Cvičenia z parciálnych diferenciálnych rovníc
Domáca úloha 9

1. Nájdite riešenie rovnice vedenia tepla na priamke so začiatočnou podmienkou

$$u_0(x) = e^{-x^2}.$$

2. Vyriešte nasledujúci príklad a zistite, či ryba prežije:

A river is defined by the domain

$$D = \{(x, y) | \quad |y| < 1, \ -\infty < x < \infty\}.$$

A factory spills a contaminant into the river. The contaminant is further spread and convected by the flow in the river. The velocity field of the fluid in the river is only in the x direction. The concentration of the contaminant at a point (x, y) in the river and at time τ is denoted by $u(x, y, \tau)$. Conservation of matter and momentum implies that u satisfies the first-order PDE

$$u_\tau - (y^2 - 1)u_x = 0.$$

The initial condition is $u(x, y, 0) = e^y e^{-x^2}$.

(a) Find the concentration u for all (x, y, τ).
(b) A fish lives near the point $(x, y) = (2, 0)$ at the river. The fish can tolerate contaminant concentration levels up to 0.5. If the concentration exceeds this level, the fish will die at once. Will the fish survive? If yes, explain why. If no, find the time in which the fish will die.