I. Derivatives, call and put options, bounds on option prices, combined strategies

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Financial derivatives

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What are financial derivatives

- Term DERIVATIVE in a dictionary:

noun

1 something which is based on another source:

the aircraft is a derivative of the Falcon 20G

- a word derived from another or from a root in the same or another language:
  'fly-tip' is a derivative of the phrase 'on the fly'
- a substance that is derived chemically from a specified compound:
  crack is a highly addictive cocaine derivative

2 (often **derivatives**) *Finance* an arrangement or product (such as a future, option, or warrant) whose value derives from and is dependent on the value of an underlying asset, such as a commodity, currency, or security:

[*as modifier*]:

the derivatives market

3 *Mathematics* an expression representing the rate of change of a function with respect to an independent variable.

http://oxforddictionaries.com/definition/derivative

Derivatives

- Aristotle writes about Thales of Miletus (Politics, Book I, Chapter XI):

  ... while it was yet winter, having got a little money, he gave earnest for all the oil works that were in Miletus and Chios, which he hired at a low price, there being no one to bid against him; but when the season came for making oil, many persons wanting them, he all at once let them upon what terms he pleased; and raising a large sum of money

  English translation: http://www.gutenberg.org

- Right to use the oil presses - its value depends on the crop in the given year
- Some presses may stay unused; Thales has a right, but not an obligation to use the presses

Derivatives

- A right but also an obligation to realize an arranged trade - some historical examples:
  - England, France, 12th century - arrangement of a future trade based on a sample, *„lettre de faire“*
  - Japan, 17th century - standardized rice trades
  - Chicago, 19th century - wheat, establishment of *Chicago Board of Trade* (1848)
  - 1898 - *Chicago Mercantile Trading*, butter and eggs, later also other agricultural commodities
  - 1978 - *International Monetary Market* as a part of *Chicago Mercantile Trading*, foreign exchange, later also (e.g.) S&P 500 derivatives

Akcie

- Mostly we will deal with derivatives in stock market
- Example: evolution of DIS (*The Walt Disney Company*) stock price

http://finance.yahoo.com

Stocks

- Evolution of a stock price consists of a trend and random fluctuations
- Example of a trend: NFLX (Netflix, Inc.):

http://finance.google.com

Stocks

- Example of fluctuations: NFLX (Netflix, Inc.)

http://finance.google.com

- Next lecture: mathematical modelling of this observation
  trend + fluctuations

Stock options

- European call option is a right - but not an obligation - to buy the asset for the predetermined price E) (*strike price, exercise price*) in the predetermined time T) (*expiration time*)
- European put option is a right - but not an obligation - to sell the asset for the predetermined price E) (*strike price, exercise price*) in the predetermined time T) (*expiration time*)
- Americal call/put options - a right to buy/sell the stock not ony at the expiration time T), but at any time prior to the expiration time

Stock options

- Example of real data: put options on Disney stock

Put Options — Expire at close Saturday, October 18, 2014

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
50.00	DIS141018P00050000	**0.03**	0.00	0.01	0.04	34	396
55.00	DIS141018P00055000	**0.05**	0.00	0.02	0.05	32	172
60.00	DIS141018P00060000	**0.09**	0.00	0.04	0.08	10	416
65.00	DIS141018P00065000	**0.15**	0.00	0.07	0.14	25	815
67.50	DIS141018P00067500	**0.15**	0.00	0.08	0.14	5	130
70.00	DIS141018P00070000	**0.17**	↓0.03	0.12	0.17	30	1,062
72.50	DIS141018P00072500	**0.23**	↓0.03	0.17	0.21	2	734

http://finance.yahoo.com

Option price

- Option price consists of two parts:
  - intrinsic value - value of the option if exercised now
  - time value - remaining part of the price:
    - holder of the option pays this value, expecting that the option brings him profit in the future
    - risk premium for the writer of the option

Intrinsic and time value: example

- Put prices from page 9 - we will use the last realized price
- Stock price: 87.40 USD

The Walt Disney Company (DIS) - NYSE Follow

87.40 0.55(0.63%) 9:53AM EDT - Nasdaq Real Time Price

Prev Close:	86.85	Day's Range:	87.05 - 87.49
Open:	87.07	52wk Range:	60.41 - 87.63
Bid:	87.33 x 200	Volume:	482,854
Ask:	87.35 x 200	Avg Vol (3m):	6,031,720

