I. Derivatives, call and put options, boundaries for option prices, combined strategies

Beáta Stehlíková

Financial derivatives, winter term 2014/2015

Faculty of Mathematics, Physics and Informatics
Comenius University, Bratislava

What are financial derivatives

- Term DERIVATIVE in a dictionary:

noun

1 something which is based on another source:

the aircraft is a derivative of the Falcon 20G

- a word derived from another or from a root in the same or another language:
  'fly-tip' is a derivative of the phrase 'on the fly'
- a substance that is derived chemically from a specified compound:
  crack is a highly addictive cocaine derivative

2 (often **derivatives**) *Finance* an arrangement or product (such as a future, option, or warrant) whose value derives from and is dependent on the value of an underlying asset, such as a commodity, currency, or security:

[*as modifier*]:

the derivatives market

3 *Mathematics* an expression representing the rate of change of a function with respect to an independent variable.

http://oxforddictionaries.com/definition/derivative

Derivatives

- Aristotle writes about Thales of Miletus (Politics, Book I, Chapter XI):

  ... while it was yet winter, having got a little money, he gave earnest for all the oil works that were in Miletus and Chios, which he hired at a low price, there being no one to bid against him; but when the season came for making oil, many persons wanting them, he all at once let them upon what terms he pleased; and raising a large sum of money

  English translation: http://www.gutenberg.org

- Right to use the oil presses - its value depends on the crop in the given year
- Some presses may stay unused; Thales has a right, but not an obligation to use the presses

Derivatives

- A right but also an obligation to realize an arranged trade - some historical examples:
  - England, France, 12th century - arrangement of a future trade based on a sample, *„lettre de faire“*
  - Japan, 17th century - standardized rice trades
  - Chicago, 19th century - wheat, establishment of *Chicago Board of Trade* (1848)
  - 1898 - *Chicago Mercantile Trading*, butter and eggs, later also other agricultural commodities
  - 1978 - *International Monetary Market* as a part of *Chicago Mercantile Trading*, foreign exchange, later also (e.g.) S&P 500 derivatives

Akcie

- Mostly we will deal with derivatives in stock market
- Example: evolution of DIS (*The Walt Disney Company*) stock price

http://finance.yahoo.com

Stocks

- Evolution of a stock price consists of a trend and random fluctuations
- Example of a trend: NFLX (Netflix, Inc.), august 2012 - august 2014):

http://finance.google.com

Stocks

- Example of fluctuations: NFLX (Netflix, Inc.)

http://finance.google.com

- Next lecture: mathematical modelling of this observation
  trend + fluctuations

Stock options

- European call option is a right - but not an obligation - to buy the asset for the predetermined price E) (*strike price, exercise price*) in the predetermined time T) (*expiration time*)
- European put option is a right - but not an obligation - to sell the asset for the predetermined price E) (*strike price, exercise price*) in the predetermined time T) (*expiration time*)
- Americal call/put options - a right to buy/sell the stock not ony at the expiration time T), but at any time prior to the expiration time

Stock options

- Example of real data: put options on Disney stock

Put Options							Expire at close Saturday, October 18, 2014
Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
50.00	DIS141018P00050000	**0.03**	0.00	0.01	0.04	34	396
55.00	DIS141018P00055000	**0.05**	0.00	0.02	0.05	32	172
60.00	DIS141018P00060000	**0.09**	0.00	0.04	0.08	10	416
65.00	DIS141018P00065000	**0.15**	0.00	0.07	0.14	25	815
67.50	DIS141018P00067500	**0.15**	0.00	0.08	0.14	5	130
70.00	DIS141018P00070000	**0.17**	↓0.03	0.12	0.17	30	1,062
72.50	DIS141018P00072500	**0.23**	↓0.03	0.17	0.21	2	734

http://finance.yahoo.com

Option price

- Option price consists of two parts:
  - intrinsic value - value of the option if exercised now
  - time value - remaining part of the price:
    - holder of the option pays this value, expecting that the option brings him profit in the future
    - risk premium for the writer of the option

Intrinsic and time value: example

- Put prices from page 9 - we will use the last realized price
- Stock price: 87.40 USD

