

CVIČENIA Z EKONOMETRIE 2006/2007

DOMÁCA ÚLOHA 2

TERMÍN ODOVZDANIA: 27.2.2007

Pokyny k DÚ:

- Ak úlohu (alebo jej časť) posielate e-mailom, pošlite ju na adresu *bs.ulohy@gmail.com* so subjectom *ekonometria - du2 - vase priezvisko*
- Neodpisujte.

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- (a) (3 body) Odvoďte vzorec na výpočet odhadu parametra λ Poissonovho rozdelenia metódou maximálnej vierohodnosti.
 - (b) (3 body) Nájdite príklad rozdelenia, pre ktoré je odhad parametra metódou maximálnej vierohodnosti vychýlený. Svoje tvrdenie dokážte.
 - (c) (3 body) Uvažujme náhodný výber z rovnomerného rozdelenia na intervale $[0, \theta]$. Nájdite nejaký nevychýlený odhad parametra θ a dokážte jeho nevychýlenosť.
- Po zavedení eura bol v *The Guardian* (4.1.2002)¹ uvedený výsledok pokusu s belgickou mincou s hodnotou jedného eura: Dvaja poľskí matematici a ich študenti hádzali mincou 250 krát. Hlava padla 140 krát.
 - (a) (3 body) Padnutie hlavy pri hode mincou modelujeme alternatívnym rozdelením - nado-búda hodnotu 1 ("padne hlava") s pravdepodobnosťou p a hodnotu 0 ("nepadne hlava") s pravdepodobnosťou $1 - p$. Nájdite nevychýlený odhad parametra p z dát, ktoré obsahujú počet pokusov, v ktorých padla hlava a počet pokusov, v ktorých nepadla hlava. Dokážte jeho nevychýlenosť a použite na dáta z uvedeného pokusu s belgickou mincou.
 - (b) (3 body) V *The Guardian* je okrem iného aj tento komentár:
It looks very suspicious to me. If the coin were unbiased the chance of getting a result as extreme as that would be less than 7%.
Zistite, o akú pravdepodobnosť tu ide a vypočítajte jej hodnotu.

Bonus (3 body) Na cvičení sme ukázali, že odhad štandardnej odchýlky

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2}$$

je vychýlený. Uvažujme teraz náhodný výber z normálneho rozdelenia. Nájdite nevychýlený odhad štandardnej odchýlky tvaru

$$\tilde{s} = c(n)s,$$

kde $c(n)$ je konštanta závislá od rozsahu náhodného výberu.

¹Pozri <http://www.guardian.co.uk/euro/story/0,,627496,00.htm>. Text článku je aj na str. 2 tejto úlohy.

Heads, Belgium wins - and wins

Charlotte Denny and Sarah Dennis Friday January 4, 2002
Guardian

Memo to all teams playing Belgium in the World Cup this year: don't let them use their own coins for the toss.

Mathematicians say the coins issued in the eurozone's administrative heartland are more likely to land heads up than down.

While the notes which began circulating in the 12 members of the eurozone on January 1 are all the same, the coins show national symbols on one side and a map of Europe on the other.

King Albert, who appears on Belgian coins, appears to be a bit of a lightweight, according to Polish mathematicians Tomasz Gliszczynski and Wacław Zawadowski. The two professors and their students at the Podlaska Academy in Siedlce spun a Belgian one euro coin 250 times, and found it landed heads up 140 times. The cent coins proved even more likely to land heads up.

"The euro is struck asymmetrically," Prof Gliszczynski, who teaches statistics, told Germany's Die Welt newspaper.

The blanks for the coins were made in Liege and then struck and engraved in the royal mint in central Brussels. In total 1.95bn coins were churned out.

The head of the mint said yesterday that the Polish mathematicians' findings were "just luck". "When the coins were made they were struck in exactly the same way on all sides and the metal was evenly distributed," said Romain Coenen. "I haven't heard of any problems with the coins."

But a variation of the experiment at the Guardian office suggested that the Polish mathematicians may be right. When tossed 250 times, the one euro coin came up heads 139 times and tails 111.

"It looks very suspicious to me," said Barry Blight, a statistics lecturer at the London School of Economics. "If the coin were unbiased the chance of getting a result as extreme as that would be less than 7%."

A trial of Belgium's soon to be defunct 20 franc coin, which bears the slimmer features of King Albert's predecessor, King Baudoin, suggests that the old currency was a fairer bet: it landed heads up 52 out of 100 times.