COMMENTARII ACADEMIAE SCIENTIARVM IMPERIALIS PETROPOLITANAE.

TOMVS VII.

AD ANNOS cIↄIↄCCXXXIV. & cIↄIↄCCXXXV.

PETROPOLI,
TYPIS ACADEMIAE. cIↄIↄCCXL.

Euler:

§. 2. Deductus ſum autem nuper omnino inopinato ad elegantem ſummae huius ſeriei $1 + \frac{1}{4} + \frac{1}{9} + \frac{1}{16}$ + etc. expreſſionem

http://eulerarchive.maa.org/pages/E041.html

Neskôr mnohí ďalší

Evaluating $\zeta(2)$

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I list several proofs of the celebrated identity:

$$\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}. \tag{1}$$

Several proofs je konkrétne 14 rôznych dôkazov

http://empslocal.ex.ac.uk/people/staff/rjchapma/etc/zeta2.pdf

Užitočná poznámka na začiatku

As it is clear that

$$\frac{3}{4}\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} - \sum_{m=1}^{\infty} \frac{1}{(2m)^2} = \sum_{r=0}^{\infty} \frac{1}{(2r+1)^2},$$

(1) is equivalent to

$$\sum_{r=0}^{\infty} \frac{1}{(2r+1)^2} = \frac{\pi^2}{8}.$$

Many of the proofs establish this latter identity first.

Náš dôkaz bude podľa článku:

Probabilistically Proving that $\zeta(2) = \pi^2/6$

Luigi Pace

Abstract. We give a short proof of the identity $\zeta(2) = \pi^2/6$ using tools from elementary probability. Related identities, due to Euler, are also briefly discussed.

The American Mathematical Monthly, Vol. 118, No. 7 (August-September 2011), pp.641-643
http://www.jstor.org/stable/10.4169/amer.math.monthly.118.07.641

Cauchyho rozdelenie

Cauchyho rozdelenie má hustotu:

$$f(x) = \frac{1}{\pi(1 + x^2)}$$

KROK 1:
Odvoďte hustotu absolútnej hodnoty Cauchyho rozdelenia.

Hustota podielu

KROK 2:
Dokážte, že podiel dvoch nezávislých náhodných premenných, pričom každá z nich je absolútna hodnota Cauchyho rozdelenia má hustotu (pre y kladné) rovnú

$$\frac{4}{\pi^2}\frac{\log(y)}{y^2-1}$$

KROK 3:
Vysvetlite, ako z toho vyplýva, že

$$\int_0^1 \frac{-\log(y)}{1-y^2}\,dy = \frac{\pi^2}{8}$$

Dokončenie dôkazu

KROK 4:
Dosaďte

$$1/(1 - y^2) = 1 + y^2 + y^4 + \cdots$$

a zintegrujte člen po člene.

DOSTANEME:

$$\sum_{k=0}^{\infty} \frac{1}{(2k+1)^2} = \frac{\pi^2}{8}$$