The Walt Disney Company (DIS) - NYSE ★ Follow

87.40 ↑0.55(0.63%) 9:53AM EDT - Nasdaq Real Time Price

Prev Close:	86.85	Day's Range:	87.05 - 87.49
Open:	87.07	52wk Range:	60.41 - 87.63
Bid:	87.33 x 200	Volume:	482,854
Ask:	87.35 x 200	Avg Vol (3m):	6,031,720

- Let us consider the put option with exercise price 70 USD which costs 0.17 USD:
  - intrinsic value: 0
  - time value: 0.17

Intrinsic and time value: example

- Questions:
  - Why do all the options (page 9) zero intrinsic value?
  - Which puts would have a positive intrinsic value?
  - How about call options? Use data below:

Toyota Motor Corporation (TM) - NYQ ★ **Follow**

119.43 ↑2.08(1.77%) Sep 18, 4:03PM EDT

Options

View By Expiration: Sep 14 | **Oct 14** | Jan 15 | Apr 15 | Jan 16 | Jan 17

Call Options

Strike	Symbol	Last	Chg
110.00	TM141018C00110000	**9.45**	↑1.79
115.00	TM141018C00115000	**4.45**	↑1.40
120.00	TM141018C00120000	**1.20**	↑0.44
125.00	TM141018C00125000	**0.20**	↑0.03
130.00	TM141018C00130000	**0.05**	0.00

Example

- We sell the DIS stock for the current *bid price* (the price a buyer is willing to pay): 87.33 USD.

The Walt Disney Company (DIS) - NYSE Follow

87.40 0.55 (0.63%) 9:53AM EDT - Nasdaq Real Time Price

Prev Close:	86.85	Day's Range:	87.05 - 87.49
Open:	87.07	52wk Range:	60.41 - 87.63
Bid:	87.33 x 200	Volume:	482,854
Ask:	87.35 x 200	Avg Vol (3m):	6,031,720

http://finance.yahoo.com

Example

- Then, we sell a put option with exercise price 60 USD and expiration in October for - we find the *bid price* - 0.04 USD

Put Options — Expire at close Saturday, October 18, 2014

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
50.00	DIS141018P00050000	0.03	0.00	0.01	0.04	34	396
55.00	DIS141018P00055000	0.05	0.00	0.02	0.05	32	172
60.00	DIS141018P00060000	0.09	0.00	0.04	0.08	10	416
65.00	DIS141018P00065000	0.15	0.00	0.07	0.14	25	815
67.50	DIS141018P00067500	0.15	0.00	0.08	0.14	5	130
70.00	DIS141018P00070000	0.17	↓0.03	0.12	0.17	30	1,062
72.50	DIS141018P00072500	0.23	↓0.03	0.17	0.21	2	734

http://finance.yahoo.com

Example

- How much are you willing to pay for a call option with the same exercise price and the same expiration time?
- Recall the evolution of DIS stock price; the data are from August 11, 2014:

http://finance.yahoo.com

Example

- Russel Sage, New York, 19th century:
  - bought a stock and a put option, sold a call with the same exercise price and the same expiration time
  - avoided bounds on interest rates given by usury laws

- EXAMPLE - CONTINUED:
  - We show that this strategy is - in fact - a loan (so called *synthetised loan*)
  - What interest rate did you agreed on by your accepted price of the call option?

Call-put parity

- Consider a portfolio:
  - write 1 call option with exercise price E
  - sell 1 put option with the same exercise price and the same expiration time
  - buy 1 stock
- What will be the portfolio value at the time of expiration?

portfolio = - 1 call + 1 put + 1 stock

$$\Rightarrow$$

payoff = - [payoff of call] + [payoff of put] + [stock price]

