.289	0.166
		23	-0.111	-0.100	31.764	0.105
		24	0.017	0.024	31.850	0.131
		25	-0.061	-0.006	32.921	0.133
		26	0.078	0.033	34.672	0.119
		27	-0.078	-0.068	36.417	0.106
		28	0.050	0.033	37.132	0.116
		29	-0.089	-0.020	39.419	0.094
		30	0.018	-0.016	39.516	0.115

- Odhadneme tento model:

Dependent Variable: VYNOS
Method: Least Squares
Date: 12/10/11 Time: 12:16
Sample (adjusted): 1/10/2007 12/07/2011
Included observations: 257 after adjustments

	Coefficient	Std. Error	t-Statistic	Prob.
C	0.000984	0.002940	0.334844	0.7380

R-squared	0.000000	Mean dependent var	0.000984
Adjusted R-squared	0.000000	S.D. dependent var	0.047124
S.E. of regression	0.047124	Akaike info criterion	-3.268165
Sum squared resid	0.568503	Schwarz criterion	-3.254356
Log likelihood	420.9592	Hannan-Quinn criter.	-3.262612
Durbin-Watson stat	2.040712		

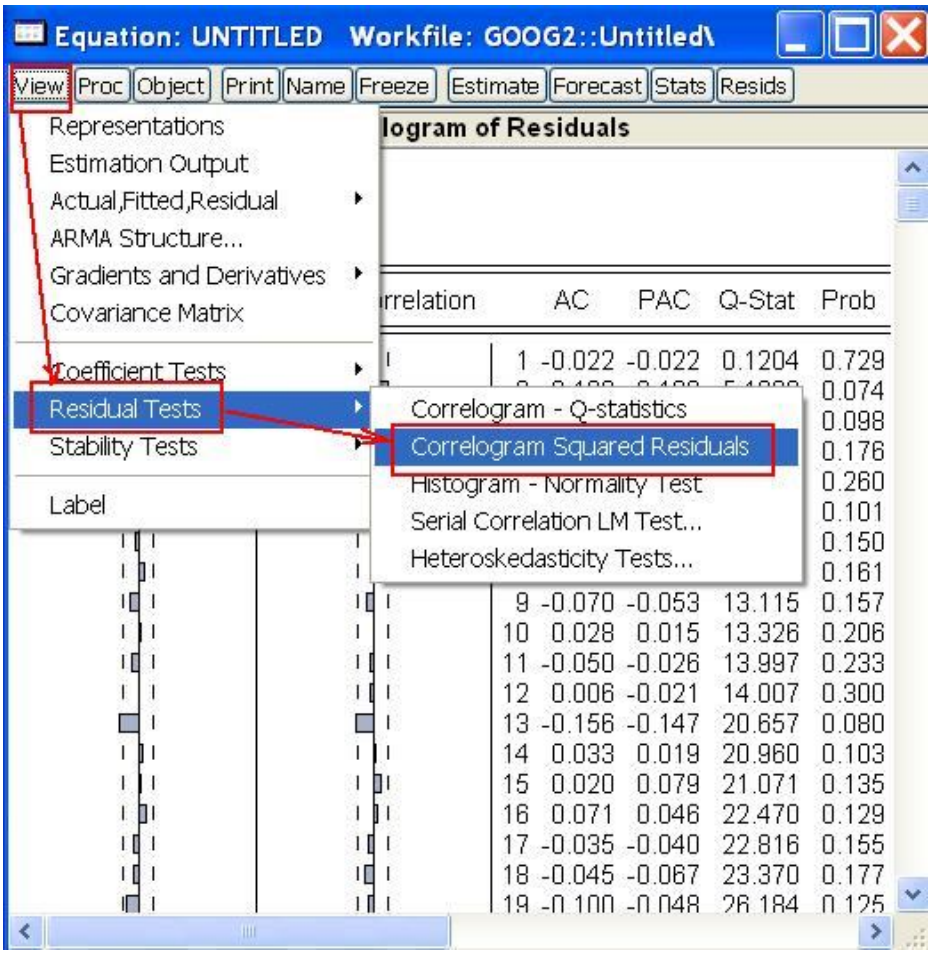
- ACF a PACF reziduí sú rovnaké ako pre pôvodný časový rad, keďže sme od neho iba odrátali konštantu.

