Opakovanie k písomke

Pravdepodobnosť a štatistika pre informatikov

3.9 $\boxdot$ A certain grapefruit variety is grown in two regions in southern Spain. Both areas get infested from time to time with parasites that damage the crop. Let A be the event that region R_1 is infested with parasites and B that region R_2 is infested. Suppose $\mathrm{P}(A) = 3/4$, $\mathrm{P}(B) = 2/5$ and $\mathrm{P}(A \cup B) = 4/5$. If the food inspection detects the parasite in a ship carrying grapefruits from R_1, what is the probability region R_2 is infested as well?

3.12 The events A, B, and C satisfy: $\mathrm{P}(A \,|\, B \cap C) = 1/4$, $\mathrm{P}(B \,|\, C) = 1/3$, and $\mathrm{P}(C) = 1/2$. Calculate $\mathrm{P}(A^c \cap B \cap C)$.

3.10 A student takes a multiple-choice exam. Suppose for each question he either knows the answer or gambles and chooses an option at random. Further suppose that if he knows the answer, the probability of a correct answer is 1, and if he gambles this probability is $1/4$. To pass, students need to answer at least 60% of the questions correctly. The student has "studied for a minimal pass," i.e., with probability 0.6 he knows the answer to a question. Given that he answers a question correctly, what is the probability that he actually *knows* the answer?

3.11 A breath analyzer, used by the police to test whether drivers exceed the legal limit set for the blood alcohol percentage while driving, is known to satisfy

$$\mathrm{P}(A \mid B) = \mathrm{P}(A^c \mid B^c) = p,$$

where A is the event "breath analyzer indicates that legal limit is exceeded" and B "driver's blood alcohol percentage exceeds legal limit." On Saturday night about 5% of the drivers are known to exceed the limit.

a. Describe in words the meaning of $\mathrm{P}(B^c \mid A)$.

b. Determine $\mathrm{P}(B^c \mid A)$ if $p = 0.95$.

c. How big should p be so that $\mathrm{P}(B \mid A) = 0.9$?

4.1 ⊞ Let Z represent the number of times a 6 appeared in two independent throws of a die, and let S and M be as in Section 4.1.

a. Describe the probability distribution of Z, by giving either the probability mass function p_Z of Z or the distribution function F_Z of Z. What type of distribution does Z have, and what are the values of its parameters?

b. List the outcomes in the events $\{M = 2, Z = 0\}$, $\{S = 5, Z = 1\}$, and $\{S = 8, Z = 1\}$. What are their probabilities?

c. Determine whether the events $\{M = 2\}$ and $\{Z = 0\}$ are independent.

S = súčet čísel na kockách
M = maximum

4.5 A fair die is thrown until the sum of the results of the throws exceeds 6. The random variable X is the number of throws needed for this. Let F be the distribution function of X. Determine $F(1)$, $F(2)$, and $F(7)$.

4.3 ⊡ Suppose that the distribution function of a discrete random variable X is given by

$$F(a) = \begin{cases} 0 & \text{for } a < 0 \\ \frac{1}{3} & \text{for } 0 \leq a < \frac{1}{2} \\ \frac{1}{2} & \text{for } \frac{1}{2} \leq a < \frac{3}{4} \\ 1 & \text{for } a \geq \frac{3}{4}. \end{cases}$$

Determine the probability mass function of X.

4.2 Let X be a discrete random variable with probability mass function p given by:

a	-1	0	1	2
$p(a)$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{2}$

and $p(a) = 0$ for all other a.

a. Let the random variable Y be defined by $Y = X^2$, i.e., if $X = 2$, then $Y = 4$. Calculate the probability mass function of Y.

b. Calculate the value of the distribution functions of X and Y in $a = 1$, $a = 3/4$, and $a = \pi - 3$.