- Let us consider the put option with exercise price 70 USD which costs 0.17 USD:
  - intrinsic value: 0
  - time value: 0.17

Intrinsic and time value: example

- Questions:
  - Why do all the options (page 9) zero intrinsic value?
  - Which puts would have a positive intrinsic value?
  - How about call options? Use data below:

Toyota Motor Corporation (TM) - NYQ ★ **Follow**

119.43 ↑2.08(1.77%) Sep 18, 4:03PM EDT

Options

View By Expiration: Sep 14 | **Oct 14** | Jan 15 | Apr 15 | Jan 16 | Jan 17

Call Options

Strike	Symbol	Last	Chg
110.00	TM141018C00110000	**9.45**	↑1.79
115.00	TM141018C00115000	**4.45**	↑1.40
120.00	TM141018C00120000	**1.20**	↑0.44
125.00	TM141018C00125000	**0.20**	↑0.03
130.00	TM141018C00130000	**0.05**	0.00

Example

- We sell the DIS stock for the current *bid price* (the price a buyer is willing to pay): 87.33 USD.

The Walt Disney Company (DIS) - NYSE ★ Follow

87.40 ↑0.55(0.63%) 9:53AM EDT - Nasdaq Real Time Price

Prev Close:	86.85	Day's Range:	87.05 - 87.49
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Bid:	87.33 x 200	Volume:	482,854
Ask:	87.35 x 200	Avg Vol (3m):	6,031,720

http://finance.yahoo.com

Example

- Then, we sell a put option with exercise price 60 USD and expiration in October for - we find the *bid price* - 0.04 USD

Put Options — Expire at close Saturday, October 18, 2014

Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
50.00	DIS141018P00050000	**0.03**	0.00	0.01	0.04	34	396
55.00	DIS141018P00055000	**0.05**	0.00	0.02	0.05	32	172
60.00	DIS141018P00060000	**0.09**	0.00	0.04	0.08	10	416
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http://finance.yahoo.com

Example

- How much are you willing to pay for a call option with the same exercise price and the same expiration time?
- Recall the evolution of DIS stock price; the options are from August:

http://finance.yahoo.com

Example

- Russel Sage, New York, 19th century:
  - bought a stock and a put option, sold a call with the same exercise price and the same expiration time
  - avoided bounds on interest rates given by usury laws

- EXAMPLE - CONTINUED:
  - We show that this strategy is - in fact - a loan (so called *synthetised loan*)
  - What interest rate did you agreed on by your accepted price of the call option?

Call-put parity

- Consider a portfolio:
  - write 1 call option with exercise price E
  - sell 1 put option with the same exercise price and the same expiration time
  - buy 1 stock
- What will be the portfolio value at the time of expiration?

portfolio = - 1 call + 1 put + 1 stock

$$\Rightarrow$$

payoff = - [payoff of call] + [payoff of put] + [stock price]

Call-put parity

- Hence, depending on the stock price S at the time of expiration:
  - if $S \leq E$:

$$payoff = -[0] + [E - S] + [S] = E$$

  - if $S \geq E$:

$$payoff = -[S - E] + [0] + [S] = E$$

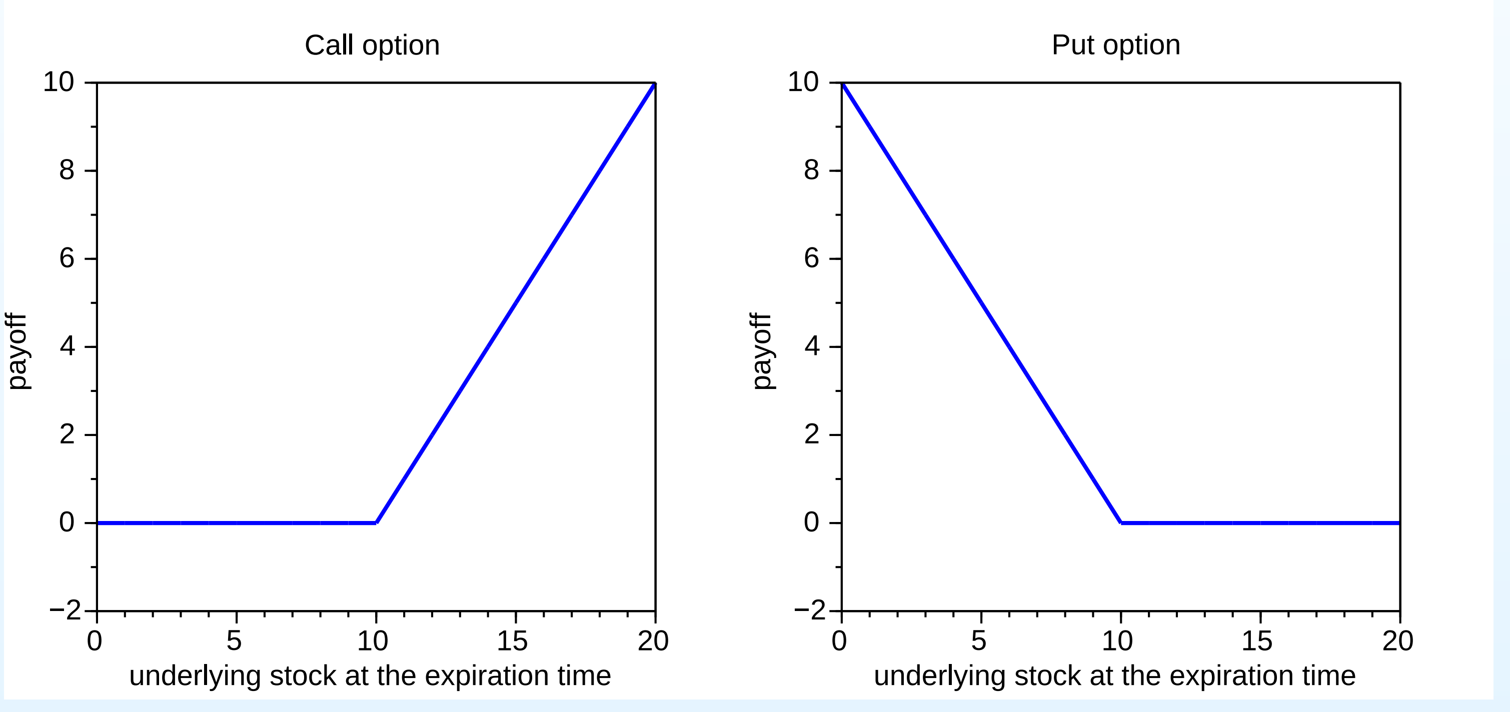
  So, without any risk we end up with E
- Therefore, the value of the portfolio today has to be

$$-c(S, E, \tau) + p(S, E, \tau) + S = Ee^{-r\tau}$$

  - we have obtained a relation between call and put prices, known as call-put parity

Payoff diagram

- Payoff diagram of an option - value of the option at the time of expiration, as a function of the stock price at this time
- Call option: $\max(0, S - E)$, put option: $\max(E - S, 0)$



Profit diagram

- Profit diagram of an option - payoff of the option minus the value of our initial investment:
  - If $r = 0$, then

    profit = payoff - costs

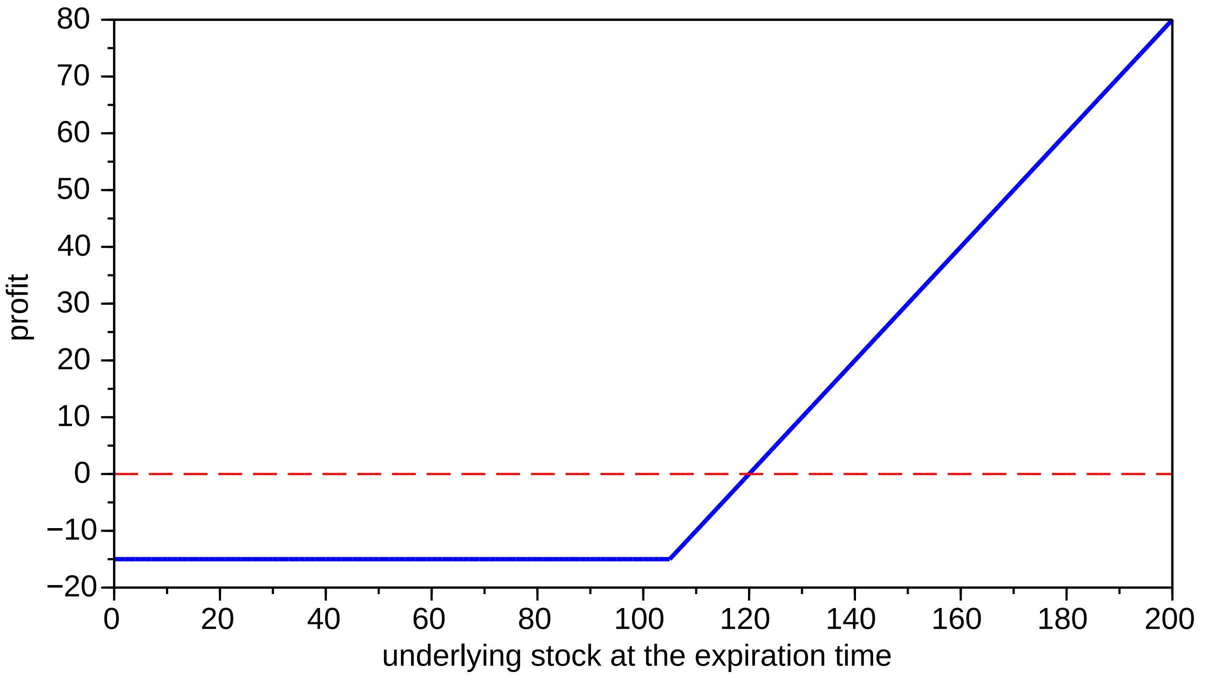
  - In general:

    profit = payoff - costs $\times e^{r\tau}$

    (to pay *costs* today is the same as to pay *costs*$\times e^{r\tau}$ at the expiration time)

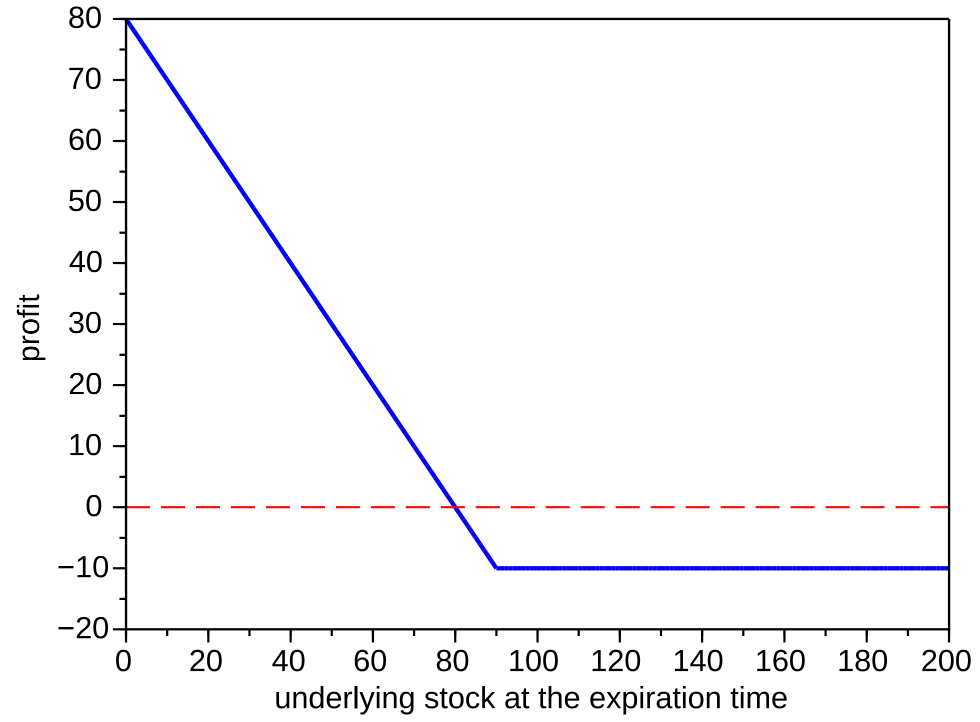
Profit diagram - example 1

- Consider a call option with exercise price 105 USD which costs 15 USD
- Profit diagram (for $r = 0$):



Profit diagram - example 2

- Analyze the following profit diagram of a put option (for $r = 0$):



Profit diagram - example 2

- Simple questions:
  - What is the exercise price of the option? How much did it cost?
  - Is the possible profit bounded? If it is bounded, when it is maximal? If it is not bounded, when it rises without bounds?
  - Is the possible loss bounded? If it is bounded, when it is maximal? If it is not bounded, when it rises without bounds?