Call-put parity

- Hence, depending on the stock price S at the time of expiration:
  - if $S \leq E$:

$$payoff = -[0] + [E - S] + [S] = E$$

  - if $S \geq E$:

$$payoff = -[S - E] + [0] + [S] = E$$

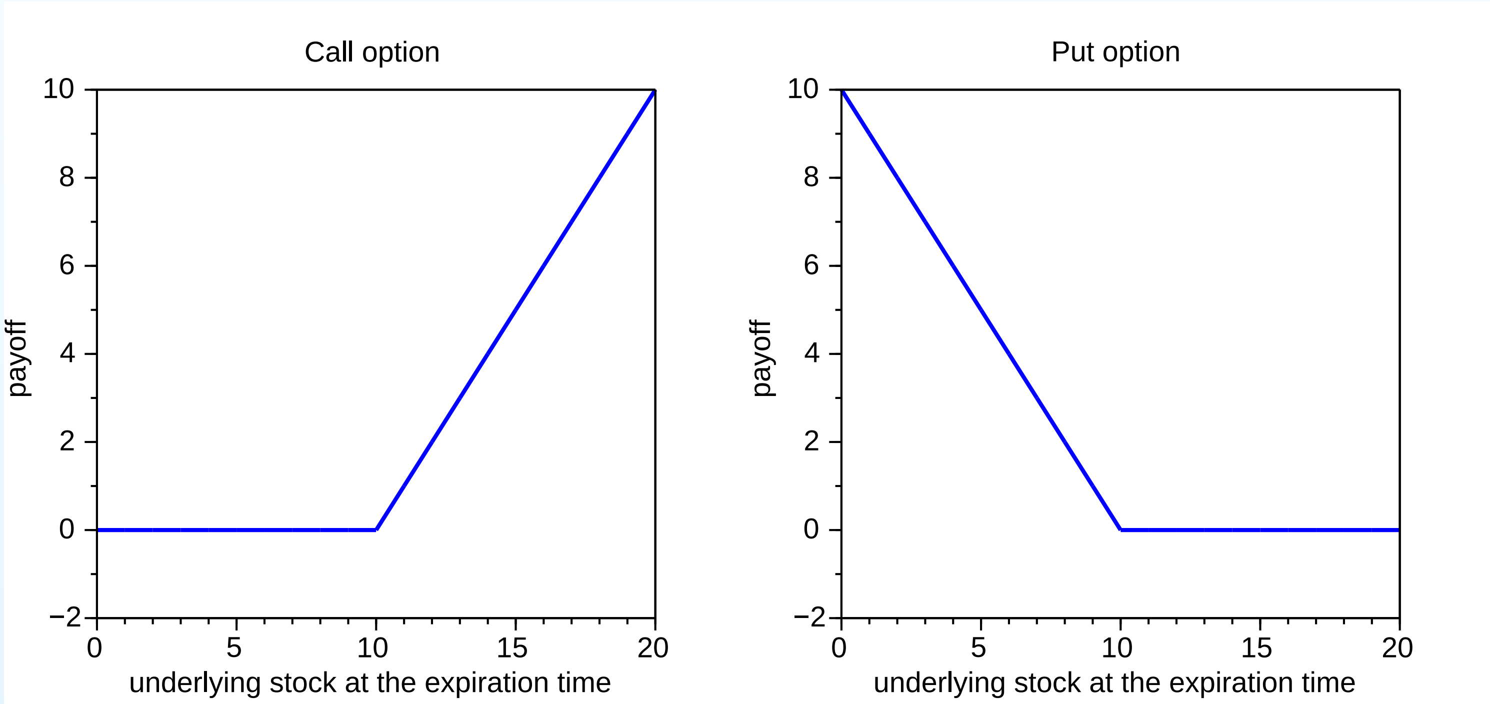
  So, without any risk we end up with E
- Therefore, the value of the portfolio today has to be

$$-c(S, E, \tau) + p(S, E, \tau) + S = Ee^{-r\tau}$$

  - we have obtained a relation between call and put prices, known as call-put parity

Payoff diagram

- Payoff diagram of an option - value of the option at the time of expiration, as a function of the stock price at this time
- Call option: $\max(0, S - E)$, put option: $\max(E - S, 0)$



