

**Financial derivatives**  
**Sample test 2: Black-Scholes and Leland model**

In questions 1-5 write only the answers, each correct answer – 1 point.

1. Compute the Black-Scholes price of a call option on a stock which does not pay dividends and has the volatility 0.2, if its exercise price is 200 USD and expiration in one year. Interest rate is zero and the price of the stock is 180 USD.
2. Consider a European call option on a non-dividend paying stock with exercise price 100 USD and expiration time in one year. Interest rate is 1 percent and the price of the stock today is 75 USD. For what price of the option is the Black-Scholes implied volatility equal to 0.35
3. Consider a European call option on a non-dividend paying stock with exercise price 100 USD and expiration in one month. Interest rate is 1 percent and volatility of the stock is 0.4. We sold 100 of these options and use Black-Scholes delta hedging to hedge our portfolio. Give an example of the stock price, for which we have more than 50 stocks in our portfolio.
4. Consider a European put option on a non-dividend paying stock with exercise price 100 USD and expiration in one month. Interest rate is 1 percent and volatility of the stock is 0.4. We sold 100 of these options and use Black-Scholes delta hedging to hedge our portfolio. The stock price equals 90 USD. Compute the number of stocks in our portfolio.
5. Consider the following inputs for the Leland model:

**Oracle Corporation (ORCL)** - NYSE ★ Watchlist  
**41.87** ↑ 0.72 (1.75%) Nov 26, 4:00PM EST  
 After Hours : **41.87** 0.00 (0.00%) Nov 26, 4:57PM EST

Prev Close:	<b>41.15</b>	Day's Range:	<b>41.18 - 41.91</b>
Open:	<b>41.19</b>	52wk Range:	<b>33.22 - 43.19</b>
Bid:	<b>41.65</b> x 1800	Volume:	<b>11,590,010</b>
Ask:	<b>41.86</b> x 2700	Avg Vol (3m):	<b>15,282,200</b>
1y Target Est:	<b>43.76</b>	Market Cap:	<b>185.54B</b>

The maturity of the following put options is 11 days (consider 252 trading days in a year) and the interest rate is equal to zero. Compute the implied volatility and implied time between two adjustments of portfolio for the put option with strike price 39 USD using data below:

Puts				
Strike	Contract Name	Last	Bid	Ask
38.00	ORCL141212P00038000	0.21	0.01	0.07
38.50	ORCL141212P00038500	0.20	0.02	0.08
39.00	ORCL141212P00039000	0.22	0.03	0.09
39.50	ORCL141212P00039500	0.12	0.04	0.09

In questions 6 and 7 write all of the computations. Also partial credit is possible in these problems.

6. Show that if the payoff is an increasing differentiable function of the stock price at expiration, that the derivative price is an increasing function of the stock price at any time before expiration. (2 points) What is the financial interpretation of this result? (0.5 points)
  
7. (2.5 points) Consider the difference between bid and ask price of an option as a function of the stock price  $S$ . (The remaining parameters - stock volatility, parameter  $c$  from the transaction costs, interest rate, strike price, expiration time - are constant). Analytically derive the stock price for which this maximum is attained (for general values of parameters).