Financial derivatives, winter term 2014/2015
Sample test 1

1. **True or false?** Write only the answer. *Correct answer +0.25 points, incorrect answer -0.25 points, no answer 0 points.*

   (a) If the underlying stock price is greater than the strike price of a call option, the option is in the money.

   (b) If the price of a call option is greater than the price of the underling stock, there is an arbitrage possibility.

   (c) Selling one call option and one put option with the same stike prices and same expiration times, written on the same underlying asset, is an example of a bearish strategy (i.e., used when expecting the asset price to fall).

   (d) Let $x(t) = e^{w(t)}$, where w is a Wiener process. Then its expected value at each time t equals 1.

   (e) Assume that the stock price S follows a geometrical Brownian motion $S(t) = S_0 e^{\mu t + \sigma w(t)}$, where w is a Wiener process and $\mu, \sigma > 0$. Then its expected value is an increasing function of time.

   (f) Let w be a Wiener process. Then the icrements $w(3) - w(1)$ and $w(2) - w(1)$ are independent random variables.

2. *Max. 1.5 points.* Consider the following pair of call and put options: they have the common strike price of 55 USD and they both expire in one year. The stock price is 53 USD and the call price is 0.1 USD higher than the put price. Determine the interest rate.

3. *Max. 1.5 points.* Compute the differential dy, if $y = t^3 e^{2w}$, where w is a Wiener process.

4. *Max. 1.5 points.* Assume that the stock price S follows a geometrical Brownian motion $S(t) = S_0 e^{\mu t + \sigma w(t)}$, where w is a Wiener process and $\mu, \sigma > 0$. Consider the probability of a negative return at time t as a function of time: $p(t) = Prob\left(\log\left(\frac{S(t)}{S(0)}\right) < 0\right)$. Compute the limit of $p(t)$ as $t \to \infty$.

5. *Max. 1.5 points.* Prove that the price of a put option is a convex function of the exercise price.

6. *Max. 0.5 points.* Find the expected value of the process $x(t) = e^{-t} w(t)^2$ at time t and sketch the graph of the expected value as a function of time.