Correlogram of Residuals

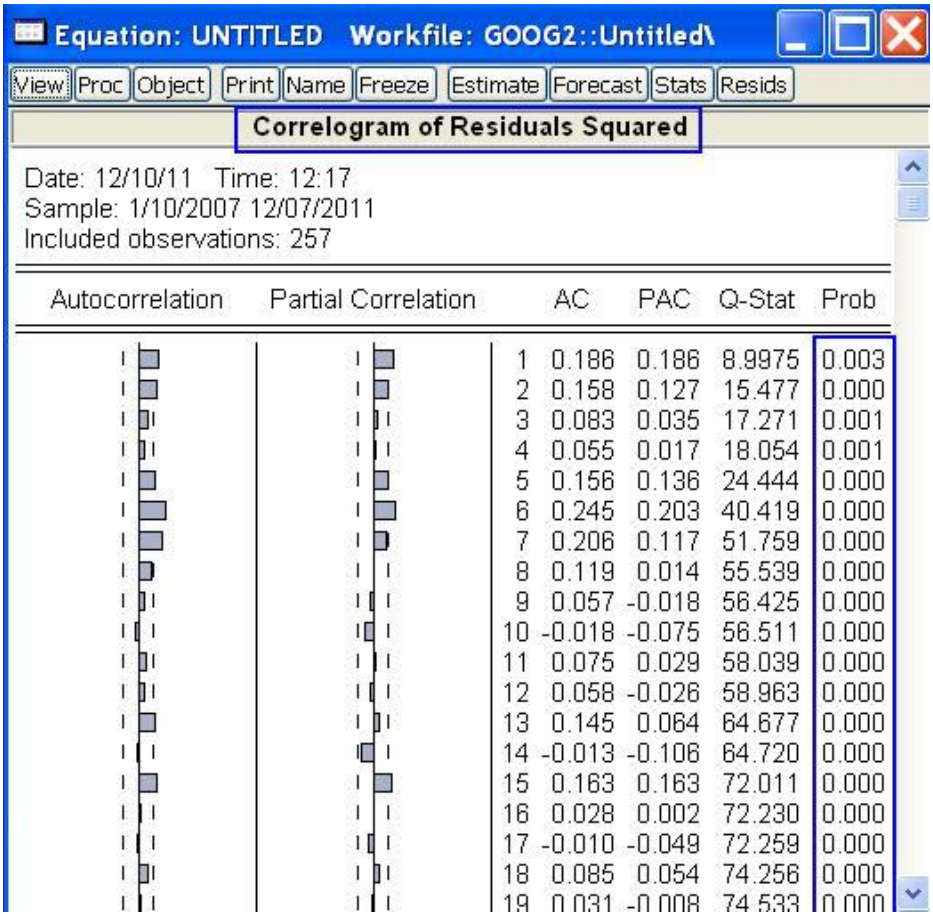
Date: 12/10/11 Time: 12:16
Sample: 1/10/2007 12/07/2011
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.022	-0.022	0.1204	0.729
		2	0.139	0.139	5.1993	0.074
		3	-0.064	-0.060	6.2864	0.098
		4	0.013	-0.008	6.3337	0.176
		5	-0.026	-0.009	6.5096	0.260
		6	0.125	0.123	10.630	0.101
		7	-0.021	-0.014	10.749	0.150
		8	0.063	0.027	11.793	0.161
		9	-0.070	-0.053	13.115	0.157
		10	0.028	0.015	13.326	0.206
		11	-0.050	-0.026	13.997	0.233
		12	0.006	-0.021	14.007	0.300
		13	-0.156	-0.147	20.657	0.080
		14	0.033	0.019	20.960	0.103
		15	0.020	0.079	21.071	0.135
		16	0.071	0.046	22.470	0.129
		17	-0.035	-0.040	22.816	0.155
		18	-0.045	-0.067	23.370	0.177
		19	0.100	0.048	26.184	0.125

- Pozrime sa ale na priebeh rezídií:



Dostaneme:



Equation: UNTITLED Workfile: GOOG2::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Correlogram of Residuals Squared

Date: 12/10/11 Time: 12:17
Sample: 1/10/2007 12/07/2011
Included observations: 257

Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.186	0.186	8.9975	0.003
		2	0.158	0.127	15.477	0.000
		3	0.083	0.035	17.271	0.001
		4	0.055	0.017	18.054	0.001
		5	0.156	0.136	24.444	0.000
		6	0.245	0.203	40.419	0.000
		7	0.206	0.117	51.759	0.000
		8	0.119	0.014	55.539	0.000
		9	0.057	-0.018	56.425	0.000
		10	-0.018	-0.075	56.511	0.000
		11	0.075	0.029	58.039	0.000
		12	0.058	-0.026	58.963	0.000
		13	0.145	0.064	64.677	0.000
		14	-0.013	-0.106	64.720	0.000
		15	0.163	0.163	72.011	0.000
		16	0.028	0.002	72.230	0.000
		17	-0.010	-0.049	72.259	0.000
		18	0.085	0.054	74.256	0.000
		19	0.031	-0.008	74.533	0.000

Ak by boli rezíduá bielym šumom, aj ich druhé mocniny by boli nekorelované. Toto nasvedčuje tomu, že nejde o biely šum, ale o proces s meniacou sa disperziou. Na prednáške sme si ukázali, že takýto priebeh sa dá modelovať ARCH a ARCH modelmi.