Profit diagram

- Profit diagram of an option - payoff of the option minus the value of our initial investment:
  - If $r = 0$, then

    profit = payoff - costs

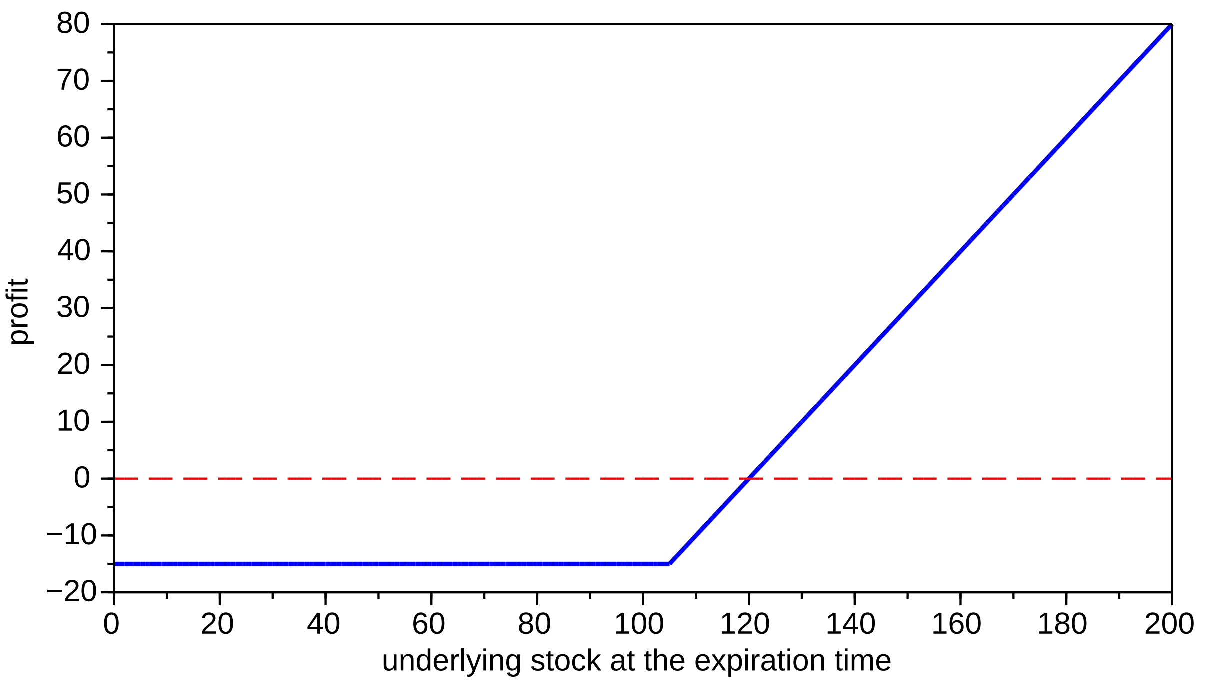
  - In general:

    profit = payoff - costs $\times e^{r\tau}$

    (to pay *costs* today is the same as to pay *costs* $\times e^{r\tau}$ at the expiration time)

