## Financial derivatives Sample test 2: Black-Scholes and Leland model

1. Compute the Black-Scholes price of a call option on a stock which does not pay dividends and has the volatility 0.2 , if its exercise price is 200 USD and expiration in one year. Interest rate is zero and the price of the stock is 180 USD.
2. Consider a European cal option on a non-dividend paying stock with exercise price 100 USD and expiration time in one year. Interest rate is 1 percent and the price of the stock today is 75 USD. For what price of the option is the Black-Scholes implied volatility equal to 0.35 ?
3. Consider a European call option on a non-dividend paying stock with exercise price 100 USD and expiration in one month. Interest rate is 1 percent and volatility of the stock is 0.4 . We sold 100 of these options and use Black-Scholes delta hedging to hedge our portfolio. Give an example of the stock price, for which we have more than 50 stocks in our portfolio.
4. Consider the delta of a put option on a stock which does not pay dividends. Find the limit of the delta as time to expiration approaches zero.
5. Consider the following inputs for the Leland model:

| Oracle Corporation (ORCL) - NYSE Watchlist |  |  |  |
| :---: | :---: | :---: | :---: |
| 41.87 + $0.72(1.75 \%)$ Nov $26,4: 00 \mathrm{PM}$ EST |  |  |  |
| After Hours : 41.870 .00 (0.00\%) Nov 26, 4:57PM EST |  |  |  |
| Prev Close: | 41.15 | Day's Range: | 41.18-41.91 |
| Open: | 41.19 | 52wk Range: | 33.22-43.19 |
| Bid: | $41.65 \times 1800$ | Volume: | 11,590,010 |
| Ask: | $41.86 \times 2700$ | Avg $\mathrm{Vol}(3 \mathrm{~m})$ : | 15,282,200 |
| 1 y Target Est: | 43.76 | Market Cap: | 185.54B |

The maturity of the following put options is 11 days (consider 252 trading days in a year) and the interest rate is equal to zero. Compute the implied volatility and implied time between two adjustments of portfolio for the option with strike price 39 USD using data below:

| Puts |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| ^ Strike | Contract Name | Last | Bid | Ask | Change | \%Change |
| *Filter |  |  |  |  |  |  |
| 38.00 | ORCL141212P00038000 | 0.21 | 0.01 | 0.07 | 0.00 | 0.00\% |
| 38.50 | ORCL141212P00038500 | 0.20 | 0.02 | 0.08 | 0.00 | 0.00\% |
| 39.00 | ORCL141212P00039000 | 0.22 | 0.03 | 0.09 | 0.00 | 0.00\% |
| 39.50 | ORCL141212P00039500 | 0.12 | 0.04 | 0.09 | 0.00 | 0.00\% |
| $4 \cap ก$ | ORC. 141210Pกกก4กกกก | $\bigcirc 13$ | ก กк | $\bigcirc 11$ | $\bigcirc \cap$ | 0 กn\% |

6. Suppose that there are 252 trading days in a year and that the stock market is opened 7 hours a day. The difference between ask and bid price of a stock is 0.5 percent of their average
values. Consider the Leland model. We would like to hedge our portfolio every 10 minutes. Give an example of a stock volatility, for which it is a feasible time interval and an example of a volatility, for which it is an infeasible time interval.
7. Consider the difference between bid and ask price of an option as a function of the stock price $S$. (The remaining parameters - stock volatility, parameter $c$ from the transaction costs, interest rate, strike price, expiration time - are constant). Derive the stock price for which the difference is maximal.
