Náhodné vektory - cvičenia 1

Všetky príklady sú zo skúšok aktuárskej spoločnosti *Society of Actuaries* (preto sú väčšinou zamerané na udalosti súvisiace s poisťovníctvom).

Cvičenie 1

A company has two electric generators. The time until failure for each generator follows an exponential distribution with mean 10. The company will begin using the second generator immediately after the first one fails. What is the variance of the total time that the generators produce electricity?

(A) 10 (B) 20 (C) 50 (D) 100 (E) 200

Cvičenie 2

An insurance policy is written to cover a loss X, where X has density function

$$f(x) = \begin{cases} \frac{3}{8}x^2 & \text{for } 0 \le x \le 2, \\ 0 & \text{otherwise.} \end{cases}$$

The time (in hours) to process a claim of size x, where $0 \le x \le 2$, is uniformly distributed on the interval from x to $2x$. Calculate the probability that a randomly chosen claim on this policy is processed in three hours or more.

(A) 0.17 (B) 0.25 (C) 0.32 (D) 0.58 (E) 0.83

Cvičenie 3

A family buys two policies from the same insurance company. Losses under the two policies are independent and have continuous uniform distributions on the interval from 0 to 10. One policy has a deductible of 1 and the other has a deductible of 2. The family experiences exactly one loss under each policy. Calculate the probability that the total benefit paid to the family does not exceed 5.

(A) 0.13 (B) 0.25 (C) 0.30 (D) 0.32 (E) 0.42

Poznámka:
Deductible je spoluúčasť. Hodnota napr. 1 znamená, že ak je škoda menšia ako 1, tak poisťovňa nevyplatí nič. Ak je väčšia ako 1, tak poisťovna vyplatí výšku škody znížený o 1.

Cvičenie 4

An insurance company sells two types of auto insurance policies: Basic and Deluxe. The time until the next Basic Policy claim is an exponential random variable with mean two days. The time until the next Deluxe Policy claim is an independent exponential random variable with mean three days. What is the probability that the next claim will be a Deluxe Policy claim?

(A) 0.172 (B) 0.223 (C) 0.400 (D) 0.487 (E) 0.500

Cvičenie 5

The total claim amount for a health insurance policy follows a distribution with density function

$$f(x) = \frac{1}{1000} e^{-x/1000}, \quad \text{for } x > 0.$$

The premium for the policy is set at 100 over the expected total claim amount. If 100 policies are sold, what is the approximate probability that the insurance company will have claims exceeding the premiums collected?

(A) 0.001 (B) 0.159 (C) 0.333 (D) 0.407 (E) 0.460

Cvičenie 6

A city has just added 100 new female recruits to its police force. The city will provide a pension to each new hire who remains with the force until retirement. In addition, if the new hire is married at the time of her retirement, a second pension will be provided for her husband. A consulting actuary makes the following assumptions:

(i) Each new recruit has a 0.4 probability of remaining with the police force until retirement.

(ii) Given that a new recruit reaches retirement with the police force, the probability that she is not married at the time of retirement is 0.25.

(iii) The number of pensions that the city will provide on behalf of each new hire is independent of the number of pensions it will provide on behalf of any other new hire.

Determine the probability that the city will provide at most 90 pensions to the 100 new hires and their husbands.

(A) 0.60 (B) 0.67 (C) 0.75 (D) 0.93 (E) 0.99