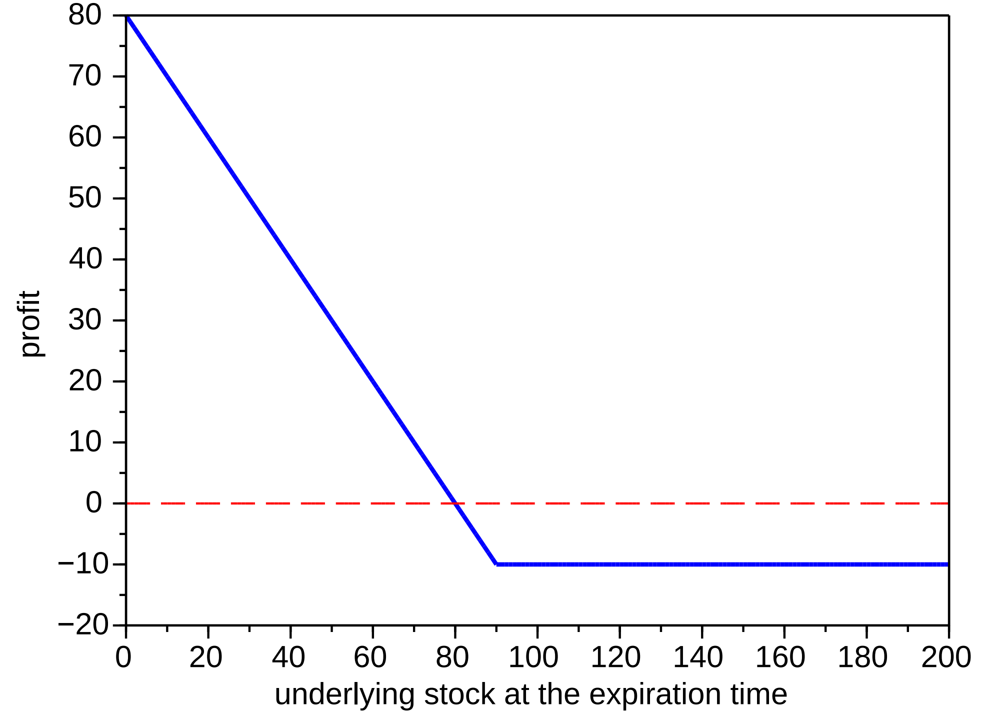
Profit diagram - example 1

- Consider a call option with exercise price 105 USD which costs 15 USD
- Profit diagram (for $r = 0$):



Profit diagram - example 2

- Analyze the following profit diagram of a put option (for $r = 0$):



Profit diagram - example 2

- SIMPLE QUESTIONS:
  - What is the exercise price of the option? How much did it cost?
  - Is the possible profit bounded? If it is bounded, when it is maximal? If it is not bounded, when it rises without bounds?
  - Is the possible loss bounded? If it is bounded, when it is maximal? If it is not bounded, when it rises without bounds?

Bounds for option prices

- We show some inequalities for prices, which have to hold - otherwise there is an arbitrage on the market
- All the options considered have the same expiration time
- We denote the riskless interest rate by r.
- Notation:
  - $c(S, E, \tau)$ is the market price of a call option with exercise price E, if the stock price today is S and time remaining to expiration is τ
  - $p(S, E, \tau)$ is the market price of a put option with exercise price E, if the stock price today is S and time remaining to expiration is τ

Bounds for option prices

- Outline:
  - We consider two portfolios - such that at the time of expiration we have:

    (value of portfolio I.) $\leq$ (value of portfolio II.)
  - To avoid a possibility of arbitrage, also today necessarily

    (value of portfolio I.) $\leq$ (value of portfolio II.);

    the portfolios are constructed in such a way that *this* is the inequality that we need to prove

Bounds for option prices - examples

EXAMPLE 1: Clearly

$$c(S, E, \tau) \geq 0, p(S, E, \tau) \geq 0$$

EXAMPLE 2: Show that

$$E_1 \geq E_2 \Rightarrow c(S, E_1, \tau) \leq c(S, E_2, \tau)$$

SOLUTION Let $E_1 \geq E_2$ Consider the following portfolios

portfolio I.: option with exercise price E_1

portfolio II.: option with exercise price E_2

We compare their value at the expiration time, depending on the stock price S at this time

Bounds for option prices - examples

	$0 \le S \le E_2$	$E_2 \le S \le E_1$	$E_1 \le S$
portfolio I.	0	0	$S - E_1$
portfolio II.	0	$S - E_2$	$S - E_2$
comparison	$0 = 0$	$0 \le S - E_2$	$S - E_1 \le S - E_2$

At the expiration time:

(value of portfolio I.) $\le$ (value of portfolio II.)

$\Rightarrow$ also today:

(value of portfolio I.) $\le$ (value of portfolio II.),

i.e.,

$c(S, E_1, \tau) \le c(S, E_2, \tau)$, QED

Bounds for option prices - examples

EXAMPLE 3:
Assume zero interest rate and the following call option prices:

exercise price	option price
10	30
15	26
20	27
25	23
30	19

Find an arbitrage.

SOLUTION: We plot the dependence of the call option price on the exercise price - its decreasing character, proved in the previous example, is not satisfied.