Bounds on option prices

- We show some inequalities for prices, which have to hold - otherwise there is an arbitrage on the market
- All the options considered have the same expiration time
- We denote the riskless interest rate by r.
- Notation:
  - $c(S, E, \tau)$ is the market price of a call option with exercise price E, if the stock price today is S and time remaining to expiration is τ
  - $p(S, E, \tau)$ is the market price of a put option with exercise price E, if the stock price today is S and time remaining to expiration is τ

Bounds on option prices

- Outline:
  - We consider two portfolios - such that at the time of expiration we have:

    (value of portfolio I.) $\leq$ (value of portfolio II.)

  - To avoid a possibility of arbitrage, also today necessarily

    (value of portfolio I.) $\leq$ (value of portfolio II.);

    the portfolios are constructed in such a way that *this* is the inequality that we need to prove

Bounds on option prices - examples

EXAMPLE 1: Clearly

$$c(S,E,\tau) \geq 0, p(S,E,\tau) \geq 0$$

EXAMPLE 2: Show that

$$E_1 \geq E_2 \Rightarrow c(S,E_1,\tau) \leq c(S,E_2,\tau)$$

SOLUTION Let $E_1 \geq E_2$ Consider the following portfolios

portfolio I.: option with exercise price E_1

portfolio II.: option with exercise price E_2

We compare their value at the expiration time, depending on the stock price S at this time

Bounds on option prices - examples

	$0 \leq S \leq E_2$	$E_2 \leq S \leq E_1$	$E_1 \leq S$
portfolio I.	0	0	$S - E_1$
portfolio II.	0	$S - E_2$	$S - E_2$
comparison	$0 = 0$	$0 \leq S - E_2$	$S - E_1 \leq S - E_2$

At the expiration time:

(value of portfolio I.) $\leq$ (value of portfolio II.)

$\Rightarrow$ also today:

(value of portfolio I.) $\leq$ (value of portfolio II.),

i.e.,

$$c(S, E_1, \tau) \leq c(S, E_2, \tau), \text{ QED}$$

Bounds on option prices - examples

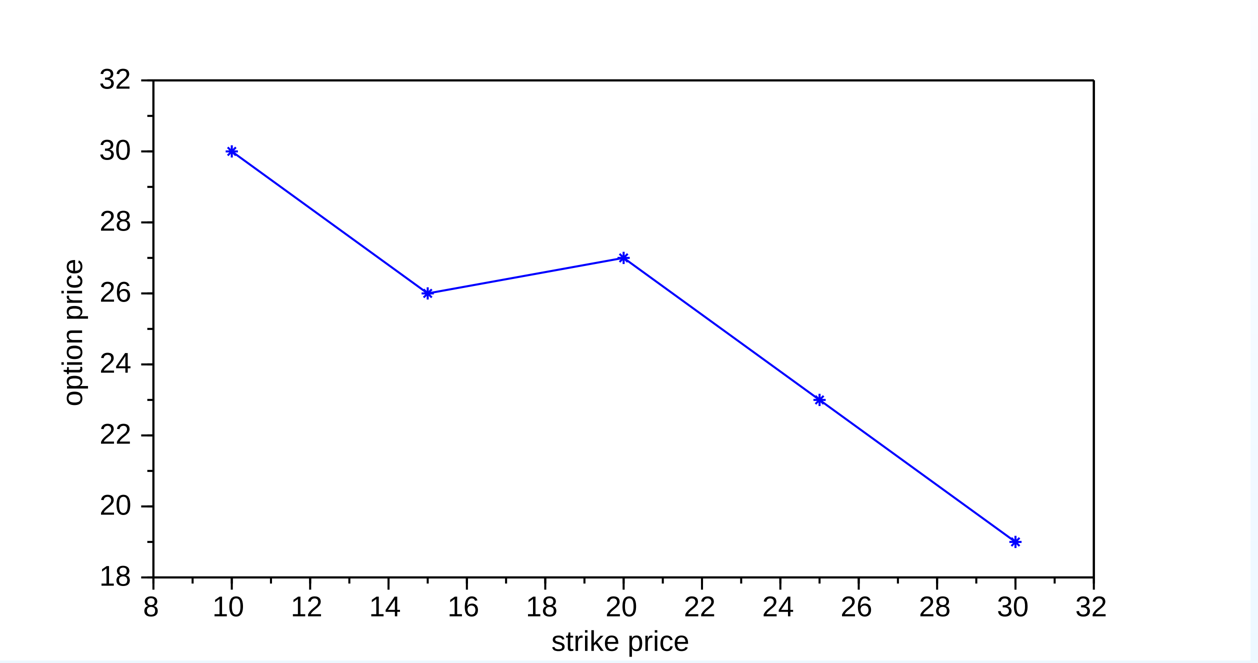
EXAMPLE 3:
Assume zero interest rate and the following call option prices:

exercise price	option price
10	30
15	26
20	27
25	23
30	19

Find an arbitrage.

