FINANČNÉ DERIVÁTY

CKLS[1] MODEL - UKÁŽKA ŠTATISTICKÝCH ANALÝZ

- **CKLS model**: short rate r sa riadi stochastickou diferenciálnou rovnicou $dr = \kappa(\theta - r)dt + \sigma r^\gamma dw$, zapisuje sa aj v tvare

$$dr = (\alpha + \beta r)dt + \sigma r^\gamma dw$$

  (je to ekvivalentný tvar, ide len o iný zápis všeobecnej lineárnej funkcie). Špeciálnymi prípadmi je **Vašíčkov model** ($\gamma = 0$) a **Cox-Ingersoll-Rossov model** ($\gamma = 1/2$), pre ktoré existujú explicitné riešenia pre cenu dlhopisu.

- Majme dáta $r_1, r_2, \ldots, r_n$ s určitým časovým krokom medzi dvoma pozorovaniami ⇒ poznáme podmienené rozdelenie r_i pri danom r_{i-1} vo Vašíčkovom modeli ⇒ vieme napísať vierohodnostnú funkciu ⇒ vieme nájsť odhady parametrov modelu metódou maximálnej vierohodnosti

- Pre všeobecnú hodnotu parametra γ: spravíme aproximáciu - budeme predpokladať, že na intervale medzi pozorovaniami r_{i-1} a r_i bola volatilita konštantná a rovnala sa σr_{i-1}^γ ⇒ pri takejto aproximácii máme znovu normálne podmienené rozdelenie ⇒ zostrojíme aproximáciu vierohodnostnej funkcie ⇒ vieme nájsť odhady parametrov modelu (tzv. **Gaussovské odhady** alebo **Nowmanova metodológia**)

- Opakovanie zo štatistiky a ekonometrie: ako testovať hypotézy pri použití metódy maximálnej vierohodnosti, **test pomerom vierohodnosti** (likelihood ratio test)

- *Athanasios Episcopos: **Further evidence on alternative continuous time models of the short-term interest rate**, Journal of International Financial Markets, Institutions and Money Volume 10, Issue 2, June 2000, pp. 199-212*
  - odhady pre 10 štátov
  - na str. 2 ukážka - výsledky pre USA (dáta: 1/1986 - 4/1998, spolu 148 pozorovaní)
  - vysvetlíme si, ako sa získali výsledky likelihood ratio testu, ktorým sa testoval (napr.) Vašíčkov a CIR model ako reštrikcia všeobecného modelu (testovacia štatistika a P-hodnota)

- *K. Ben Nowman: **Gaussian estimation of continuous time diffusions of UK interest rates**, Mathematics and Computers in Simulation Volume 81, Issue 8, April 2011, pp. 1618-1624*
  - štatistický model pre dvojicu úrokových mier r_1, r_2:

$$\begin{aligned} dr_1 &= (\alpha_1 + \beta_1 r_1 + \beta_2 r_2)dt + \sigma_1 r_1^{\gamma_1} dw_1, \\ dr_2 &= (\alpha_2 + \beta_3 r_1 + \beta_4 r_2)dt + \sigma_1 r_2^{\gamma_2} dw_2, \end{aligned}$$

    pričom korelácia medzi prírastkami Wienerovych procesov dw_1 a dw_2 je ρ
  - ukážka odhadov na str. 3 hore - úrokové miery Bank of England, 1/1970 - 3/2010
  - testujme hypotézu, že $\gamma_1 = \gamma_2 = 0$

[1]Chan-Karolyi-Longsfaff-Sanders

Table 3 (Continued)

Country	Model[b]	α	β	σ^2	γ	Avg. Log L	χ^2-test[c]	df	S-test
	BR-SC	0.0023	−0.0237	0.0061	1	4.3824	10.6625	1	22.13
		(1.9612)	(−1.7192)	(13.2670)			(0.0011)		
	CIR VR	0	0	0.0711	1.5	4.2984	57.554	3	21.71
				(34.6877)			(0.0000)		
	CEV	0	−0.0013	0.0008	0.5605	4.3962	3.0149	1	21.13
			(−0.2985)	(1.7082)	(4.5247)		(0.0825)		
USA	Unrestricted	0.0013	−0.0234	0.0001	0.4239	5.1768			60.51
		(1.4696)	(−1.5710)	(1.0309)	(2.5099)				
	Vasicek	0.0013	−0.0235	0.0000	0	5.1569	5.8655	1	55.37
		(4.3499)	(−5.6112)	(16.1077)			(0.0154)		
	CIR SR	0.0013	−0.0241	0.0002	0.5	5.1761	0.201	1	61.74
		(4.6916)	(−5.8893)	(13.0558)			(0.6539)		
	BR-SC	0.0014	−0.0255	0.0038	1	5.1365	11.8748	1	67.81
		(6.1214)	(−6.1221)	(14.9738)			(0.0006)		
	CIR VR	0	0	0.0794	1.5	5.0220	45.529	3	72.29
				(21.0933)			(0.0000)		
	CEV	0	−0.003	0.0001	0.4063	5.1705	1.8477	1	59.12
			(−0.6401)	(24.7981)	(31.0657)		(0.1740)		

