Finančné deriváty: príklady

Príklad 1: Delta hedžing

Assume the Black-Scholes framework. You are given:

(i) $S(t)$ is the price of a nondividend-paying stock at time t.

(ii) $S(0) = 10$

(iii) The stock's volatility is 20%.

(iv) The continuously compounded risk-free interest rate is 2%.

At time $t = 0$, you write a one-year European option that pays 100 if $[S(1)]^2$ is greater than 100 and pays nothing otherwise.

You delta-hedge your commitment.

Calculate the number of shares of the stock for your hedging program at time $t = 0$.

(A) 20

(B) 30

(C) 40

(D) 50

(E) 60

Príklad je zo skúšok *Society of Actuaries* (zbierka na webstránke predmetu)

Príklad 2: Závislosť vegy od času

European option Vega with respect to expiry and implied volatility

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I was told that the Vega of an European option always increases when its time to expiry increases (all else equal). I found this confusing and potentially wrong, but there doesn't seem to be relevant sources online about this. Let's take an ATM option for simplicity, its Vega is: $S\sqrt(\tau)N'(d1)$, which is just $\frac{1}{\sqrt(2\pi)}S\sqrt(\tau)e^{\frac{-(r+\frac{\sigma^2}{2})^2\tau}{2}}$. Now as τ increases to a large range, I found that Vega certainly decreases as we have a $-\tau$ term in the exponent. However in small ranges of τ for example between 0 and 1, Vega does increases as τ increases. Am I mistaken here?

I also would like to see the relationship of European option price with respect to volatility and plotted a graph where the y axis is the option price computed from Black-Scholes, and the x axis is σ (also holding everything else equal and use ATM options for simplicity). The graph surprisingly looks like a straight line. But from the formula above, the local slope of this line should just be Vega at different values of σ, and thus should be decreasing, so theoretically the line should be concave. Am I mistaken here?

options black-scholes european-options vega

Budeme uvažovať cenu akcie rovnú *odúročenej* exspiračnej cene, teda **S = E exp(-r * (T-t))**, otázka je z quant.stackexchange.com

Príklad 3: Odvodenie PDR pre derivát

- Louis Bachelier na začiatku 20. storočia modeloval vývoj ceny akcie Brownovým pohybom (nie geometrickým Brownovým pohybom ako to poznáme z Black-Scholesovho modelu).
- Odvoďte PDR pre cenu derivátu pri takomto predpoklade. *Ktoré kroky treba zmeniť v pôvodnom odvodení z prednášky?*

Príklad 4: Odvodenie PDR pre derivát

- Uvažujme Black-Scholesove predpoklady.
- Majme derivát, ktorý v čase exspirácie nevyplatí nič, ale do tohto času vypláca spojité platby: na intervale
- (t, t+dt) vyplatí C(S, t)dt, kde C je zadaná funkcia.
- Odvoďte PDR pre cenu takéhoto derivátu.

Príklad 5: Odvodenie PDR pre derivát

- Uvažujme swap: strana A bude strane B platiť fixný spojitý úrok u, strana B bude platiť strane A trhový úrok r.
- Ekvivalentne: uvažujme kupónový dlhopis s náhodným kupónom r – u, ktorý v čase maturity nevyplatí nič
- Predpokladajme, že vývoj úrokovej miery r sa riadi Vašíčkovym modelom.
- Odvoďte PDR pre cenu takéhoto dlhopisu.