SOLUTION: We plot the dependence of the call option price on the exercise price - its decreasing character, proved in the previous example, is not satisfied.

Bounds on option prices - examples

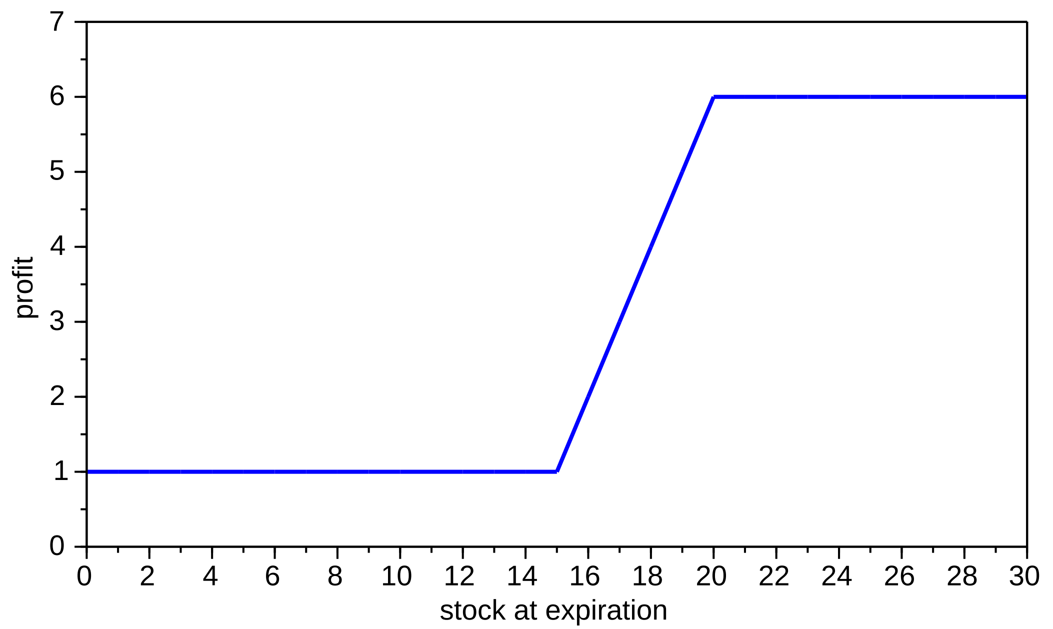


We should have $c(S, 15, \tau) \geq c(S, 20, \tau)$; however, here $c(S, 15, \tau) < c(S, 20, \tau)$. Therefore:

- *we buy the option, which costs less than it is supposed to*, in this case the option with exercise price $E = 15$,
- *we sell the option, which costs more than it is supposed to*, in this case the option with exercise price $E = 20$.

Bounds on option prices - examples

- Resulting profit diagram:



$\Rightarrow$ this strategy is indeed an arbitrage

- Exercises this week: More practice with proving bounds for option prices and finding arbitrage opportunities

Combined strategies

- In the previous (theoretical) example we combined the options to construct an arbitrage
- This idea of buying and sellig several options can be used also with real option prices - based on our expectations about future behavious of the stock price

Combined strategies

EXAMPLE:

- Consider the MCD (Mac Donald's Corp.) stock prices

http://finance.yahoo.com

and suppose (for this exercise) that we expect the stock price to be falling

Combined strategies

- The stock price is 93.72 USD and some of the available options are (data from August 11):