[a] The general model to be estimated is given by: $dr_t = (\alpha + \beta r_t)dt + \sigma r_t^{\gamma} dW_t$; where r_t is the 1-month interbank rate, dW_t is the increment of the Weiner process, α and β are the drift and mean reversion parameters, and σ and γ specify the conditional variance of r_t. Maximizing the log of the Gaussian likelihood function yields estimates for the four parameters α, β, σ^2 and γ.

[b] CIR SR, CIR VR: Cox et al. (1980, 1985), respectively. BR-SC: Brennan and Schwartz (1980). Avg. Log$L$: the average of the estimated maximum of the log-likelihood function for each country. χ^2-test: the likelihood ratio test for each restricted model. The test statistic is computed as $2[L(b) - L(b^*)]$, where b is the parameter vector (α, β, σ^2, γ), b^* is the restricted vector and $L(b)$ the log-likelihood function evaluated at the maximum. The statistic follows a χ^2 distribution with degrees of freedom (df) equal to the number of restrictions in each model. S-test: the Bergstrom (1990) dynamic specification test for white noise residuals. The statistic is given by

$$S = \frac{1}{n(N-l)} \sum_{k=1}^{l} \sum_{t=l+1}^{N} (z_t z_{t-k})^2,$$

where $n = 1$, $l = 12$, N is the number of observations and z are standardized residuals and follows a χ^2 with 12 degrees of freedom. The critical value at the 5% significance level is 21.03.

[c] T-statistics in parentheses except for the χ^2 Test column, which shows P-values in parentheses.

[d] 0.0000 denotes numbers less than 10^{-4}.

Table 4
Gaussian estimates of two factor models: 2 and 5 year rates.

Parameters	CKLS	Vasicek	CIRSR	BRSC
α_1	−0.0011 (0.0004)	−0.0001 (0.0006)	−0.0010 (0.0005)	−0.0013 (0.0004)
α_2	−0.0003 (0.0004)	0.0004 (0.0005)	−0.0003 (0.0004)	−0.0004 (0.0003)
β_1	−0.0835 (0.0272)	−0.1084 (0.0225)	−0.0799 (0.0380)	−0.0703 (0.0239)
β_2	0.0910 (0.0292)	0.1030 (0.0227)	0.0864 (0.0405)	0.0832 (0.0274)
β_3	0.0405 (0.0246)	0.0372 (0.0159)	0.0341 (0.0377)	0.0341 (0.0240)
β_4	−0.0399 (0.0234)	−0.0448 (0.0162)	−0.0339 (0.0360)	−0.0309 (0.0219)
γ_1	0.5968 (0.0510)	0.0	0.5	1.0
γ_2	0.7240 (0.0565)	0.0	0.5	1.0
σ_1^2	0.0048 (0.0001)	0.00003 (0.000002)	0.0003 (0.00002)	0.0042 (0.0002)
σ_2^2	0.0006 (0.0001)	0.00002 (0.000001)	0.0002 (0.00001)	0.0025 (0.0001)
ρ_{12}	0.8872 (0.0098)	0.9018 (0.0086)	0.8906 (0.0095)	0.8725 (0.0109)
Log LF	10350.6900	10195.9368	10335.717	10270.9950

Table 5
Gaussian estimates of two factor models: 2 and 10 year rates.

Parameters	CKLS	Vasicek	CIRSR	BRSC
α_1	−0.0008 (0.0018)	0.0001 (0.0006)	−0.0007 (0.0004)	−0.0010 (0.0004)
α_2	0.0002 (0.0014)	0.0005 (0.0005)	0.0002 (0.0004)	0.0001 (0.0003)
β_1	−0.0304 (0.1153)	−0.0497 (0.0700)	−0.0310 (0.0151)	−0.0240 (0.0269)
β_2	0.0355 (0.1280)	0.0429 (0.0669)	0.0345 (0.0159)	0.0335 (0.0284)
β_3	0.0017 (0.1251)	−0.0112 (0.0389)	−0.0042 (0.0100)	0.0026 (0.0418)
β_4	−0.0047 (0.1144)	0.0048 (0.0410)	0.0015 (0.0101)	−0.0052 (0.0439)
γ_1	0.6193 (0.0565)	0.0	0.5	1.0
γ_2	0.7718 (0.0822)	0.0	0.5	1.0