Bounds for option prices - examples

We should have $c(S, 15, \tau) \geq c(S, 20, \tau)$; however, here $c(S, 15, \tau) < c(S, 20, \tau)$. Therefore:

- *we buy the option, which costs less than it is supposed to*, in this case the option with exercise price $E = 15$,
- *we sell the option, which costs more than it is supposed to*, in this case the option with exercise price $E = 20$.

Bounds for option prices - examples

- We check the result of our strategy:

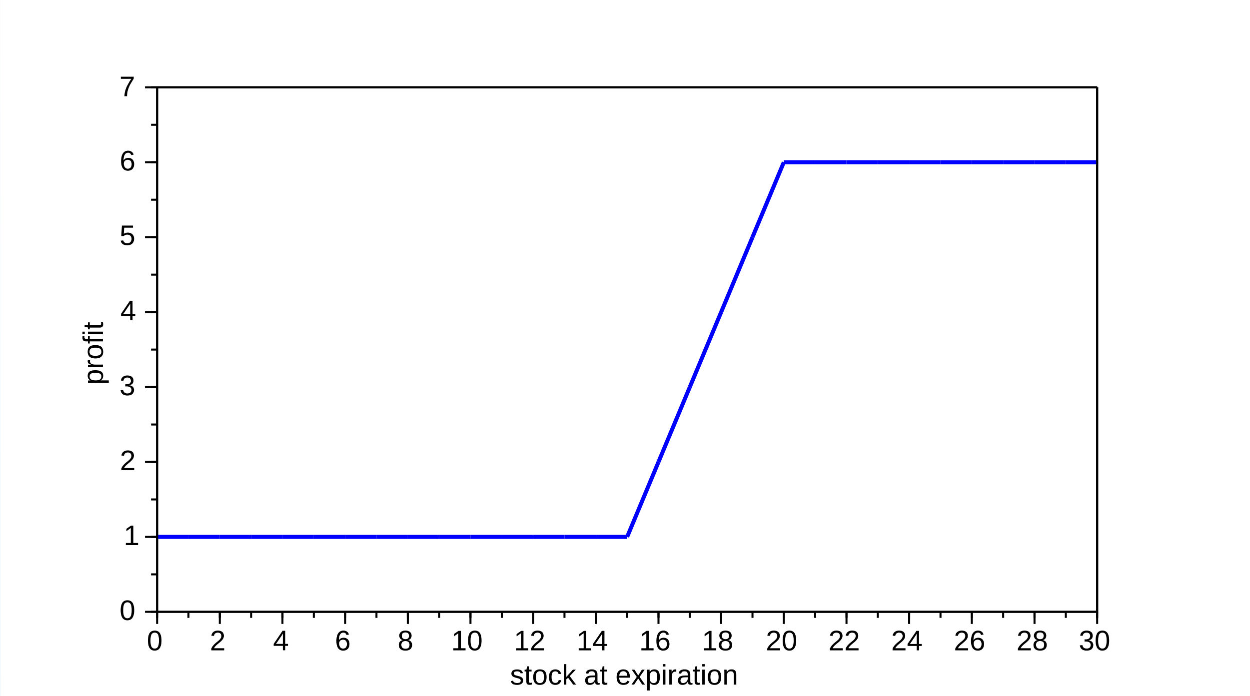
```
S=0:30;
payoff=max(S-15,0)-max(S-20,0);
price=26-27; // price of the strategy = initial investment
profit=payoff-price;

figure(1);
plot(S,profit);
xlabel("stock at expiration"); ylabel("profit");
```

- What do we expect as a results? How does a profit diagram of an arbitrage look like?

Bounds for option prices - examples

- Resulting profit diagram:



$\Rightarrow$ this strategy is indeed an arbitrage

- EXERCISES SESSION NEXT WEEK: More practice with proving bounds for option prices and finding arbitrage opportunities

Combined strategies

- In the previous (theoretical) example we combined the options to construct an arbitrage
- This idea of buying and sellig several options can be used also with real option prices - based on our expectations about future behavious of the stock price

Combined strategies

EXAMPLE:

- Consider the MCD (Mac Donald's Corp.) stock prices

http://finance.yahoo.com

and suppose (for this exercise) that we expect the stock price to be falling

Combined strategies

- The stock price is 93.72 USD and some of the available options are (data from August 11):