:: Definícia modelov a ich odhadovanie ::

- Opakovanie z prednášky:

ARCH a GARCH modely

- u nie je biely šum, ale

$$u_t = \sqrt{\sigma_t^2}\,\eta_t,$$

kde η je biely šum s jednotkovou disperziou; teda

$$u_t \sim N(0, \sigma_t^2)$$

- ARCH model (autoregressive conditional heteroskedasticity) - rovnica pre disperziu σ_t^2:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \ldots \alpha_p u_{t-p}^2$$

- Ohraničenia na parametre:
  - na zabezpečenie kladnosti disperzie:

$$\omega > 0, \alpha_1, \ldots, \alpha_{p-1} \geq 0, \alpha_p > 0$$

  - kvôli stacionarite:

$$\alpha_1 + \ldots + \alpha_p < 1$$

Modelovanie volatility - ARCH a GARCH modely – p.8/1

ARCH a GARCH modely

- GARCH model (generalized autoregressive conditional heteroskedasticity) rovnica pre disperziu σ_t^2:

$$\sigma_t^2 = \omega + \alpha_1 u_{t-1}^2 + \ldots + \alpha_p u_{t-p}^2 + \beta_1 \sigma_{t-1}^2 + \ldots + \beta_q \sigma_{t-q}^2$$

- Ohraničenia na parametre:
  - na zabezpečenie kladnosti disperzie:

$$\omega > 0, \alpha_1, \ldots, \alpha_{p-1} \geq 0, \alpha_p > 0$$

$$\beta_1, \ldots, \beta_{q-1} \geq 0, \beta_q > 0$$

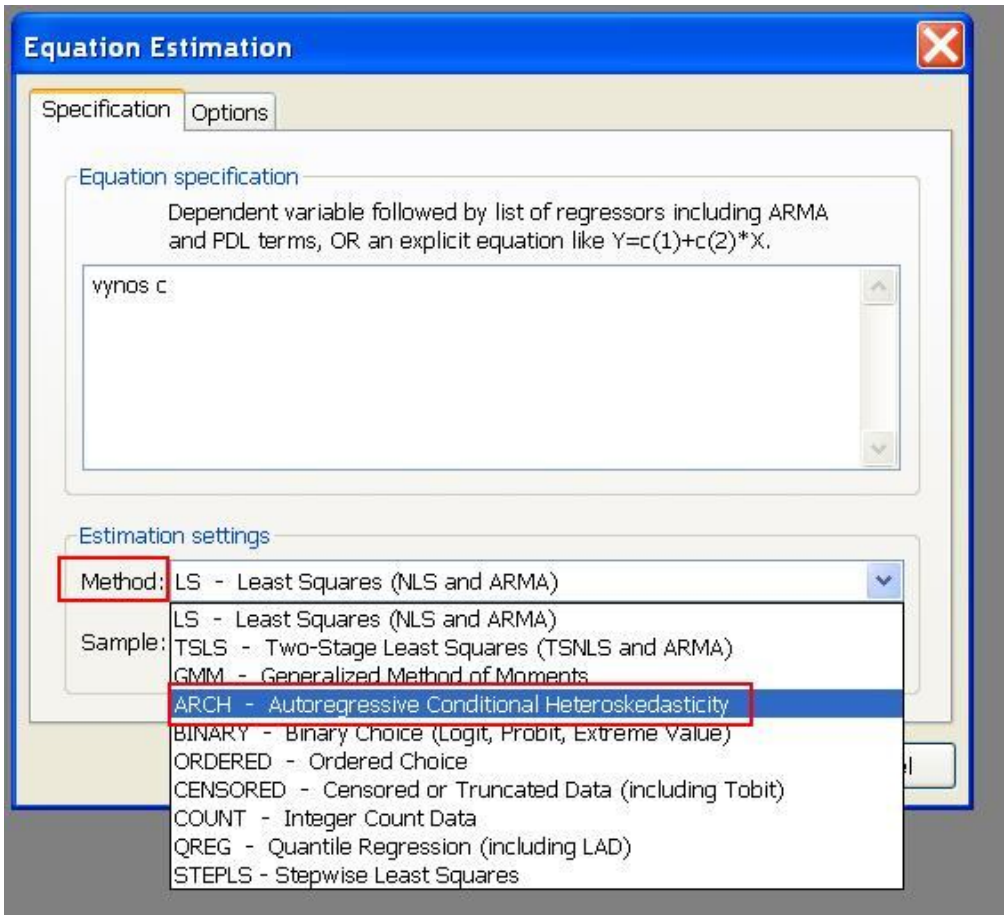
  - kvôli stacionarite:

$$(\alpha_1 + \ldots + \alpha_p) + (\beta_1 + \ldots \beta_q) < 1$$

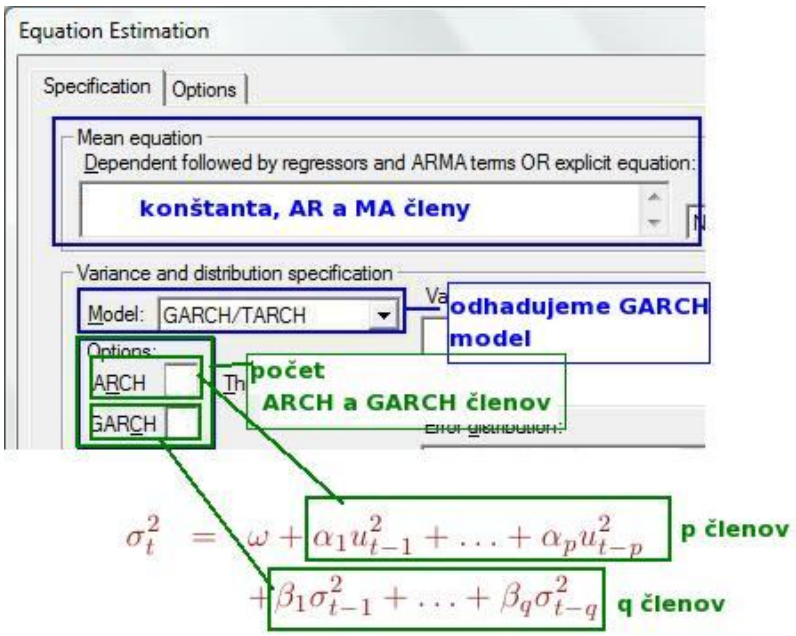
- Často sa používa GARCH(1,1).

Modelovanie volatility - ARCH a GARCH modely – p.11/15

- Odhadovanie (G)ARCH modelov v EViews:
  - Začneme rovnako ako doteraz **- Quick - Estimate Equation**. Tu z možností v časti **Methods** vyberieme **ARCH - Autoregressive Conditional Heteroskedasticity**:



  - Potom zadáme počet ARCH a GARCH členov:



:: Príklad - pokračovanie ::

- Odhadneme ARCH(1):
  - Ohraničenia na parametre sú splnené:

Equation: UNTITLED Workfile: GOOG2::Untitled\

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: VYNOS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/10/11 Time: 12:20
Sample (adjusted): 1/10/2007 12/07/2011
Included observations: 257 after adjustments
Convergence achieved after 7 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2

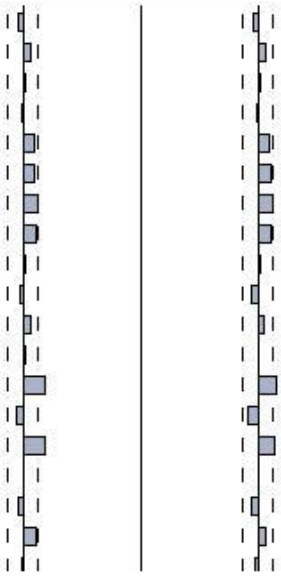
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003426	0.002936	1.166832	0.2433
Variance Equation				
C	0.001737	0.000176	9.851405	0.0000
RESID(-1)^2	0.223464	0.090924	2.457688	0.0140

R-squared	-0.002694	Mean dependent var	0.000984
Adjusted R-squared	-0.010589	S.D. dependent var	0.047124
S.E. of regression	0.047373	Akaike info criterion	-3.290493
Sum squared resid	0.570035	Schwarz criterion	-3.249064
Log likelihood	425.8283	Hannan-Quinn criter.	-3.273832
Durbin-Watson stat	2.035229		

  - Korelogram druhých mocnín rezíduí však nevyhovuje:

Correlogram of Standardized Residuals Squared

Date: 12/10/11 Time: 12:20
Sample: 1/10/2007 12/07/2011
Included observations: 257



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.042	-0.042	0.4587	0.498
		2	0.081	0.080	2.1851	0.335
		3	0.025	0.032	2.3545	0.502
		4	-0.000	-0.004	2.3545	0.671
		5	0.100	0.096	4.9782	0.419
		6	0.098	0.108	7.5456	0.273
		7	0.138	0.136	12.640	0.081
		8	0.117	0.117	16.290	0.038
		9	0.032	0.026	16.557	0.056
		10	-0.032	-0.060	16.835	0.078
		11	0.081	0.052	18.618	0.068
		12	0.028	0.008	18.833	0.093
		13	0.204	0.161	30.184	0.004
		14	-0.054	-0.086	30.981	0.006
		15	0.190	0.147	40.943	0.000
		16	0.013	0.013	40.988	0.001
		17	-0.034	-0.053	41.307	0.001
		18	0.114	0.067	44.948	0.000
		19	-0.000	-0.022	44.948	0.001

- Podobne ak odhadneme ARCH(2):
  - Ohraničenia na parametre sú splnené:

Equation: UNTITLED Workfile: GOOG2::Untitled

View | Proc | Object | Print | Name | Freeze | Estimate | Forecast | Stats | Resids

Dependent Variable: VYNOS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/10/11 Time: 12:21
Sample (adjusted): 1/10/2007 12/07/2011
Included observations: 257 after adjustments
Convergence achieved after 9 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2

