FINANCIAL DERIVATIVES - SAMPLE TEST 1

$W(t)$ denotes a Wiener process.

In problems 1-5, write only the answers. Each correct answer = 1 point.

1. What is the variance of the process $X(t) = e^{-t} W(t)$ at time t?
2. Stock price $S(t) = S_0 e^{\mu t + \sigma W(t)}$. The price today $S_0 = 150$ and the parameters are $\mu = 0.3$, $\sigma = 0.2$. Compute the expected value of the stock price at time $t = 1$.
3. Consider the stock from the previous problem. Compute the probability that monthly return will be positive.
4. Find an arbitrage for the following option prices (all of them are written on the same stock, and have the same expiration time):

strike price	put option price
20	10
25	12
40	15

5. Find differential of the process $y = \log(x)$ (log is the natural logarithm), where x satisfies the stochastic differential equation $dx = 3x\,dt + 5x\,dw$.

In problems 6-8 write the whole solution, partial credit is possible.

6. (1 point) Derive the variance of the process

$$B(t) = W(t) - t \cdot W(1) \qquad t \in [0,1]$$

at time $t \in [0,1]$.

7. (2 points) $S(t)$ satisfies the stochastic differential equation

$$dS = 0.3\,S\,dt - \sigma S\,dw,$$

where $\sigma > 0$ is a constant. There is a real number a, such that

$$dS^a = -0.66\,S^a\,dt + 0.6\,S^a\,dw.$$

Determine σ.

8. (2 points) Suppose that the stock price follows a geometric Brownian motion. Expected value of the stock in one year is 120 USD and the median is 115 USD. What is the probability, that it will be greater than 100 USD?