Put Options							Expire at close Friday, September 5, 2014
Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
85.00	MCD140920P00085000	**0.22**	0.00	0.17	0.22	56	1,471
86.00	MCD140905P00086000	**0.16**	0.00	0.11	0.18	2	5
87.50	MCD140920P00087500	**0.33**	↓0.03	0.31	0.33	17	1,575
88.00	MCD140926P00088000	**0.59**	0.00	0.41	0.49	2	2
89.00	MCD140905P00089000	**0.36**	0.00	0.28	0.34	26	965
89.00	MCD140912P00089000	**0.45**	0.00	0.37	0.44	13	83
89.00	MCD140926P00089000	**0.66**	0.00	0.56	0.63	4	4
90.00	MCD140920P00090000	**0.65**	↓0.07	0.62	0.66	21	6,135

http://finance.yahoo.com

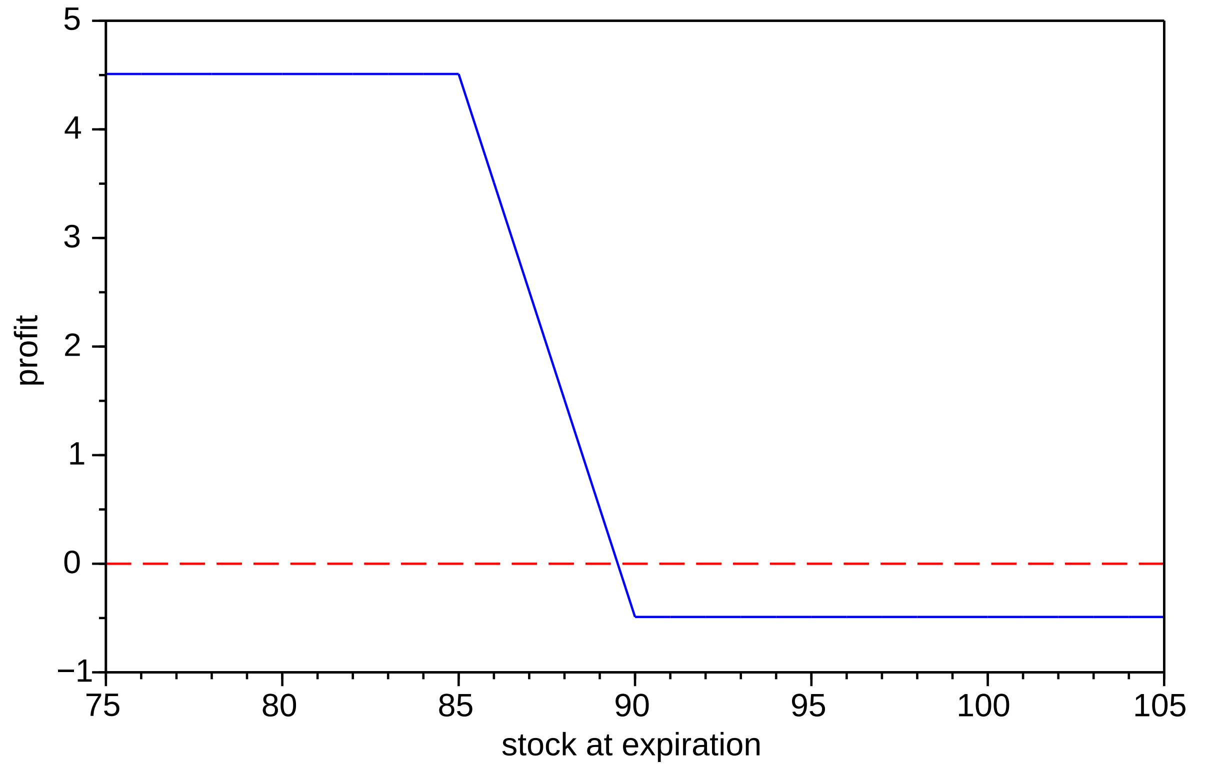
- We expect the stock to fall $\Rightarrow$ we buy a put option, for example one with exercise price 90 USD
- Howver, we don't expect it to fall too low $\Rightarrow$ we sell a put option with a lower expiration price, for example 85 USD
- We expect that the latter will not be exercised, but by writing it, we lower the initial investment