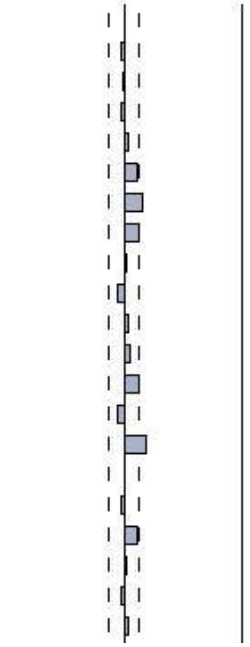
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002286	0.002846	0.803189	0.4219
Variance Equation				
C	0.001494	0.000178	8.387210	0.0000
RESID(-1)^2	0.130801	0.076967	1.699430	0.0892
RESID(-2)^2	0.202684	0.084328	2.403536	0.0162

R-squared	-0.000766	Mean dependent var	0.000984
Adjusted R-squared	-0.012632	S.D. dependent var	0.047124
S.E. of regression	0.047421	Akaike info criterion	-3.305799
Sum squared resid	0.568938	Schwarz criterion	-3.250561
Log likelihood	428.7952	Hannan-Quinn criter.	-3.283585
Durbin-Watson stat	2.039151		

  - Na základe korelogramu druhých mocnín rezíduí však model zamietneme:

Correlogram of Standardized Residuals Squared

Date: 12/10/11 Time: 12:21
Sample: 1/10/2007 12/07/2011
Included observations: 257



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.001	0.001	0.0001	0.991
		2	-0.023	-0.023	0.1389	0.933
		3	-0.015	-0.015	0.1947	0.978
		4	-0.024	-0.024	0.3446	0.987
		5	0.037	0.037	0.7137	0.982
		6	0.118	0.117	4.3850	0.625
		7	0.167	0.171	11.824	0.106
		8	0.129	0.146	16.295	0.038
		9	0.021	0.047	16.413	0.059
		10	-0.057	-0.039	17.278	0.068
		11	0.042	0.045	17.753	0.087
		12	0.054	0.035	18.532	0.100
		13	0.136	0.100	23.608	0.035
		14	-0.055	-0.113	24.433	0.041
		15	0.204	0.175	35.884	0.002
		16	0.012	0.002	35.925	0.003
		17	-0.032	-0.016	36.209	0.004
		18	0.111	0.102	39.612	0.002
		19	0.018	-0.012	39.706	0.004
		20	-0.017	-0.062	39.787	0.005
		21	0.034	-0.009	40.114	0.007

- Pri procesoch vyššieho rádu už vznikajú problémy aj s podmienkami na parametre - výstup pre ARCH(4):

Dependent Variable: VYNOS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/10/11 Time: 12:23
Sample (adjusted): 1/10/2007 12/07/2011
Included observations: 257 after adjustments
Convergence achieved after 14 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*RESID(-2)^2 + C(5)
*RESID(-3)^2 + C(6)*RESID(-4)^2 + C(7)*RESID(-5)^2

	Coefficient	Std. Error	z-Statistic	Prob.
C	0.002971	0.002952	1.006513	0.3142
Variance Equation				
C	0.001391	0.000233	5.979092	0.0000
RESID(-1)^2	0.119433	0.072629	1.644440	0.1001
RESID(-2)^2	0.158631	0.078728	2.014918	0.0439
RESID(-3)^2	-0.007896	0.062820	-0.125693	0.9000
RESID(-4)^2	0.008261	0.072009	0.114717	0.9087
RESID(-5)^2	0.088025	0.064445	1.365888	0.1720

R-squared	-0.001785	Mean dependent var	0.000984
Adjusted R-squared	-0.025828	S.D. dependent var	0.047124
S.E. of regression	0.047729	Akaike info criterion	-3.294299
Sum squared resid	0.569518	Schwarz criterion	-3.197631
Log likelihood	430.3174	Hannan-Quinn criter.	-3.255424
Durbin-Watson stat	2.037076		

Dependent Variable: VYNOS
Method: ML - ARCH (Marquardt) - Normal distribution
Date: 12/10/11 Time: 12:23
Sample (adjusted): 1/10/2007 12/07/2011
Included observations: 257 after adjustments
Convergence achieved after 32 iterations
Presample variance: backcast (parameter = 0.7)
GARCH = C(2) + C(3)*RESID(-1)^2 + C(4)*GARCH(-1)

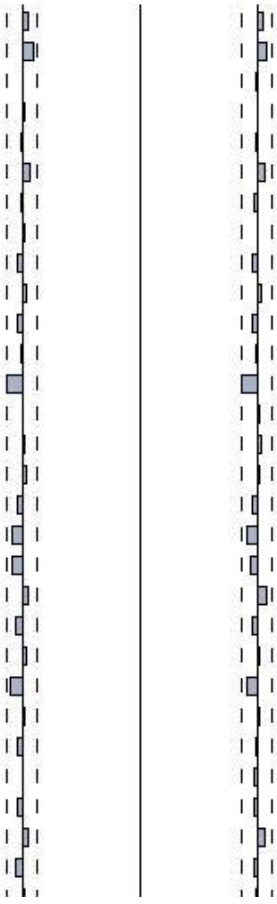
	Coefficient	Std. Error	z-Statistic	Prob.
C	0.003345	0.002649	1.262965	0.2066
Variance Equation				
C	9.07E-05	6.81E-05	1.330921	0.1832
RESID(-1)^2	0.109708	0.048577	2.258448	0.0239
GARCH(-1)	0.855146	0.063325	13.50401	0.0000

