$$2\int_0^1 \sin(k\pi x)\,dx = 2\cdot\left[\frac{-\cos(k\pi x)}{k\pi}\right]_0^1 = \frac{2}{k\pi}\Big(\underbrace{-\cos(k\pi)}_{=(-1)^k}+1\Big) =$$

$$= \frac{2}{k\pi}\left((-1)^{k+1}+1\right)$$

$$2\int_0^1 x\cdot\sin(k\pi x)\,dx = \left|\begin{matrix} u=x & v'=\sin(k\pi x) \\ u'=1 & v=-\frac{\cos(k\pi x)}{k\pi}\end{matrix}\right| =$$

$$= 2\left[-x\,\frac{\cos(k\pi x)}{k\pi}\right]_0^1 + \frac{2}{k\pi}\int_0^1 \cos(k\pi x)\,dx =$$

$$= \frac{2}{k\pi}\Big(-\underbrace{\cos(k\pi)}_{=(-1)^k}-0\Big) + \frac{2}{k\pi}\underbrace{\left[\frac{-\sin(k\pi x)}{k\pi}\right]_0^1}_{=0} =$$

$$= \frac{2}{k\pi}(-1)^{k+1}$$

$$\Rightarrow f_k(t) = (-\cos t)\cdot\frac{2}{k\pi}\left((-1)^{k+1}+1\right) + (\sin t+\cos t)\cdot\frac{2}{k\pi}(-1)^{k+1} =$$

$$= \sin t\cdot\left(\frac{2}{k\pi}(-1)^{k+1}\right) + \cos t\left(-\frac{2}{k\pi}\left((-1)^{k+1}+1\right)+\frac{2}{k\pi}(-1)^{k+1}\right) =$$

$$= \underbrace{\boxed{\frac{2}{k\pi}(-1)^{k+1}}}_{A_k}\cdot\sin t + \underbrace{\boxed{\frac{(-2)}{k\pi}}}_{B_k}\cdot\cos t =$$

$$= \frac{2}{k\pi}(-1)^{k+1}\cdot\sin t - \frac{2}{k\pi}\cdot\cos t$$

$$= A_k\cdot\sin t + B_k\cos t\,,\quad \text{kde}\quad A_k = \frac{2}{k\pi}(-1)^{k+1}$$

$$B_k = -\frac{2}{k\pi}$$