Put Options						Expire at close Friday, September 5, 2014	
Strike	Symbol	Last	Chg	Bid	Ask	Vol	Open Int
85.00	MCD140920P00085000	**0.22**	0.00	0.17	0.22	56	1,471
86.00	MCD140905P00086000	**0.16**	0.00	0.11	0.18	2	5
87.50	MCD140920P00087500	**0.33**	↓0.03	0.31	0.33	17	1,575
88.00	MCD140926P00088000	**0.59**	0.00	0.41	0.49	2	2
89.00	MCD140905P00089000	**0.36**	0.00	0.28	0.34	26	965
89.00	MCD140912P00089000	**0.45**	0.00	0.37	0.44	13	83
89.00	MCD140926P00089000	**0.66**	0.00	0.56	0.63	4	4
90.00	MCD140920P00090000	**0.65**	↓0.07	0.62	0.66	21	6,135

http://finance.yahoo.com

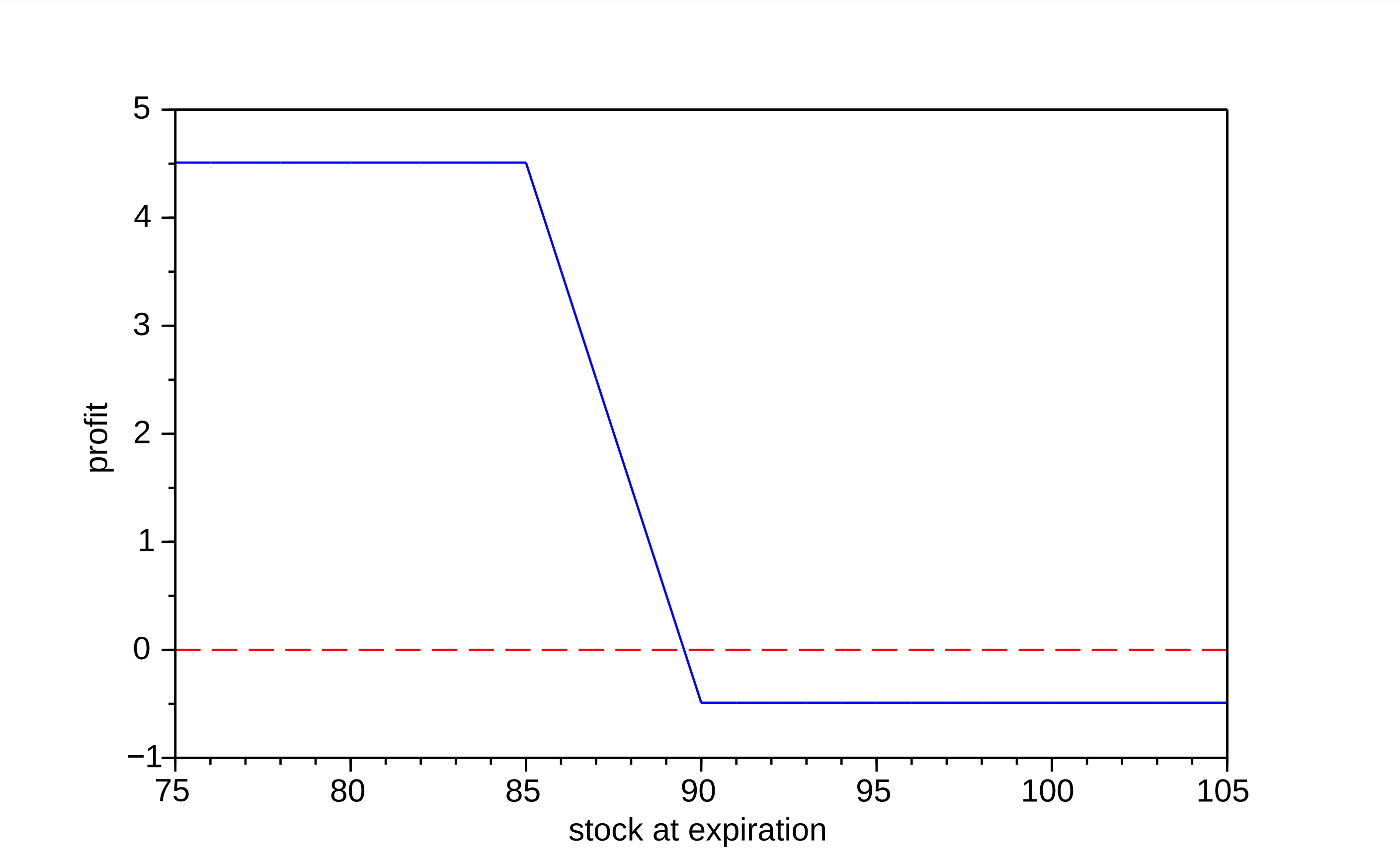
- We expect the stock to fall $\Rightarrow$ we buy a put option, for example one with exercise price 90 USD
- Howver, we don't expect it to fall too low $\Rightarrow$ we sell a put option with a lower expiration price, for example 85 USD
- We expect that the latter will not be exercised, but by writing it, we lower the initial investment