4.4 You toss n coins, each showing heads with probability p, independently of the other tosses. Each coin that shows tails is tossed again. Let X be the total number of heads.

a. What type of distribution does X have? Specify its parameter(s).

b. What is the probability mass function of the total number of heads X?

4. A sequence of independent Bernoulli trials, each with success probability p, is undertaken. Find the probability that the second success occurs after exactly n trials (where n is an integer greater than 1).

3. Let U be a discrete random variable with $P(U = u) = p(1-p)^{u-1}$, for $u = 1, 2, 3, \ldots$ where $0 < p < 1$. Find $P(U \leq u)$.

11. The performance of a computer is studied over a period of time, during which it makes 10^{15} computations. If the probability that a particular computation is incorrect equals 10^{-14}, independent of other computations, write down an expression for the probability that fewer than 10 errors occur and calculate an approximation to its value

16. Let X be a random variable with pmf:

$$f(x) = C \frac{e^{-\lambda}\lambda^{x}}{x!}, \quad \lambda > 0, \; x = 1, 2, \ldots$$

This is a truncated Poisson distribution.

(a) Find the value of C.

(b) Find $E(X)$.

4.10 ⊞ Early in the morning, a group of m people decides to use the elevator in an otherwise deserted building of 21 floors. Each of these persons chooses his or her floor independently of the others, and—from our point of view—completely at random, so that each person selects a floor with probability $1/21$. Let S_m be the number of times the elevator stops. In order to study S_m, we introduce for $i = 1, 2, \ldots, 21$ random variables R_i, given by

$$R_i = \begin{cases} 1 & \text{if the elevator stops at the } i\text{th floor} \\ 0 & \text{if the elevator does not stop at the } i\text{th floor.} \end{cases}$$

a. Each R_i has a $Ber(p)$ distribution. Show that $p = 1 - \left(\frac{20}{21}\right)^m$.

b. From the way we defined S_m, it follows that

$$S_m = R_1 + R_2 + \cdots + R_{21}.$$

Can we conclude that S_m has a $Bin(21, p)$ distribution, with p as in part a? Why or why not?

c. Nájdeme strednú hodnotu počtu poschodí, na ktorých výťah zastane.

4.12 ⊡ You and a friend want to go to a concert, but unfortunately only one ticket is still available. The man who sells the tickets decides to toss a coin until heads appears. In each toss heads appears with probability p, where $0 < p < 1$, independent of each of the previous tosses. If the number of tosses needed is odd, your friend is allowed to buy the ticket; otherwise you can buy it. Would you agree to this arrangement?

7.3 For a certain random variable X it is known that $\mathrm{E}[X] = 2$, $\mathrm{Var}(X) = 3$. What is $\mathrm{E}\left[X^2\right]$?

7.4 Let X be a random variable with $\mathrm{E}[X] = 2$, $\mathrm{Var}(X) = 4$. Compute the expectation and variance of $3 - 2X$.

5.5 ⊡ The probability density function f of a continuous random variable X is given by:

$$f(x) = \begin{cases} cx + 3 & \text{for } -3 \leq x \leq -2 \\ 3 - cx & \text{for } 2 \leq x \leq 3 \\ 0 & \text{elsewhere.} \end{cases}$$

a. Compute c.

b. Compute the distribution function of X.

7.6 $\boxplus$ The random variable Z has probability density function $f(z) = 3z^2/19$ for $2 \leq z \leq 3$ and $f(z) = 0$ elsewhere. Determine $\mathrm{E}[Z]$. Before you do the calculation: will the answer lie closer to 2 than to 3 or the other way around?

7.7 Given is a random variable X with probability density function f given by $f(x) = 0$ for $x < 0$, and for $x > 1$, and $f(x) = 4x - 4x^3$ for $0 \leq x \leq 1$. Determine the expectation and variance of the random variable $2X + 3$.

