73. You are given:

(i) $$\frac{\mathrm{d}S(t)}{S(t)} = 0.3\,\mathrm{d}t - \sigma\,\mathrm{d}Z(t), \qquad t \geq 0,$$

where $Z(t)$ is a standard Brownian motion and σ is a positive constant.

(ii) There is a real number a such that

$$\frac{\mathrm{d}[S(t)]^a}{[S(t)]^a} = -0.66\,\mathrm{d}t + 0.6\,\mathrm{d}Z(t), \quad t \geq 0.$$

Calculate σ.

(A) 0.16

(B) 0.20

(C) 0.27

(D) 0.60

(E) 1.60

36. Assume the Black-Scholes framework. Consider a derivative security of a stock. You are given:

(i) The continuously compounded risk-free interest rate is 0.04.

(ii) The volatility of the stock is σ.

(iii) The stock does not pay dividends.

(iv) The derivative security also does not pay dividends.

(v) $S(t)$ denotes the time-t price of the stock.

(iv) The time-t price of the derivative security is $[S(t)]^{-k/\sigma^2}$, where k is a positive constant.

Find k.

(A) 0.04

(B) 0.05

(C) 0.06

(D) 0.07

(E) 0.08

14. You are using the Vasicek one-factor interest-rate model with the short-rate process calibrated as

$$\mathrm{d}r(t) = 0.6[b - r(t)]\mathrm{d}t + \sigma \mathrm{d}Z(t).$$

For $t \le T$, let $P(r, t, T)$ be the price at time t of a zero-coupon bond that pays \$1 at time T, if the short-rate at time t is r. The price of each zero-coupon bond in the Vasicek model follows an Itô process,

$$\frac{\mathrm{d}P[r(t),t,T]}{P[r(t),t,T]} = \alpha[r(t), t, T]\,\mathrm{d}t - q[r(t), t, T]\,\mathrm{d}Z(t), \qquad t \le T.$$

You are given that $\alpha(0.04, 0, 2) = 0.04139761$.

Find $\alpha(0.05, 1, 4)$.

8. You are considering the purchase of a 3-month 41.5-strike American call option on a nondividend-paying stock.

You are given:

(i) The Black-Scholes framework holds.

(ii) The stock is currently selling for 40.

(iii) The stock's volatility is 30%.

(iv) The current call option delta is 0.5.

Determine the current price of the option.

(A) $20 - 20.453\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x$

(B) $20 - 16.138\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x$

(C) $20 - 40.453\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x$

(D) $16.138\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x - 20.453$

(E) $40.453\int_{-\infty}^{0.15} e^{-x^2/2}\,\mathrm{d}x - 20.453$