Combined strategies

- Our strategy:
  we buy a put with $E = 16$ *and sell a put with* $E = 14$
- Recall bid and ask prices:
  - bid price (the lower one) - the price a buyer is willing to pay $\rightarrow$ I can sell the option for *bid*
  - ask price (the higher one) - the price a seller is willing to accept $\rightarrow$ I can see the option for *ask*
- Therefore our initial investment is 0.49, since:
  - we buy the put with $E = 16$ for 0.66
  - we sell the put with $E = 13$ for 0.17

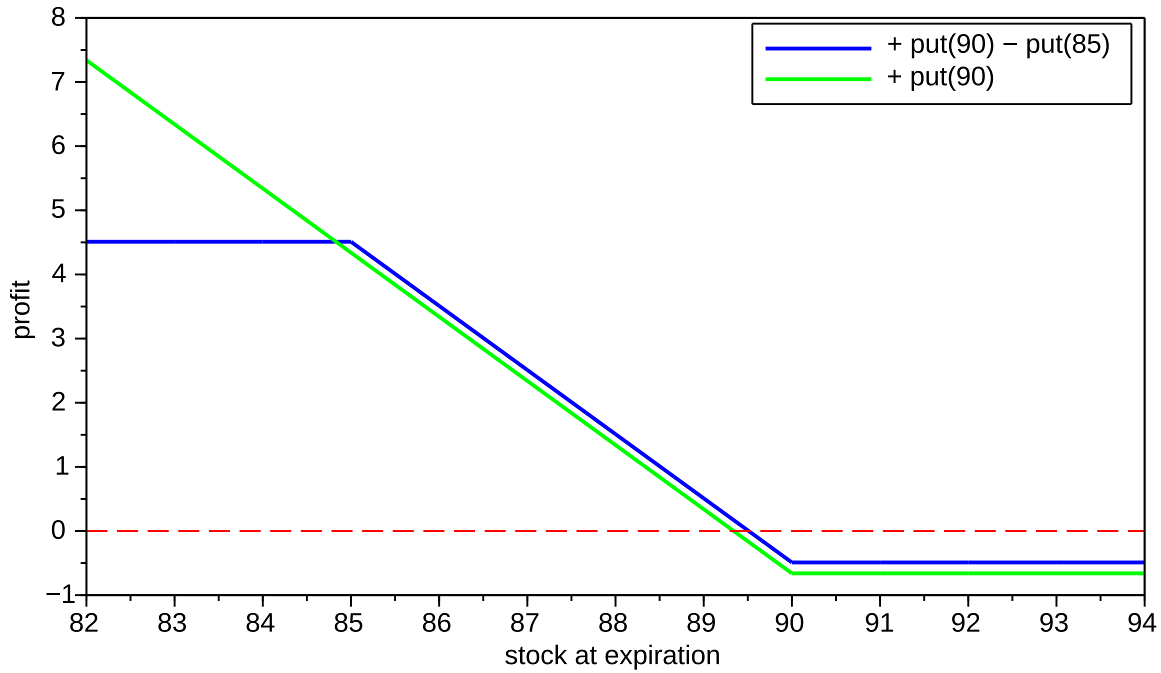
Combined strategies

- Profit diagram:



Combined strategies

- Comparison - with only buying the put with $E = 90$:



Combined strategies

- EXTRA CREDIT 1:
  Construction of a combined strategy using the real data, aiming to achieve the highest profit
- Overview of combined strategies:
  - Ševčovič, Stehlíková, Mikula: **Analytické a numerické metódy oceňovania finančných derivátov**. STU 2009. (In Slovak) - chapter 2.3.3.
  - Ševčovič, Stehlíková, Mikula: **Analytical and numerical methods for pricing financial derivatives.** Nova Science Publishers, Inc., Hauppauge, 2011. - chapter 2.3.2
  - http://www.theoptionsguide.com/option-trading-strategies.aspx