7.8 $\boxdot$ Given is a continuous random variable X whose distribution function F satisfies $F(x) = 0$ for $x < 0$, $F(x) = 1$ for $x > 1$, and $F(x) = x(2 - x)$ for $0 \leq x \leq 1$. Determine $\mathrm{E}[X]$.

6.8 A random variable X has a $Par(3)$ distribution, so with distribution function F with $F(x) = 0$ for $x < 1$, and $F(x) = 1 - x^{-3}$ for $x \geq 1$. For details on the Pareto distribution see Section 5.4. Describe how to construct X from a $U(0, 1)$ random variable.

6.6 ⊡ Somebody messed up the random number generator in your computer: instead of uniform random numbers it generates numbers with an $Exp(2)$ distribution. Describe how to construct a $U(0, 1)$ random variable U from an $Exp(2)$ distributed X.

Hint: look at how you obtain an $Exp(2)$ random variable from a $U(0, 1)$ random variable.

8.4 Transforming exponential distributions.

a. Let X have an $Exp(\frac{1}{2})$ distribution. Determine the distribution function of $\frac{1}{2}X$. What kind of distribution does $\frac{1}{2}X$ have?

b. Let X have an $Exp(\lambda)$ distribution. Determine the distribution function of λX. What kind of distribution does λX have?

8.18 $\boxdot$ Let $X_1, X_2, \ldots, X_n$ be independent random variables, all with an $Exp(\lambda)$ distribution. Let $V = \min\{X_1, \ldots, X_n\}$. Determine the distribution function of V. What kind of distribution is this?

8.5 $\boxdot$ Let X be a continuous random variable with probability function

$$f_X(x) = \begin{cases} \frac{3}{4}x(2-x) & \text{for } 0 \le x \le 2 \\ 0 & \text{elsewhere.} \end{cases}$$

a. Determine the distribution function F_X.

b. Let $Y = \sqrt{X}$. Determine the distribution function F_Y.

c. Determine the probability density of Y.

Two instruments are used to measure the height, h, of a tower. The error made by the less accurate instrument is normally distributed with mean 0 and standard deviation $0.0056h$. The error made by the more accurate instrument is normally distributed with mean 0 and standard deviation $0.0044h$. Assuming the two measurements are independent random variables, what is the probability that their average value is within $0.005h$ of the height of the tower?

(A) 0.38 (B) 0.47 (C) 0.68 (D) 0.84 (E) 0.90

6. There are six hundred students in the business school of a university who all are studying the course *Statistics 101* and *Statistics Made Simple* is the prescribed textbook for this course. Suppose that the probability is 0.05 that any randomly chosen student will need to borrow a copy of the text book from the university library. How many copies of the text book *Statistics Made Simple* should be kept in the library so that the probability may be greater than 0.90 that none of the students needing a copy from the library has to come back disappointed? Assume that `qnorm(0.90) = 1.282`.

For Company A there is a 60% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean $10,000$ and standard deviation $2,000$. For Company B there is a 70% chance that no claim is made during the coming year. If one or more claims are made, the total claim amount is normally distributed with mean $9,000$ and standard deviation $2,000$. Assume that the total claim amounts of the two companies are independent. What is the probability that, in the coming year, Company B's total claim amount will exceed Company A's total claim amount?

(A) 0.180 (B) 0.185 (C) 0.217 (D) 0.223 (E) 0.240

16. [3-16]

An insurance policy is written to cover a loss, X, where X has a uniform distribution on $[0, 1000]$. At what level must a deductible be set in order for the expected payment to be 25% of what it would be with no deductible?

(A) 250 (B) 375 (C) 500 (D) 625 (E) 750

10. [3-10]

An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} 3x^{-4} & \text{for } x > 1, \\ 0 & \text{otherwise.} \end{cases}$$

Given that a randomly selected home is insured for at least 1.5, what is the probability that it is insured for less than 2?

(A) 0.578 (B) 0.684 (C) 0.704 (D) 0.829 (E) 0.875

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