R-squared	-0.002520	Mean dependent var	0.000984
Adjusted R-squared	-0.014408	S.D. dependent var	0.047124
S.E. of regression	0.047463	Akaike info criterion	-3.356874
Sum squared resid	0.569936	Schwarz criterion	-3.301636
Log likelihood	435.3583	Hannan-Quinn criter.	-3.334660
Durbin-Watson stat	2.035583		

- Korelogram rezíduí je v poriadku:

Correlogram of Standardized Residuals

Date: 12/10/11 Time: 12:24
Sample: 1/10/2007 12/07/2011
Included observations: 257

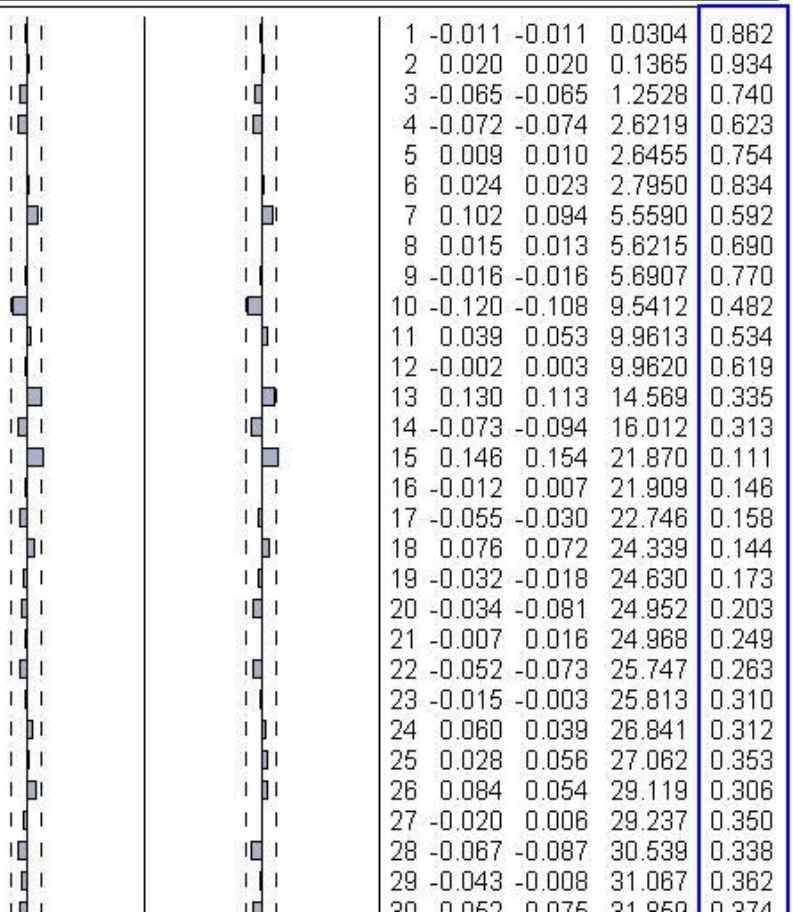


Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	0.064	0.064	1.0492	0.306
		2	0.094	0.090	3.3440	0.188
		3	0.003	-0.008	3.3461	0.341
		4	0.017	0.009	3.4240	0.490
		5	-0.009	-0.010	3.4466	0.631
		6	0.079	0.078	5.0813	0.533
		7	-0.010	-0.017	5.1069	0.647
		8	0.019	0.007	5.2072	0.735
		9	-0.036	-0.035	5.5485	0.784
		10	0.037	0.038	5.9255	0.821
		11	-0.036	-0.033	6.2823	0.854
		12	-0.003	-0.012	6.2842	0.901
		13	-0.131	-0.124	10.957	0.614
		14	0.015	0.029	11.016	0.685
		15	0.020	0.048	11.123	0.744
		16	0.045	0.031	11.675	0.766
		17	-0.040	-0.046	12.121	0.793
		18	-0.085	-0.096	14.148	0.719
		19	-0.090	-0.052	16.436	0.628
		20	0.062	0.085	17.527	0.619
		21	-0.051	-0.046	18.249	0.633
		22	0.043	0.021	18.781	0.659
		23	-0.101	-0.091	21.665	0.541
		24	0.018	0.031	21.753	0.594
		25	-0.041	-0.015	22.231	0.622
		26	0.001	-0.028	22.231	0.676
		27	-0.039	-0.024	22.663	0.703
		28	0.058	0.069	23.630	0.701
		29	-0.054	-0.029	24.478	0.705

- Korelogram druhých mocnín reziduí tiež:

Correlogram of Standardized Residuals Squared

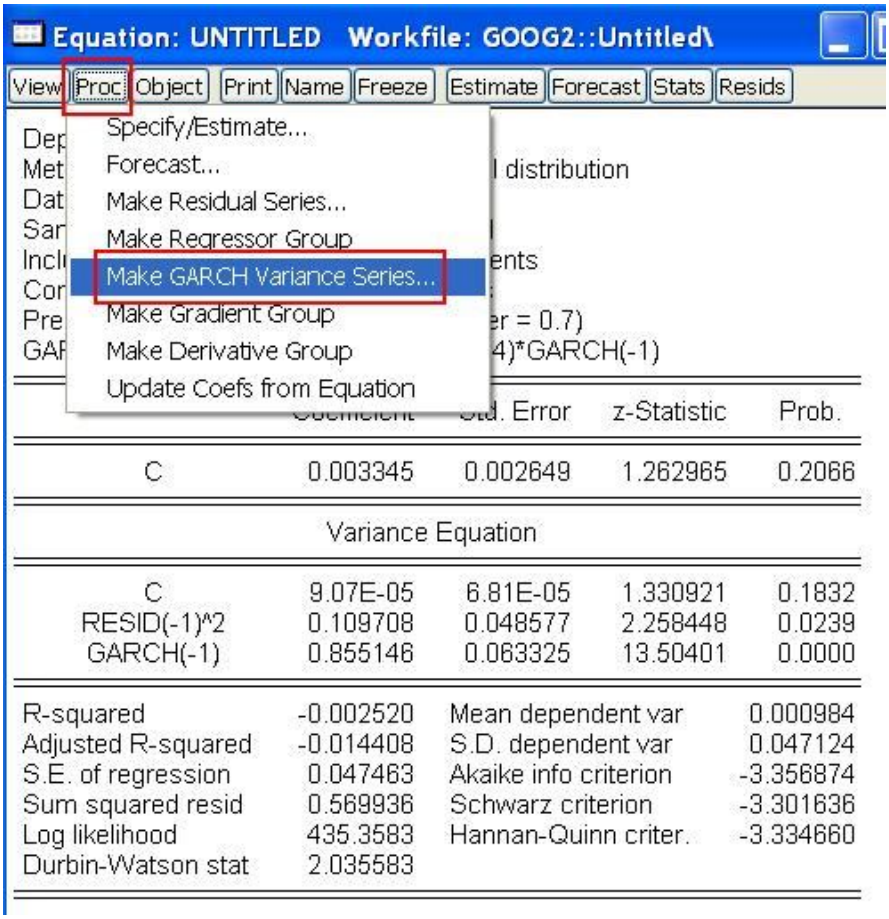
Date: 12/10/11 Time: 12:23
Sample: 1/10/2007 12/07/2011
Included observations: 257



Autocorrelation	Partial Correlation		AC	PAC	Q-Stat	Prob
		1	-0.011	-0.011	0.0304	0.862
		2	0.020	0.020	0.1365	0.934
		3	-0.065	-0.065	1.2528	0.740
		4	-0.072	-0.074	2.6219	0.623
		5	0.009	0.010	2.6455	0.754
		6	0.024	0.023	2.7950	0.834
		7	0.102	0.094	5.5590	0.592
		8	0.015	0.013	5.6215	0.690
		9	-0.016	-0.016	5.6907	0.770
		10	-0.120	-0.108	9.5412	0.482
		11	0.039	0.053	9.9613	0.534
		12	-0.002	0.003	9.9620	0.619
		13	0.130	0.113	14.569	0.335
		14	-0.073	-0.094	16.012	0.313
		15	0.146	0.154	21.870	0.111
		16	-0.012	0.007	21.909	0.146
		17	-0.055	-0.030	22.746	0.158
		18	0.076	0.072	24.339	0.144
		19	-0.032	-0.018	24.630	0.173
		20	-0.034	-0.081	24.952	0.203
		21	-0.007	0.016	24.968	0.249
		22	-0.052	-0.073	25.747	0.263
		23	-0.015	-0.003	25.813	0.310
		24	0.060	0.039	26.841	0.312
		25	0.028	0.056	27.062	0.353
		26	0.084	0.054	29.119	0.306
		27	-0.020	0.006	29.237	0.350
		28	-0.067	-0.087	30.539	0.338
		29	-0.043	-0.008	31.067	0.362
		30	0.052	0.076	31.859	0.374