Combined strategies

- Our strategy:
  we buy a put with $E = 16$ *and sell a put with* $E = 14$
- Recall bid and ask prices:
  - bid price (the lower one) - the price a buyer is willing to pay $\rightarrow$ I can sell the option for *bid*
  - ask price (the higher one) - the price a seller is willing to accept $\rightarrow$ I can see the option for *ask*
- Therefore our initial investment is 0.49, since:
  - we buy the put with $E = 16$ for 0.66
  - we sell the put with $E = 13$ for 0.17

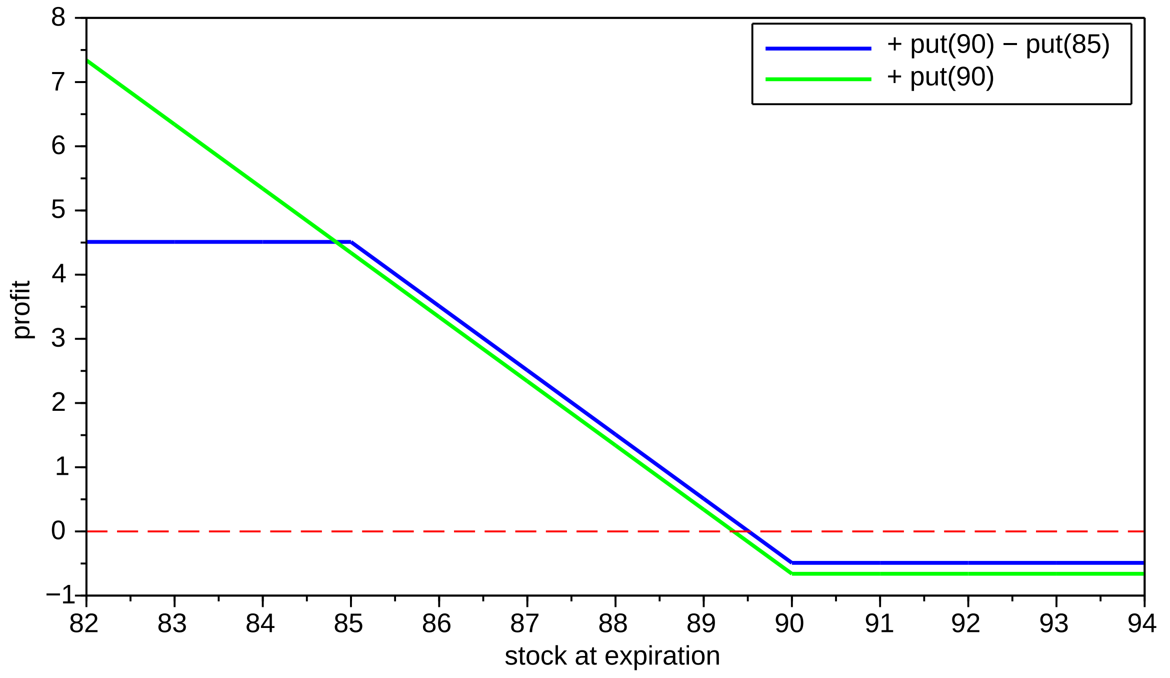
Combined strategies

- Profit diagram:



Combined strategies

- Comparison - with only buying the put with $E = 90$:



Combined strategies

- EXTRA CREDIT:
  Construction of a combined strategy using the real data, aiming to achieve the highest profit
- Overview of combined strategies:
  - Ševčovič, Stehlíková, Mikula: **Analytické a numerické metódy oceňovania finančných derivátov**. STU 2009. (In Slovak) - chapter 2.3.3.
  - Ševčovič, Stehlíková, Mikula: **Analytical and numerical methods for pricing financial derivatives.** Nova Science Publishers, Inc., Hauppauge, 2011. - chapter 2.3.2
  - http://www.theoptionsguide.com/option-trading-strategies.aspx