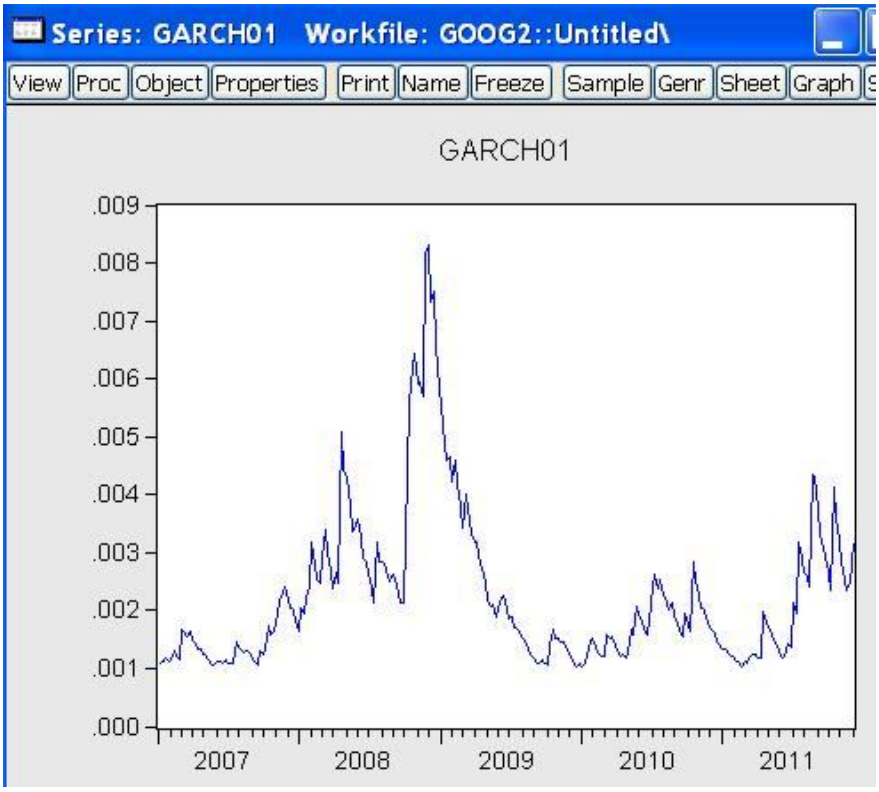
:: Odhadnutá disperzia ::

- Budeme pracovať s GARCH(1,1) modelom:
- V okne s odhadnutou rovnicou klikneme na **Proc**, odkiaľ vyberieme **Make GARCH variance series**

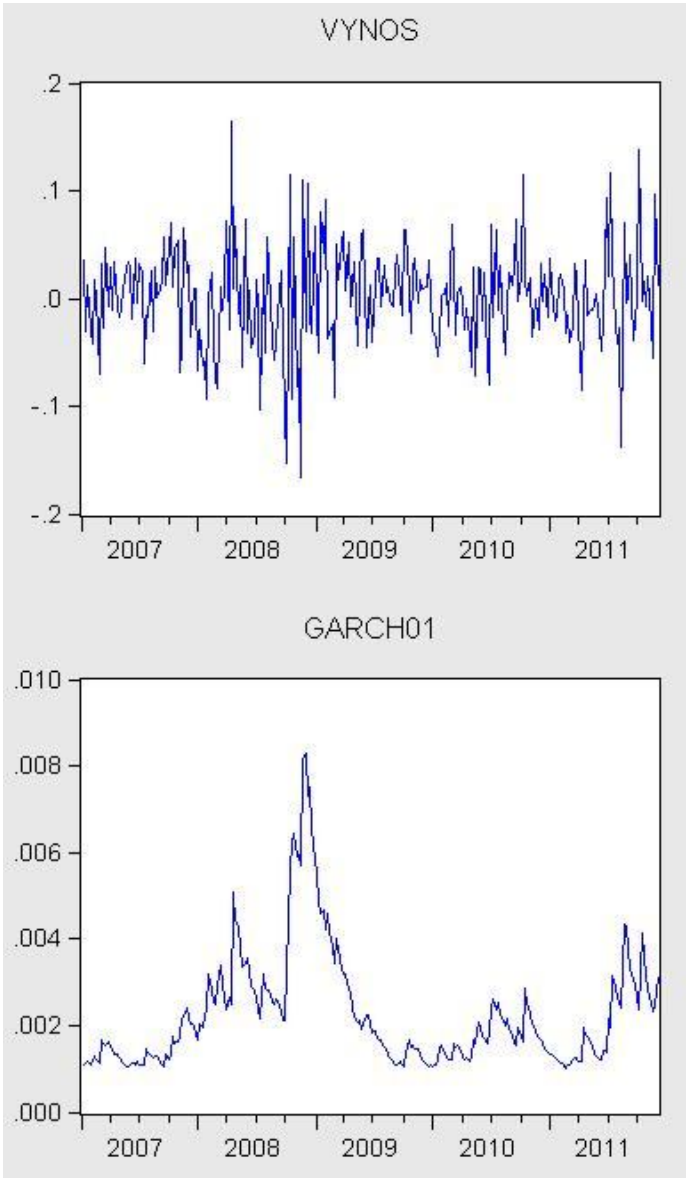


a zadáme názov premennej, do ktorej sa majú tieto variancie uložiť':

- Dostaneme:



- Môžeme teraz porovnať priebeh tejto disperzie s vývojom výnosov - vidíme, ako sa zhodujú stabilné obdobia s malou odhadnutou varianciou a nestabilné obdobia s veľkou.



E-mail: stehlikova@pc2.iam.fmph.uniba.sk
Web: http://pc2.iam.fmph.uniba.sk/institute/stehlikova/